Modeling: optimal marathon performance on the basis of physiological factors

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JOYNER, MICHAEL J. *Modeling: optimal marathon performance on the basis of physiological factors.* J. Appl. Physiol. 70(2): 683-687, 1991.—This paper examines current concepts concerning "limiting" factors in human endurance performance by modeling marathon running times on the basis of various combinations of previously reported values of maximal O_2 uptake ($\dot{V}o_{2\,max}$), lactate threshold, and running economy in elite distance runners. The current concept is that $\dot{V}o_{2\,max}$ sets the upper limit for aerobic metabolism while the blood lactate threshold is related to the fraction of $\dot{V}o_{2\,max}$ that can be sustained in competitive events greater than ~3,000 m. Running economy then appears to interact with $\dot{V}o_{2\,max}$ and blood lactate threshold to determine the actual running speed at lactate threshold, which is generally a speed similar to (or slightly slower than) that sustained by individual runners in the marathon. A variety of combinations of these variables from elite runners results in estimated running times that are significantly faster than the current world record (2:06:50). The fastest time for the marathon predicted by this model is 1:57:58 in a hypothetical subject with a $\dot{V}o_{2\,max}$ of 84 $ml \cdot kg^{-1} \cdot min^{-1}$, a lactate threshold of 85% of $\dot{V}o_{2\,max}$, and exceptional running economy. This analysis suggests that substantial improvements in marathon performance are "physiologically" possible or that current concepts regarding limiting factors in endurance running need additional refinement and empirical testing.

maximal oxygen uptake; lactate threshold; running economy; human performance

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PHYSIOLOGISTS have long been interested in modeling optimal human performance in various running events on the basis of world records (23, 26). Although a variety of approaches to this problem has been used (23, 26), recent mathematical models of running performance have been improved by the recognition that maximal O_2 uptake ($\dot{V}o_{2\,max}$) cannot be sustained in competition for >5-10 min (23, 26). This approach is supported by experimental data demonstrating that submaximal variables including the blood lactate threshold and running economy (O_2 cost to run a given speed) are powerful predictors of endurance running performance (3-5, 7, 11, 14, 16, 22, 25). Along these lines, this paper attempts to extend the current models of human distance running performance by considering how $\dot{V}o_{2\,max}$, blood lactate threshold, and running economy interact as determinants of performance in the marathon.

The emerging concepts concerning the limits of marathon performance are that $\dot{V}o_{2\,max}$ sets the upper limit for aerobic metabolism and that the blood lactate threshold is related to the fraction of $\dot{V}o_{2\,max}$ that can be sustained in competitive events of 2–3 h. Running economy then appears to interact with $\dot{V}o_{2\,max}$ and blood lactate threshold to determine the actual running speed at lactate threshold, which is generally a speed similar to (or slightly slower than) that sustained by individual runners in the marathon (14).

With this information as a background, the purpose of this paper is to estimate an "optimal" human performance in the marathon on the basis of the following simple physiological model

$$\begin{aligned}&\text{marathon running speed}\\&\quad= \dot{V}o_{2\,max}\,(ml \cdot kg^{-1} \cdot min^{-1}) \times \%\dot{V}o_{2\,max}\\&\qquad \text{at LT} \times \text{RE}\,[km \cdot h^{-1} \cdot \dot{V}o_2^{-1}\,(ml \cdot kg^{-1} \cdot min^{-1})]\end{aligned}$$

where LT is lactate threshold, RE is running economy, and $\dot{V}o_2$ is O_2 uptake. This model is then used in conjunction with a range of well-accepted values for these variables in elite male distance runners to estimate the physiologically optimal marathon performance.

METHODS

Previously reported values for $\dot{V}o_{2\,max}$, lactate threshold, and running economy in highly trained and elite endurance athletes were used to establish three estimates (low, average, and high) for each of the three variables. These values were used in different combinations to estimate the running speed at lactate threshold.

Maximal O_2 Uptake

The $\dot{V}o_{2\,max}$ values from Pollock's study (24) of 19 elite runners were used. These ranged from 71.3 to 84.4 $ml \cdot kg^{-1} \cdot min^{-1}$ and averaged 76.9 $ml \cdot kg^{-1} \cdot min^{-1}$. On the basis of these data, the low, average, and high $\dot{V}o_{2\,max}$ values used for predictions of running speed at lactate threshold were 70, 77, and 84 $ml \cdot kg^{-1} \cdot min^{-1}$. It is recognized that there have been occasional reports of $\dot{V}o_{2\,max}$ values >84 $ml \cdot kg^{-1} \cdot min^{-1}$ in humans; however, this value is at or near the upper limit of values usually reported for elite runners in textbooks (1). The 70 $ml \cdot kg^{-1} \cdot min^{-1}$ value used for the lower limit was selected *1*) so that symmetrical changes in $\dot{V}o_{2\,max}$ among the low, average, and high values would occur and *2*) because there have been reports of a marathon world record holder with a $\dot{V}o_{2\,max}$ value of ~70 $ml \cdot kg^{-1} \cdot min^{-1}$ (5, 24).

TABLE 1. *Predicted $\dot{V}o_2$ at lactate threshold*

	$\dot{V}o_{2\,\mathrm{max}}$, $\mathrm{ml \cdot kg^{-1} \cdot min^{-1}}$		
$\%\dot{V}o_{2\,\mathrm{max}}$ at LT	70	77	84
75	52.5	57.8	63.0
80	56.0	61.6	67.2
85	59.5	65.5	71.4

Values are expressed in $\mathrm{ml \cdot kg^{-1} \cdot min^{-1}}$. LT, lactate threshold.

Blood Lactate Threshold

Lactate threshold data from several sources were used (4, 7, 14, 20, 22, 25). Numerous criteria, techniques, and nomenclature systems for the lactate threshold and related physiological events have been used (12). However, it appears that, regardless of definition, most runners sustain a pace in the marathon that elicits blood lactate levels between 2 and 3 mmol/dl (4, 14). This may explain why actual running speed for the marathon is generally above the onset of plasma lactate accumulation value (first increase in blood lactate above baseline) used by Farrell et al. (14) and generally below the onset of blood lactate accumulation (i.e., 4 mmol/dl) value used by Sjodin and Jacobs (25). Additionally, regardless of the nomenclature system used, it does appear that elite runners are able to run the marathon at speeds that require ≈85% of $\dot{V}o_{2\,\mathrm{max}}$ (7, 14, 23–25). On the basis of these considerations, it seemed reasonable to set the low, average, and high values for the lactate threshold at 75, 80, and 85% of $\dot{V}o_{2\,\mathrm{max}}$, respectively.

It should be noted that there are anecdotal reports of elite runners who appear to be able to sustain roughly 90% of their $\dot{V}o_{2\,\mathrm{max}}$ values during the marathon (5). Furthermore, slower runners may not be able to sustain running speeds associated with increases in blood lactate levels much above those observed at rest. This factor could act to lower the percent $\dot{V}o_{2\,\mathrm{max}}$ utilized to run the race and further prolong the time required to complete the distance by such individuals (4, 14).

The O_2 uptake ($\dot{V}o_2$, $\mathrm{ml \cdot kg^{-1} \cdot min^{-1}}$) at lactate threshold was then calculated by multiplying the percent $\dot{V}o_{2\,\mathrm{max}}$ at lactate threshold and $\dot{V}o_{2\,\mathrm{max}}$ (Table 1) (7, 14, 25). These values then served as estimates of the relative $\dot{V}o_2$ values that could be sustained for a marathon.

Running Economy

Three running economy curves relating running speed to $\dot{V}o_2$ were established. Raw data for each of the 12 subjects studied by Conley and Krahenbuhl (G. S. Krahenbuhl, personal communication; see Ref. 3, Fig. 1) were examined. Running economy values from the two most economical subjects (lowest O_2 cost for a given running speed) were averaged, and a linear regression equation between running speed and $\dot{V}o_2$ was calculated. Likewise, running economy values from the two least economical subjects (highest O_2 cost for a given running speed) were averaged, and a regression equation was calculated. Finally, an average linear regression equation between running speed and $\dot{V}o_2$ was established using the mean values of all 12 subjects (Fig. 1). This allowed estimates of running economy at faster speeds. Use of a linear model appears justified on the basis of the work of Hagen et al. (15).

Because the highest speed used in the measurement of running economy by Conley was 17.7 km/h (which is below the current world record speed of ~20 km/h for the marathon) and because reports of running economy data at higher speeds are generally anecdotal, individual examples of running economy in world class athletes were obtained to support the extrapolation of the Conley data to higher speeds. These individual values are plotted in Fig. 1 and represent previously unpublished observations for best running economy observations made in several individuals between ~19 and ~24 km/h (J. T. Daniels, personal communication). These values and those of Conley were obtained during brief periods (5–10 min) of treadmill running, so the effects of wind resistance and the upward drift in $\dot{V}o_2$ that occurs during prolonged exercise are not considered in the regression equations used to construct Fig. 1 (10, 17).

Running speed at lactate threshold for each of the three values of running economy was then estimated using the $\dot{V}o_2$ values at lactate threshold (Table 1) and three regression equations relating running speed and $\dot{V}o_2$ (see RESULTS). These estimates of running speed and time were then slowed by ~10% to account for added effects of the *1*) 7–8% reduction in running economy that would probably occur as a result of wind resistance during overground (compared with treadmill) running (10) and *2*) the 2–3% increase in $\dot{V}o_2$ that would occur from 10 min to 2 h of running (2, 6, 8, 17). This resulted in 27 estimates of running speed at lactate threshold (Table 2) ranging from a combination of the three "lowest" values to a combination of the three "highest" values for each variable. Marathon time was estimated by dividing the marathon distance (42.195 km) by the calculated values for running speed at lactate threshold and converting to

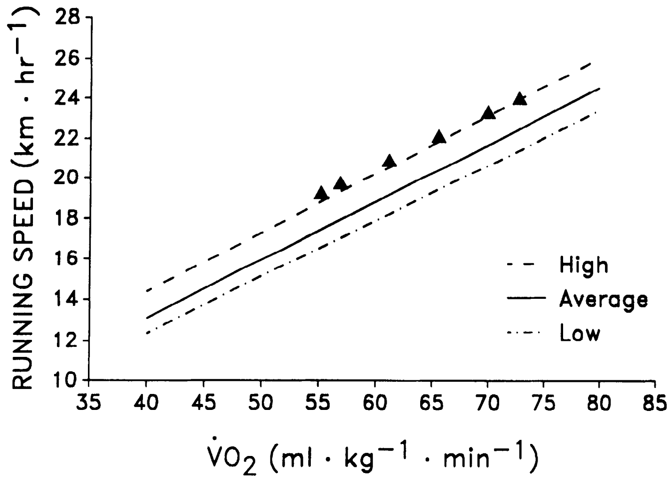

FIG. 1. High, average, and low running economy curves plotted from raw data of Conley and Krahenbuhl (Ref. 3, Fig. 1). Actual data were collected on treadmill at speeds of 14.48, 16.09, and 17.70 km/h. ▲, Unpublished observations of best individual running economy values for running speeds >17.7 km/h observed by Dr. J. T. Daniels in more than 20 years of testing elite runners. These data suggest that linear extrapolation of treadmill running economy curves to speeds >17.70 km/h is generally appropriate and that regression equation for high running economy values used in calculations is accurate for high-speed running. A discussion of how treadmill and overground running may differ is included in text.

TABLE 2. *Estimated marathon running speeds and times on the basis of $\dot{V}O_2$ lactate threshold and running economy*

$\dot{V}O_2$ at LT, ml · kg^{-1} · min^{-1}	Running Speed, km/h Low RE	Avg RE	High RE
52.5	14.40 (2:55:49)	15.18 (2:46:47)	16.42 (2:34:11)
56.0	15.28 (2:45:41)	16.10 (2:37:15)	17.35 (2:25:55)
57.8	15.74 (2:40:51)	16.56 (2:32:53)	17.84 (2:21:55)
59.5	16.16 (2:36:40)	17.00 (2:28:55)	18.29 (2:18:25)
61.6	16.70 (2:31:36)	17.56 (2:24:10)	18.85 (2:14:18)
63.0	17.05 (2:28:29)	17.93 (2:21:12)	19.23 (2:11:39)
65.5	17.68 (2:23:12)	18.58 (2:16:16)	19.89 (2:07:17)
67.2	18.11 (2:19:48)	19.03 (2:13:02)	20.35 (2:04:24)*
71.4	19.17 (2:12:45)	20.13 (2:05:46)*	21.46 (1:57:48)*

Values in parentheses (hours: minutes: seconds) represent time to complete a marathon at estimate of marathon running speed directly above. All values were obtained using data from Table 1 along with *Eqs. 1–3*. They were corrected (slowed) by ~10% to account for effects of wind resistance and upward drift in $\dot{V}O_2$ that occurs with time. For details see text. * Performances that exceeded current world record.

hours, minutes, and seconds. It is assumed for discussion purposes that the weather and race course would also be "ideal."

RESULTS

The calculations of $\dot{V}O_2$ at the lactate threshold are presented in Table 1. Values ranged from 52.5 ml · kg^{-1} · min^{-1}, when the lowest estimates of $\dot{V}O_{2\,\max}$ and lactate threshold were used, to 61.6 ml · kg^{-1} · min^{-1}, when the "average" estimates for $\dot{V}O_{2\,\max}$ and lactate threshold were used, and 71.4 ml · kg^{-1} · min^{-1}, when the highest estimates were used. Three running economy regression equations relating treadmill running speed (RS) to $\dot{V}O_2$ were generated on the basis of the data of Conley and Krahenbuhl (3) "high" running economy

$$\text{RS (km/h)} = \dot{V}O_2\ (\text{ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}) \times 0.2936 + 2.6481 \tag{1}$$

average running economy

$$\text{RS (km/h)} = \dot{V}O_2\ (\text{ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}) \times 0.2878 + 1.5867 \tag{2}$$

"low" running economy

$$\text{RS (km/h)} = \dot{V}O_2\ (\text{ml}\cdot\text{kg}^{-1}\cdot\text{min}^{-1}) \times 0.2779 + 1.2499 \tag{3}$$

Data from the treadmill running economy regression curves and predicted values above 17.70 km/h are plotted in Fig. 1. Also plotted are the previously unpublished individual data points collected at faster running speeds in elite runners. The correlation coefficient for *Eqs. 1–3* was 0.99.

Running speed at lactate threshold during actual marathon running was then estimated for each of the nine $\dot{V}O_2$ values at lactate threshold in Table 1 by using the three equations above and correcting the estimates by 10% to account for the effects of wind resistance and $\dot{V}O_2$ drift (2, 6, 10, 17). The 27 combinations of running speed at lactate threshold (3 × 3 × 3) and estimated marathon time on the basis of these values are presented in Table 2. Values ranged from 14.40 km/h (estimated marathon time = 2:55:49) for the three low values to 17.56 km/h (2:24:10) for the three average values and 21.46 km/h (1:57:58) when the high values were used.

DISCUSSION

In this paper, a set of reasonable but upper-limit assumptions was used to predict the fastest possible marathon time on the basis of currently available information about the interplay of $\dot{V}O_{2\,\max}$, lactate threshold, and running economy as "limiting" factors in endurance exercise performance (4, 5, 11, 14, 15, 20, 22–25). This approach yielded a predicted "best" marathon time nearly 9 min faster than the present world record, with 3 of the 27 estimates of marathon time being faster than the present world record (2:06:50).

On the basis of these estimates, two basic interpretations of the model presented in this paper seem reasonable: First, one could argue that substantial improvements in the marathon world record are "physiologically" possible at this time. Second, one could argue that the disparity between the present world record and the predictions in Table 2 suggests that either factors in addition to the three explored in this paper limit elite running performance or the linear extrapolation of data collected at slower running speeds in less-gifted athletes is open to question. In either case, the possible shortcomings of the proposed model must be addressed.

Potential Limits of the Model

Genetics. Little information is available concerning the genetic factors required to attain the extremely high $\dot{V}O_{2\,\max}$, lactate threshold, and running economy values needed to run the faster times in Table 2. Although a large part of championship athletic performance has been attributed to genetic endowment, there is no information about the population frequency of the characteristics that, in combination with prolonged intense training, predispose individuals for success in endurance running. If, for example, the genetic likelihood of a very high $\dot{V}O_{2\,\max}$, lactate threshold, or running economy value is 1×10^{-3}, then the probability of the same individual having all three values is 1×10^{-9}! (For discussion see Ref. 1, p. 291–292, and Ref. 5.)

Are exceptional values for more than one variable mutually exclusive? It may be that exceptional values for one variable are mutually exclusive of exceptional values for another of the factors. Two possibilities come to mind. First, the ideal marathoner may fail to achieve the extremely fast predicted times (<2 h) because of an inability to increase fat utilization enough to avoid glycogen depletion or intracellular acidosis during an effort requiring a $\dot{V}O_2$ of >70 ml · kg^{-1} · min^{-1} for several hours (18, 19). Second, it may be that high $\dot{V}O_{2\,\max}$ values are incompatible with excellent running economy or lactate threshold values (5, 22). In the study of Pollock (24), a group of elite runners successful at 1,500–10,000 m had higher ($\sim$79 $ml \cdot kg^{-1} \cdot min^{-1}$) values for $\dot{V}O_{2\,max}$ than elite marathon runners ($\sim$74 $ml \cdot kg^{-1} \cdot min^{-1}$), with the marathon runners averaging $\sim$2 $ml \cdot kg^{-1} \cdot min^{-1}$ of $\dot{V}O_2$ less than the distance runners at 19.3 km/h. In summary, on the basis of currently available information, it is not possible to determine whether exceptional values for one variable exclude exceptional values for another variable.

Running economy. Although the assumptions about $\dot{V}O_{2\,max}$ and lactate threshold are well documented in the literature, less is known about running economy. It must be emphasized that the $\dot{V}O_2$ vs. running speed regression lines in Fig. 1 are based on treadmill running data collected during 10-min trials at speeds of 14.48, 16.09, and 17.70 km/h (241, 268, 295 m/min, and 9, 10, and 11 mph, respectively) (3). Figure 1 and the equations used to calculate running speed at lactate threshold are based on the assumption that running economy continues to increase in a linear manner at speeds >17.70 km/h (15). This assumption, although supported by limited data (J. T. Daniels, unpublished observations; Refs. 21, 24), is tenuous because of the lack of systematically collected running economy data on large numbers of elite subjects at running speeds between 18.0 and 22.5 km/h (300–375 m/min, 12–14 mph).

In this context, it was assumed that two factors would distort the running economy curves during faster running for periods >10 min. First, there is a slow upward drift in $\dot{V}O_2$ during prolonged exercise (2, 17), while the running economy data used in this model were collected during 5- to 10-min exercise bouts. The experimental observations suggest that $\dot{V}O_2$ during cycling or running at 70% of maximum increases $\sim$0.1 l/min (6, 8) from 10 min to 2 h of exercise. Such an increase might slow the time required to complete the marathon by 2–3%. Second, the best available evidence suggests that the O_2 cost of sustained (5–7 min) high-speed overground running in elite runners is 7–8% higher than for treadmill running, probably because of the addition of wind resistance (10, 11). It would therefore seem likely that these two factors might operate together to slow the ideal runner on the order of 10% during overground running for 2 h. Additionally, when the estimates of performance are not corrected for the effects of $\dot{V}O_2$ drift and wind resistance, the fastest predicted time falls to an unrealistically low 1:47:13 and 9 of the 27 estimates of performance surpass the current world record. Finally, this discussion demonstrates the need for the systematic collection of running economy data at speeds >300 m/min in elite performers. The effects of wind resistance and the duration of the exercise bout on running economy at these speeds should also be considered.

Additional sites of fatigue. The traditional concept has been that metabolic processes are the key determinants of fatigue during marathon running (18, 19). Although the large training-induced increases in muscle fiber oxidative enzymes (18, 19) seen across fiber types in the trained muscle of endurance athletes suggest a high degree of fatigue resistance, there are other potentially important sites of fatigue causing failure upstream from the muscle fiber in the neuromuscular apparatus (13). Along these lines, *1)* Are the "highest threshold" motor units recruited during fast running? and *2)* If recruited, can they be trained to fire and contract repeatedly for several hours without fatigue?

Summary

A variety of approaches has been used in previous attempts to model human performance. These approaches have recently been evaluated in detail, and the newer models have been improved by the recognition that $\dot{V}O_{2\,max}$ cannot be sustained indefinitely (23, 26). However, in these models, further improvement in world record marathon running is predicated on increases in $\dot{V}O_{2\,max}$, because the values for the fraction of sustainable $\dot{V}O_{2\,max}$ and running economy presented are held fairly constant (23, 26). In the model presented in this paper, the effects of altering various submaximal variables (running economy and lactate threshold) known to affect performance have been evaluated along with changes in $\dot{V}O_{2\,max}$ (3–5, 7, 9, 12, 15, 17, 21, 23). The principal new concept advanced using this approach is that, on the basis of a set of well-documented and reasonable assumptions, an ideal marathon runner may be able to run substantially faster than the present world record. The fact that the current world record (2:06:50) is nearly 9 min slower than the predicted best time indicates that either the genetic probabilities against such a performance are immense or our level of knowledge about the determinants of human performance is inadequate. In either case, studies of how $\dot{V}O_{2\,max}$, lactate threshold, and (particularly) running economy interact as possible determinants of performance in elite athletes are needed to provide new insight into the physiological determinants and limitations of human performance.

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