

RUHR-UNIVERSITÄT BOCHUM

Fundamental Aspects of Materials Science and Engineering

Summer term 2020

Prof. Dr.-Ing. Alfred Ludwig

Chair for Materials Discovery and Interfaces

Institute for Materials

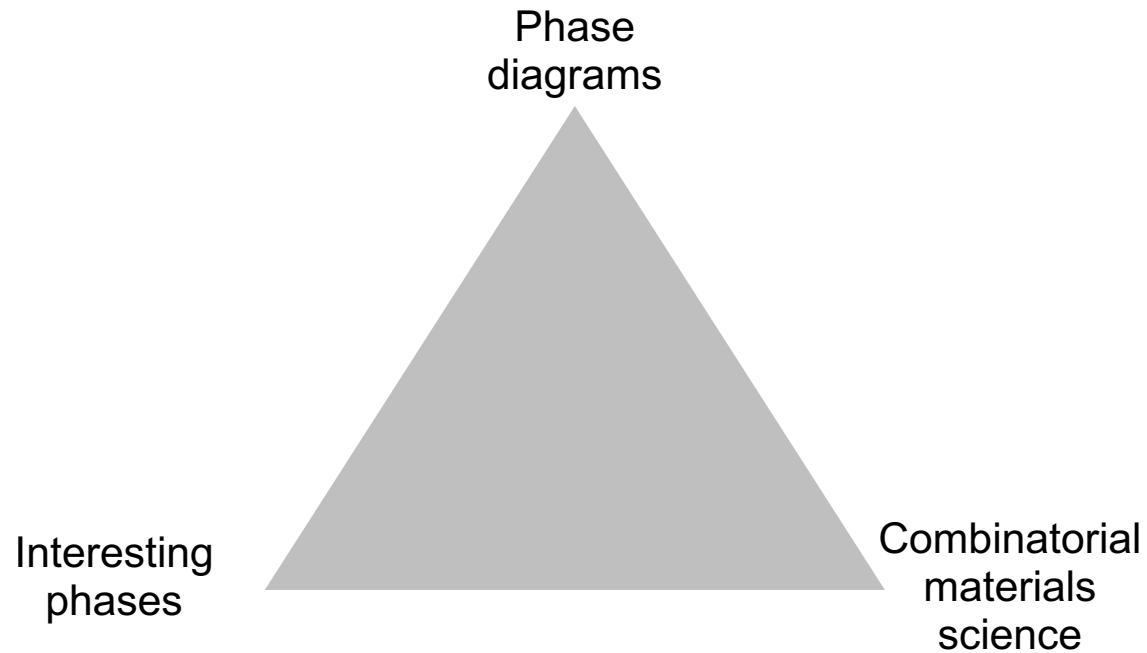
Lecture overview:

Binary and ternary phase diagrams

Intermetallic compounds

Combinatorial materials science

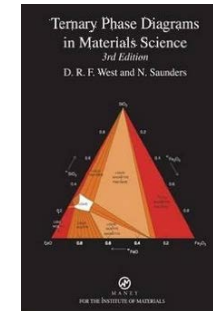
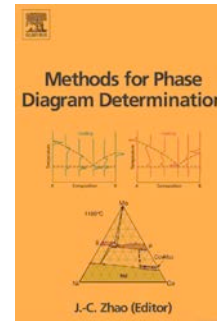
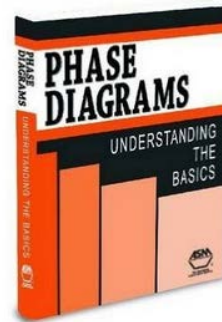
Exercises (Dr. Dennis Naujoks)



Literature

Text books

Lecture slides are based mainly on the following textbooks and databases. Slides are your personal copies and it is not allowed to distribute them further.



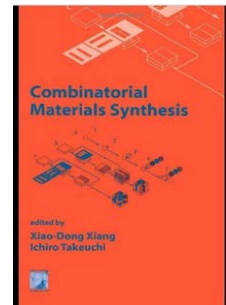
RUB

Phase diagrams

- F. C. Campbell, Phase Diagrams: Understanding the Basics, ASM International, 2012 (ISBN: 1615039864)
- J.-C. Zhao, Methods for Phase Diagram Determination, Elsevier 2011 (ISBN: 0080549969)
- D. R. F. West and N. Saunders, Ternary Phase Diagrams in Materials Science, Maney Publishing 2002 (ISBN:1902653521)

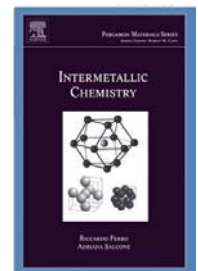
Combinatorial materials science

- X.-D. Xiang and I. Takeuchi, Combinatorial Materials Synthesis, Marcel Dekker Inc 2003 (ISBN: 0824741196)



Intermetallic compounds

- R. Ferro and A. Saccone, Intermetallic Chemistry, Pergamon 2008 (ISBN: 0080440991)
- G. Sauthoff, Intermetallics, Wiley-VCH 1995 (ISBN: 3527293205)



Chapters in many textbooks: Haasen, Gottstein, Eggeler, ...

Examples of scientific journals

Phase diagrams

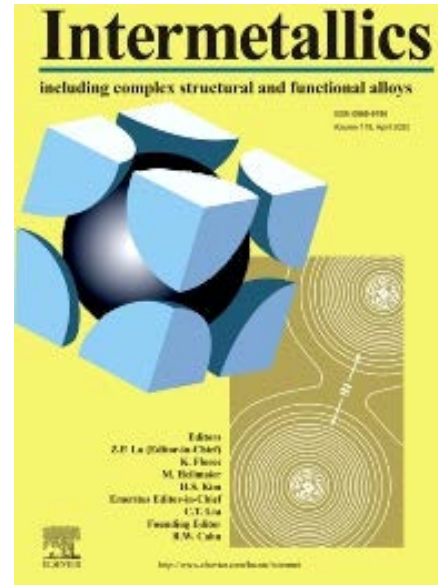
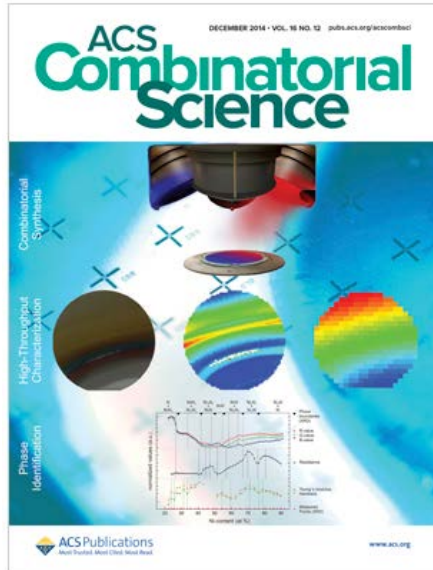
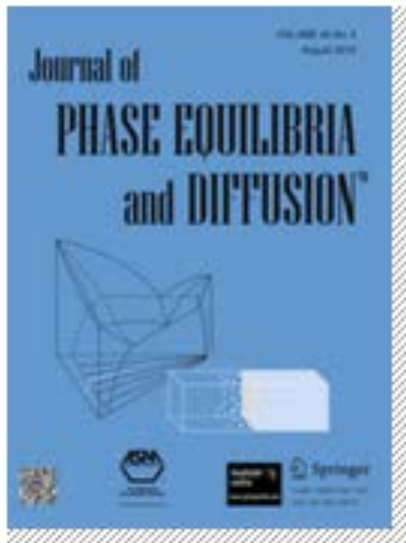
- Journal of Phase Equilibria and Diffusion, ASM International (ISSN: 1863-7345)

Combinatorial materials science

- ACS Combinatorial Science, American Chemical Society (ISSN: 2156-8952)

Intermetallic compounds

- Intermetallics, Elsevier (ISSN: 0966-9795)



Open access article on phase diagrams:
<http://link.springer.com/article/10.1007/s11669-014-0343-5/fulltext.html>

Introduction

Materials Databases: which materials exist?

- MatNavi: http://mits.nims.go.jp/index_en.html
- Pearson's Crystal Database: <http://www.crystalimpact.com/pcd/>
- Alloy Phase Diagram Database: <http://www1.asminternational.org/asmenterprise/APD/default.aspx>
- Computational Materials Repository: <https://cmr.fysik.dtu.dk/>
- The Materials Project: <https://materialsproject.org/>
- The Pauling file <https://paulingfile.com/>

MatNavi is one of the world's largest materials databases of polymer, ceramic, alloy, superconducting material, composite and diffusion.

MatNavi NIMS Materials Database

Japanese For New User National Institute for Materials Science, Materials Information Station

Home About us MITS Symposium Link Contact us NIMS

"MatNavi" is one of the world's largest materials databases provided by NIMS

Database

- Basic Properties
 - Polymer Database (PolInfo)
 - Inorganic Material Database (AtomWork)
 - Computational Phase Diagram Database (CPDD) NEW!
 - Computational Electronic Structure Database (CompES)
 - Database of Promising Adsorbents for Decontamination of Radioactive Substances (READS)
 - Neutron Transmutation Database (NuiTran)
 - Interfacial Thermal Conductance Database (ITC)
 - Diffusion Database (Kakusan)
 - Superconducting Material Database (SuperCon)
- Engineering
 - Metallic Material Database (Kinokoku)
 - CCT Diagram Database (CCTD)
- Applications
 - Composite Design & Property Prediction System (CompoTherm)
- NIMS Structural Materials Data Sheet Online
 - Creep Data Sheet (CDS)
 - Fatigue Data Sheet (FDS)
 - Corrosion Data Sheet (CoDS)
 - Space Use Materials Strength Data Sheet (SDS)
 - Metallic Material Microstructure Database (Kinso)

[Printed copy](#)

Users

New Users:
The use of "MatNavi" is free. (Free of charge)
All you need to do is register.
[Register](#) [For New User!](#)

Registered Users:
Please select a database and login from the "Enter" on each page. E-mail address and password are necessary.
[Forgot your Password?](#)
[Update registration](#)
[Close your account](#)

MatNavi Search

Keyword Search

AND OR OR Contain

Tree Search

- Material
- Element

CRYSTAL IMPACT About us Diamond Endeavour Match! Pearson's CD

About Pearson's CD
Data Information...
Software Functions...
Features...
Brochure (PDF)...
References...

Get Pearson's CD
Order Now
Demo Version
A free-of-charge demo version with a few thousand datasets can be downloaded. More...
Quickstart
A quickstart manual is also available in Spanish. More...

Support
Updates...
Known Bugs...
HowTo...
Frequently Asked Questions...
User Group...

Pearson's Crystal Data Crystal Structure Database for Inorganic Compounds

Pearson's Crystal Data is a crystallographic database published by ASM International (Materials Park, Ohio, USA), edited by Pierre Villars and Karin Cenzual. It has its roots in the well-known PAULING FILE project and contains crystal structures of a large variety of inorganic materials and compounds. The "PCD" (as it is typically abbreviated) is a collaboration between ASM International and Material Phases Data System, Vitznau, Switzerland (MPDS), aiming to create and maintain the world's largest critically evaluated "Non-organic database".

Pearson's CD News
September 14, 2015
Release 2015/15 of Pearson's Crystal Data has become available with a total entry count of about 274,000. More...
Aug 25, 2014
The new release 2014/15 of Pearson's Crystal Data contains nearly 258,500 entries. More...
Aug 27, 2013
A new release 2013/14 of Pearson's Crystal Data has just become available, with a total entry count of more than 242,600. More...
Aug 6, 2012
Release 2012/13 of Pearson's Crystal Data has just become available with a total entry count of more than 227,000. More...
Jun 11, 2012
A software update is available for release 2011/12 of Pearson's Crystal Data fixing some bugs. More...
Es steht kein Speicherplatz zur Verfügung. Klicken Sie hier, um diesen Link zu diesem Laufwerk zur Verfügung stellen können.

Phase diagrams, intermetallic phases and combinatorial materials research

Alloy Phase Diagram Database™

Home Explore Search Tools Help

APD Login

ID:
Password:
Login

Welcome to Alloy Phase Diagram Database™

The **ASM Alloy Phase Diagram Database** allows subscribers to *explore, search and view* more than **36,500 binary and ternary phase diagrams** and associated phase data for more than **6200 systems** from their Web browsers.

To get started, enter one or more elements in the boxes at the top of the screen and click **Go**, or select **Explore** to browse by elements and systems, or select **Search** to build targeted queries.

Did you know?

The **crosshair tool** helps users determine position. To use, click on a phase diagram. Next, click on the phase diagram. A magnified image of the phase diagram will appear in a separate window. Check the checkbox to enable the crosshair tool. When the crosshair tool is enabled, the center point will follow your mouse. To place the center point at the desired location and click. Click anywhere on the diagram to enable the crosshair tool.

Please read there: Resources, Intro to Phase Diagrams, Glossary, ...

[About this resource](#)
[Conventions Used](#)
[How To Cite](#)
[Other Resources](#)

<https://www.asminternational.org/>

AFLOW
Automatic - FLOW for Materials Discovery

HOME | CONSORTIUM | PUBLICATIONS | SEARCH

Welcome to the AFLOW distributed materials property repository: share with us your passion for innovation and technology.

Aflow is a globally available database of **706,808** material compounds - with over **63,612,720** calculated properties (and growing).

Try our Materials Database Search, use our online apps, consult our wiki and publications.

Enter a Compound Name, ICSD Number, **Aflowlib Unique Identifier** or advanced search string (ie. Mg & Sn & Cu).

ICSD#, AUID#, element combos...

Quick Search

Advanced search

DISCOVER OUR STORY

Interview

RECENT PUBLICATIONS

The high-throughput highway to computational materials design

High-Throughput Computational Screening of thermal conductivity, Debye temperature and Grunisen parameter using a quasi-harmonic Debye Model

Nanograined Half-Heusler Semiconductors as Advanced Thermoelectrics: An Ab Initio High-Throughput Statistical Study

<http://aflowlib.org/>

COMPUTATIONAL MATERIALS REPOSITORY
PROJECTS

Contents ... [Organometal Halide Perovskites](#) ...

Projects

Organometal Halide Perovskites
We have performed electronic structure calculations of 240 perovskites composed of Cs, CH₃NH₃, and HC(NH₂)₂ as A-cation, Sn and Pb as B-ion, and a combination of Cl, Br, and I as anions.

Porphyrin based dyes
We present a computational screening study of more than 12,000 porphyrin-based dyes obtained by modifying the porphyrin backbone (metal center and axial ligands), substituting hydrogen by fluorine, and adding different side and anchoring groups.

New Light Harvesting Materials
Electronic bandgap calculations are presented for 2400 experimentally known materials from the Materials Project database and the bandgaps, obtained with different types of functionals within density functional theory and (partial) self-consistent GW approximation, are compared for 20 randomly chosen compounds forming an unconventional set of ternary and quaternary materials.

Perovskite water-splitting
We perform computational screening of around 19,000 oxides, oxyhydrides, oxyfluorides, oxyfluorides, and oxyfluorides in the cubic perovskite structure with photoelectrochemical cell applications in mind.

<https://cmr.fysik.dtu.dk/>

Home About Apps Documentation API Login

The Materials Project

Harnessing the power of supercomputing and state of the art electronic structure methods, the Materials Project provides open web-based access to computed information on known and predicted materials as well as powerful analysis tools to inspire and design novel materials.

[Learn more](#) [Tutorials](#) [Sign in or Register](#) to start using

Electronic Structure
Click and drag to pan

Density of States

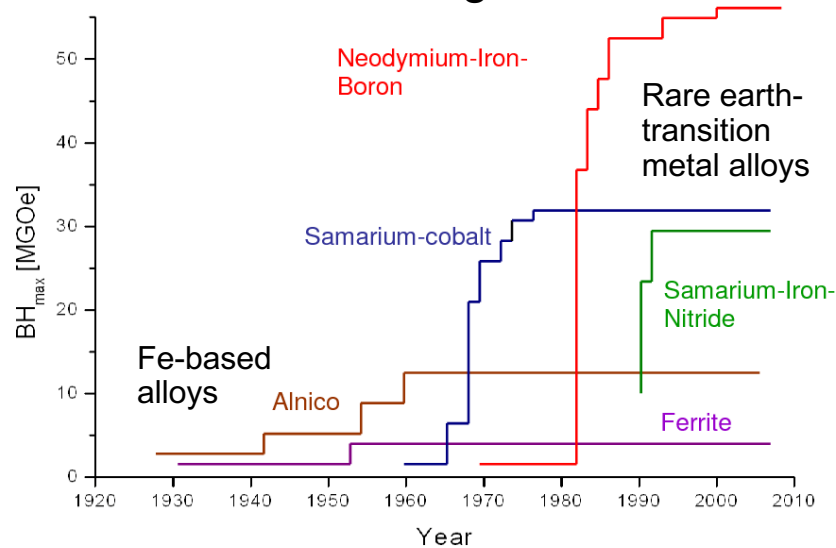
Material Details
Final Magnetic Moment
0.0000 μ_B
Formation Energy/Atom
-4.1920 eV
Energy Above Hull
0.0000 eV

<https://materialsproject.org/>

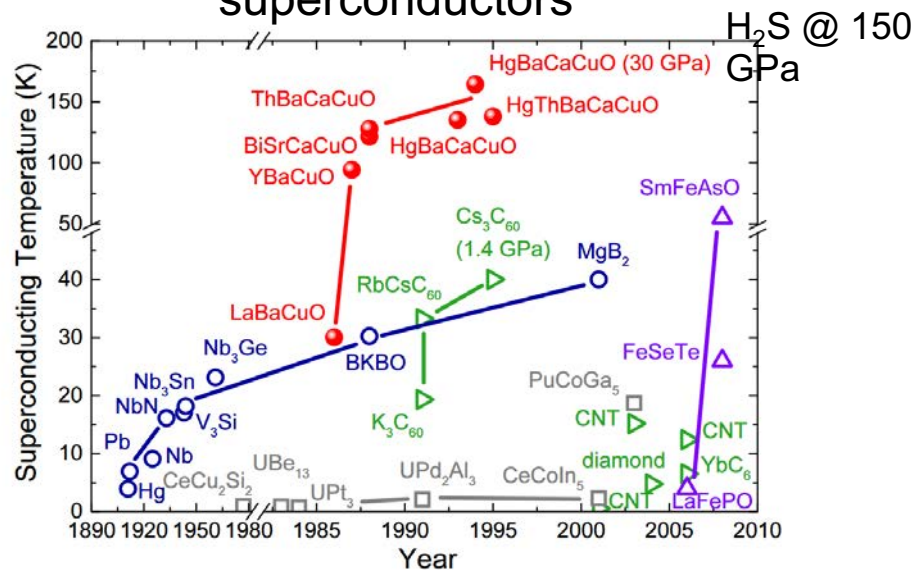
Introduction

Development of new materials frequently based on new phases

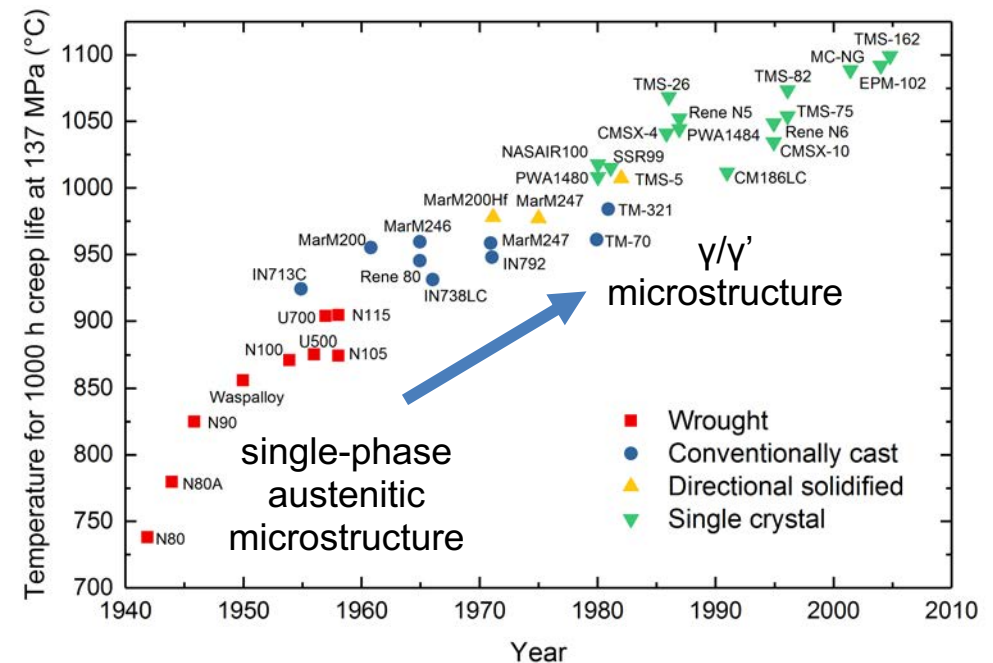
hard magnets



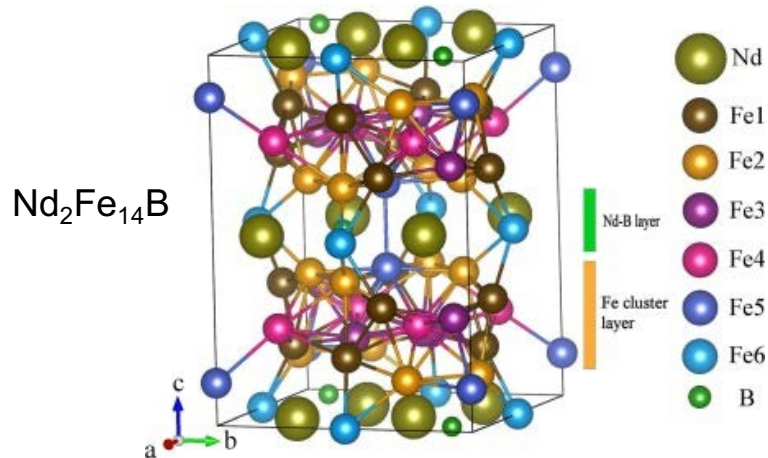
superconductors



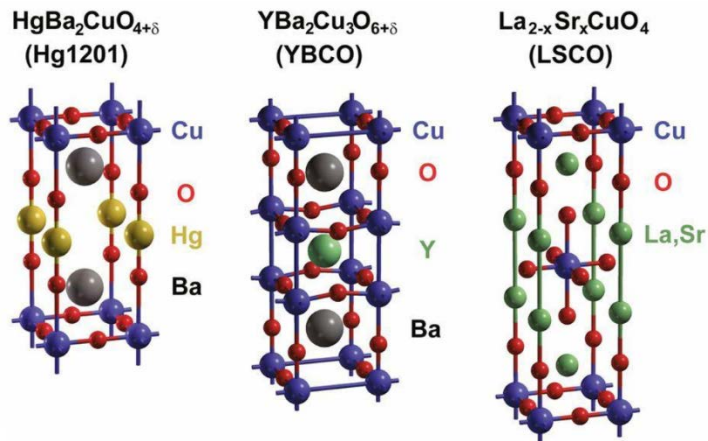
superalloys



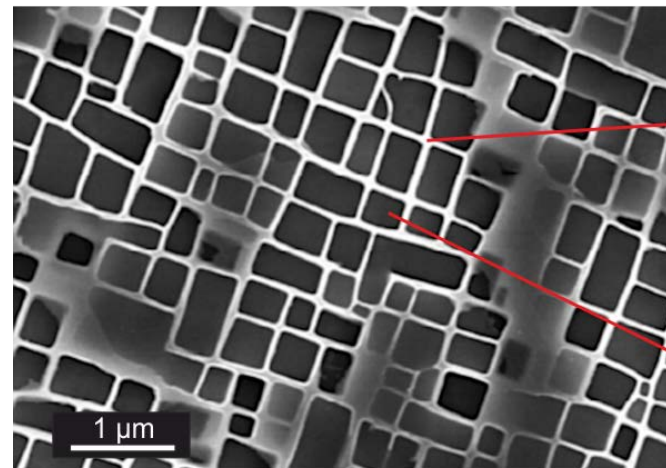
hard magnets



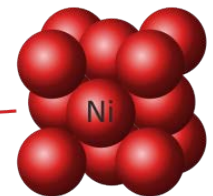
superconductors



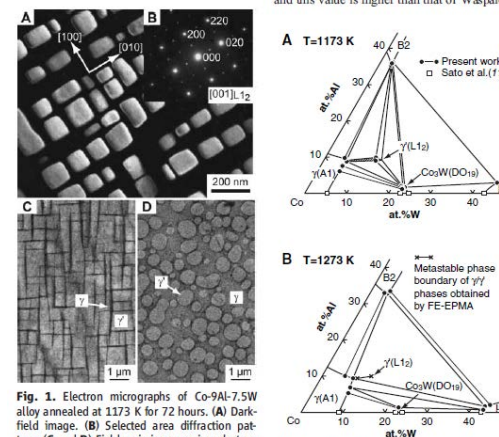
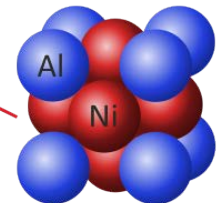
superalloys



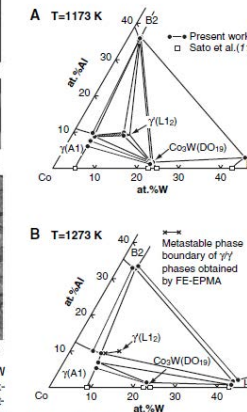
γ - Al (fcc)



γ' - L1₂



and this value is higher than that of γ/γ'.

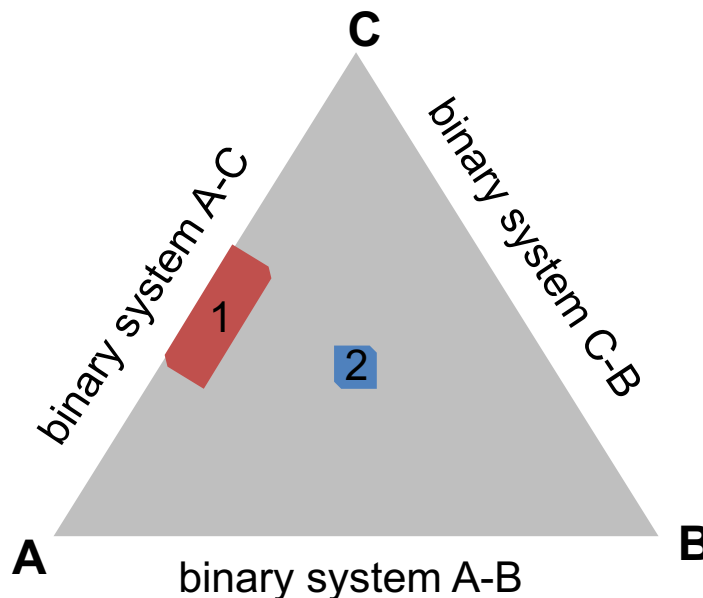


Introduction

Phase diagrams and existence diagrams

Visualizations of existence ranges of phases:

- **phase diagrams:** thermodynamic stable phases
- **existence diagrams:** metastable phases



1: binary phase with solubility of 3rd component

2: „real“ ternary phase

composition-processing-structure-maps

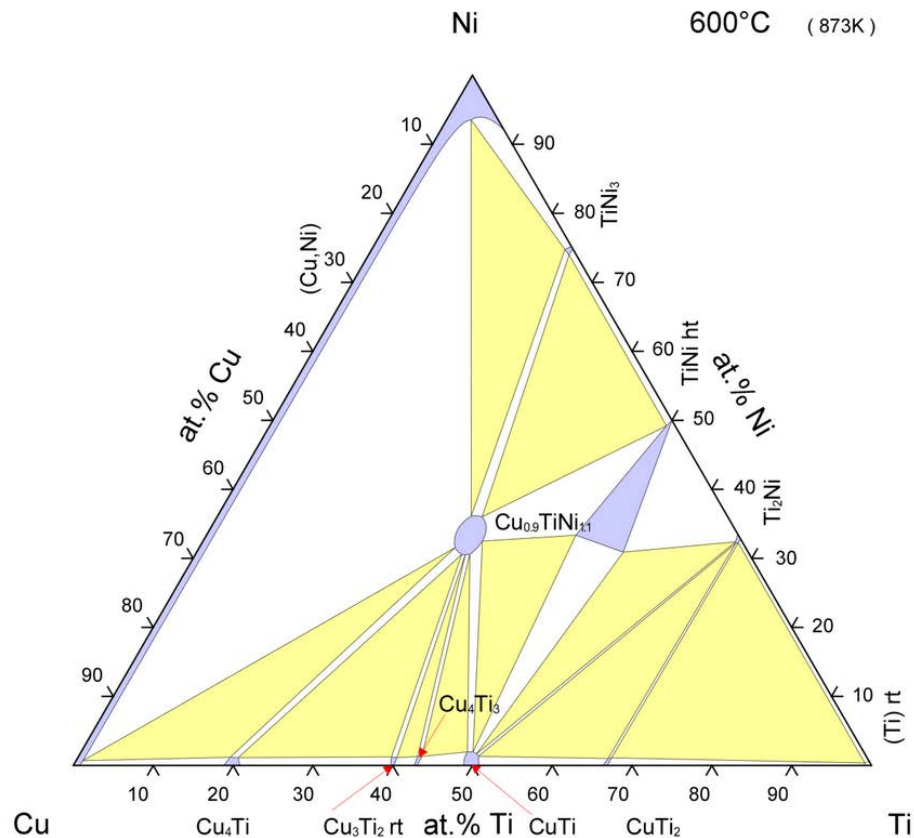
Processing: temperature, pressure

Structure:

crystallographic structure (phase)
and phase constitution
(single or multiple phases)

Introduction

Example of a phase diagram



© ASM International 2007. Diagram No. 200884

binary phase with solubility of 3rd phase

„real“ ternary phase

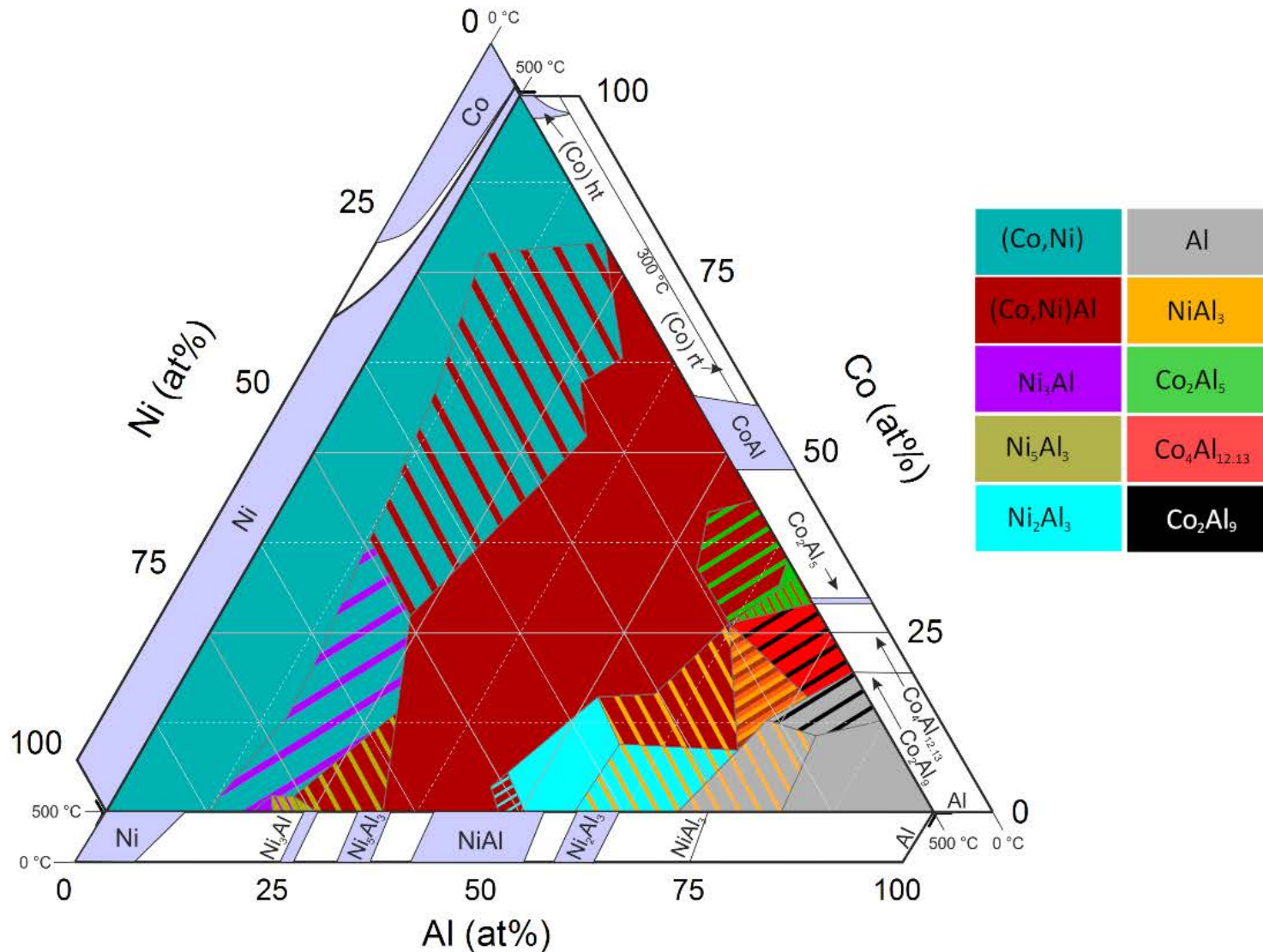
composition-processing-structure-maps

Processing: temperature, pressure

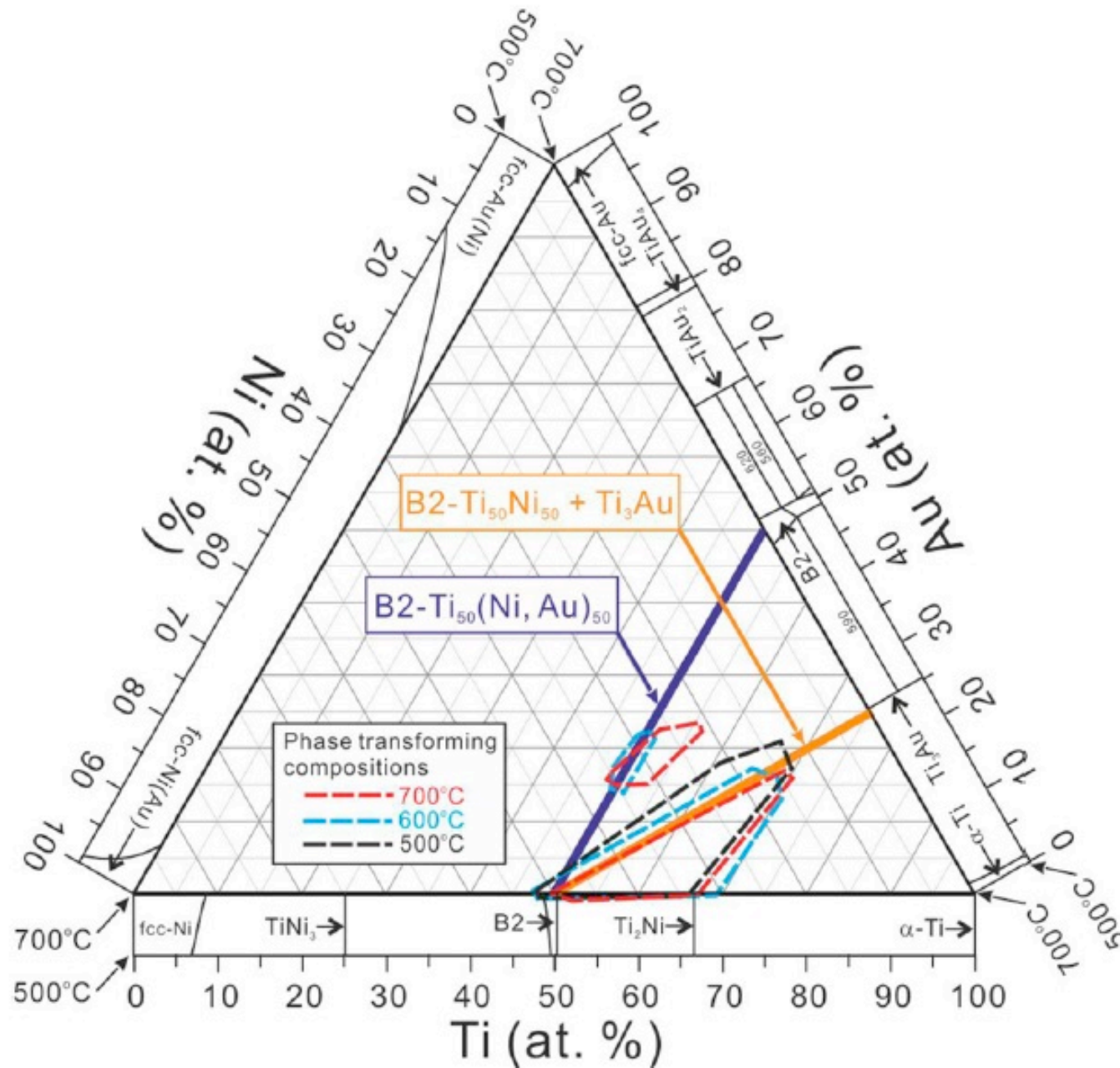
Structure:

crystallographic structure (phase)
and phase constitution
(single or multiple phases)

Assessment of phase diagrams by combinatorial materials science



Assessment of phase diagrams by combinatorial materials science



Introduction

Combinatorial Materials Science

Number of combinations of $n=50$ elements

$$(n/k) = n!/(k!(n-k)!)$$

Binaries: 1225

Ternaries:

19600

(information on 7380 systems)

Quaternaries: 230000

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba		72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra		104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Uuq	115 Uup	116 Uuh	117 Uus	118 Uuo

Check databases for
existing materials

Lanthanoide	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
Actinoide	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

+ compositional and structural diversity
(processing)

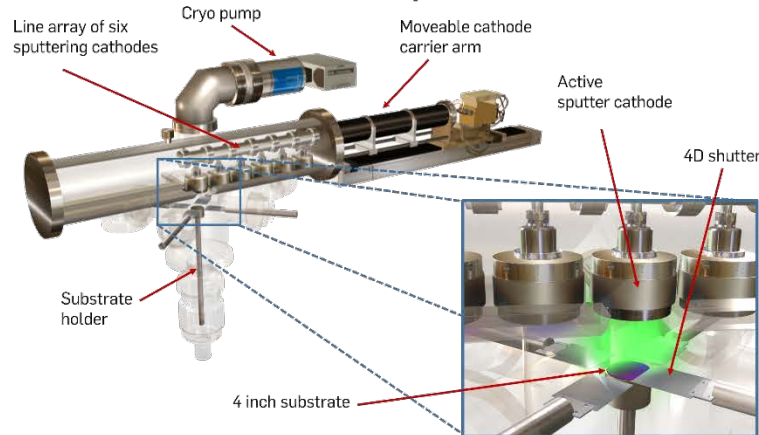
A_1B_{99} to $A_{99}B_1$

$A_{50}B_{50}(\text{bulk}) \neq A_{50}B_{50}(\text{thin film})$

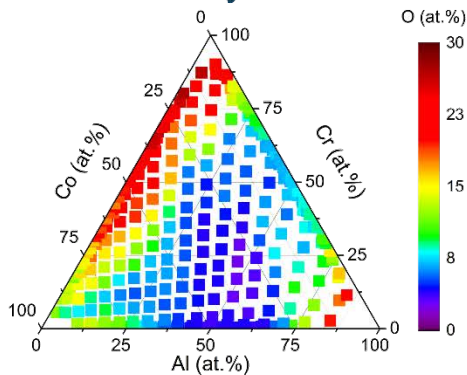
Introduction

Combinatorial materials science

Thin film deposition

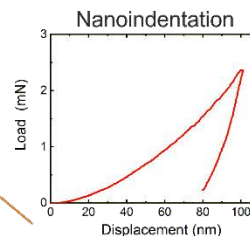
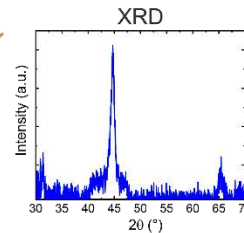
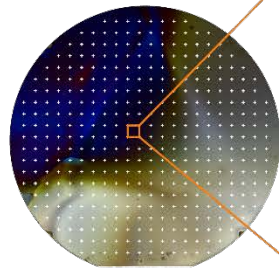


Data visualization & analysis

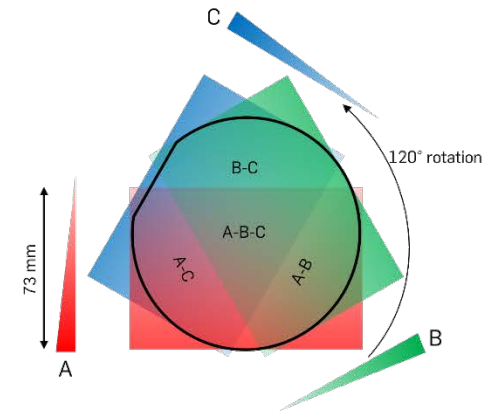


Combinatorial Material Research

High-throughput characterization



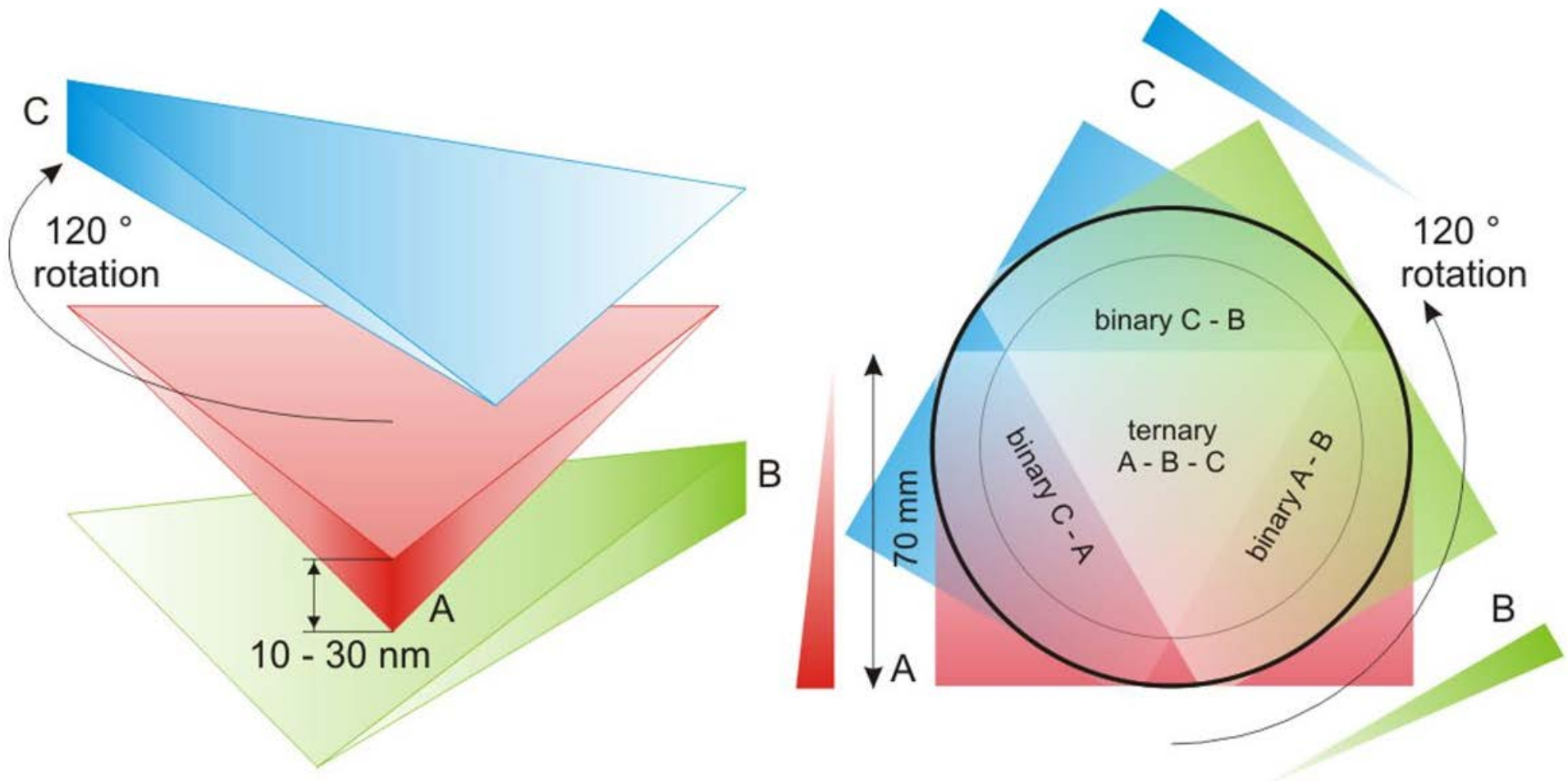
Combinatorial materials libraries



Introduction

Combinatorial materials science:

Synthesis of complete binary and ternary
thin film materials libraries by magnetron sputtering



Introduction

Combinatorial materials science

Establish

Composition-Structure-Property Correlation Maps

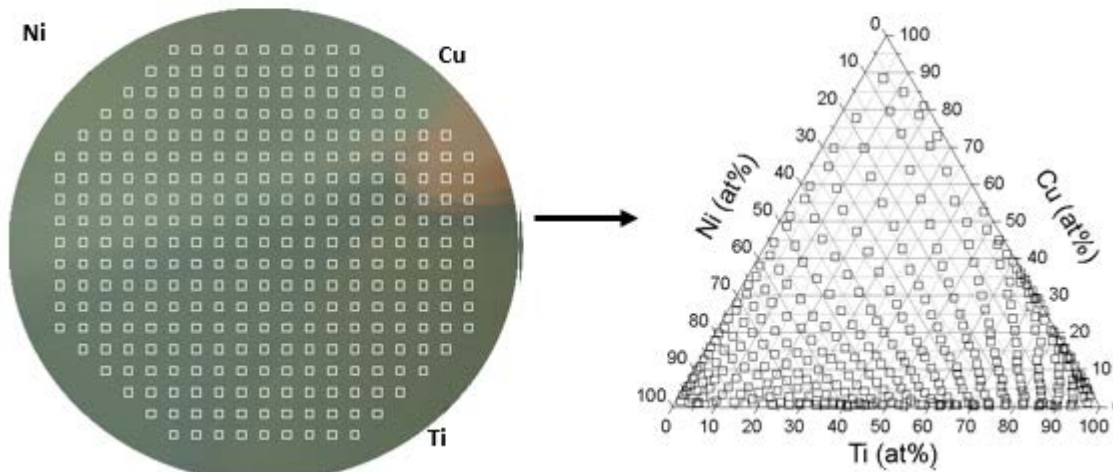
Trendlines: Property = $f(\text{composition})$

Hits: compositions with unique properties

Composition: automated Energy Dispersive X-ray analysis (EDX)

- over-night measurements (90-120 s/point – # of points 301)

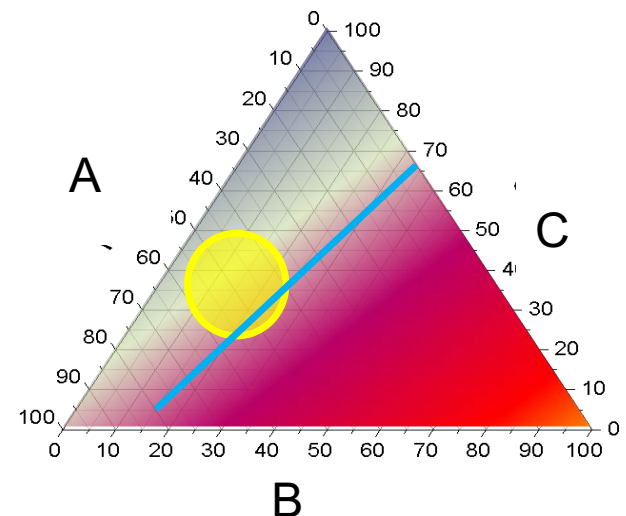
- spacing in x-, y- direction 4.5 mm



Fabricate
materials library


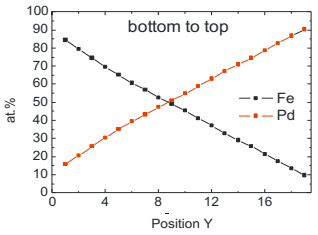
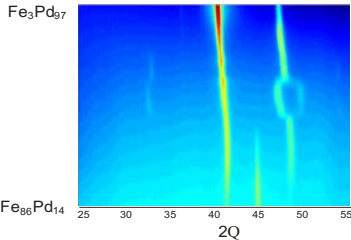

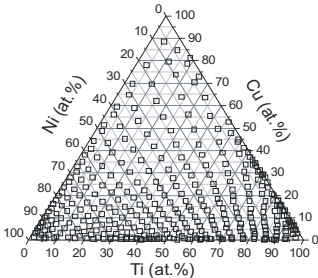
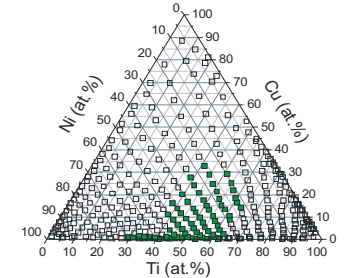
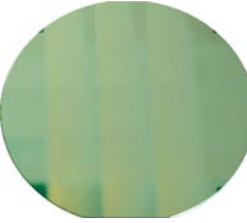
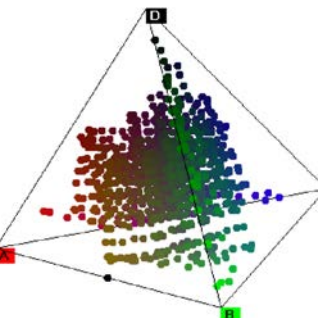
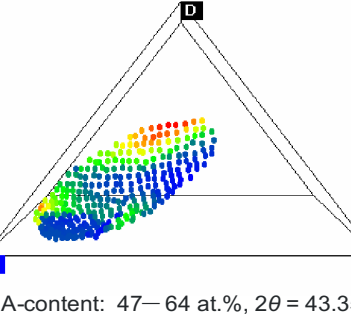
High-throughput
characterization

Property I II



Introduction

Combinatorial materials science

Type of materials library	Wafer appearance	Screening results / Visualization	
		Composition (EDX)	Crystallinity (XRD) Functional Properties
Binary composition spread			
Ternary composition spread			
Quaternary composition spread			 A-content: 47–64 at.%, $2\theta = 43.35 \pm 0.05$, color-code: intensity

Introduction

Aim of the lecture

Questions and tasks (examples):

- What are phases and phase diagrams?
- Define solid solutions, alloys and compounds.
- What are phase transformations?
- What are polymorphism and allotropy?
- How are phase diagrams determined / assessed?
- How can I calculate the fractions and compositions of phases in multi-phase regions?
- Apply the phase rule in a ternary system for different equilibria.
- Deduce isothermal and vertical sections from ternary phase diagrams.
- Know rules for formation of different classes of intermediate phases.
- Understand crystal structure and properties of important intermetallics.
- Why are many intermetallics brittle?
- Explain an example of the fabrication of a materials library.
- How is compositional and structural data acquired on materials libraries?

Aim:

You should be able to answer important scientific and technical questions regarding these topics.

Phase diagrams

Phase diagrams

Definition of a phase

A phase is:

- a homogeneous part of a system, consisting of components
- a physically homogenous state of matter with a given chemical composition and structure
- different phases have different arrangement of atoms
- phases have phase boundaries

Examples of solid phases:

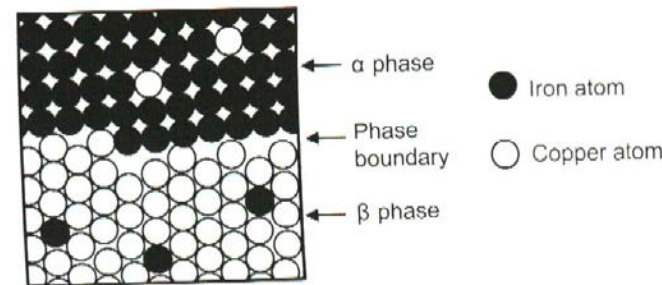
- solid solution
- line compound
- intermetallic phase
- alloy

States of matter:

solid, liquid, gaseous

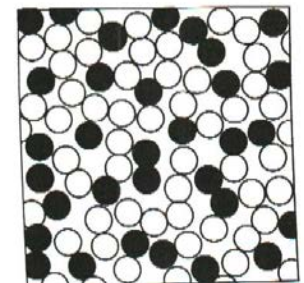
State variables:

composition, temperature, pressure, ...

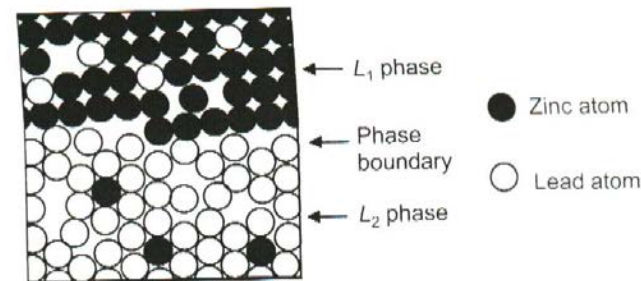


(a) solid

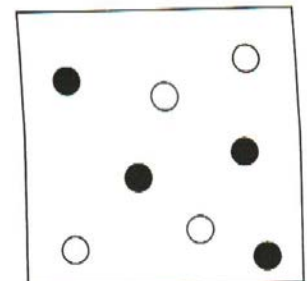
liquid



(b)



(c)



(d) gas

Phase diagrams


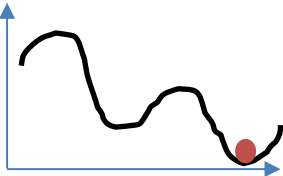
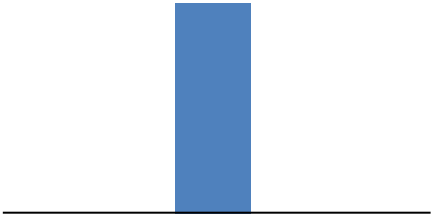
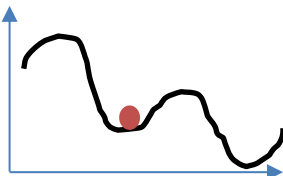

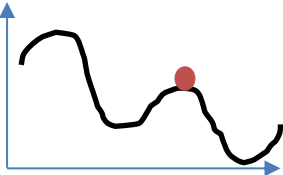
Phase diagrams are:

- important and useful as „tools“ in materials science, physical chemistry, geosciences, ...
- graphical representations of possible phases and phase changes
- showing information on existing phases and their existence ranges
- visualizing material behavior and phase constitution, often: during heating/cooling, under equilibrium conditions
- showing occurring material reactions
- displaying thermodynamic data
- give guidance on microstructures forming at different conditions (e.g. on cooling from melt, during heat treatment)
- shows constitution information: type of phases, composition of phases, fraction of phases
- roadmaps to understand conditions for phase formation or transformation caused by a change of state variables
- starting points for materials design and process optimization
- ...

also called: *constitutional diagram*, *equilibrium diagram*,
in case of non-equilibrium conditions also: *existence diagram*

Phase equilibrium

Three types of equilibria

	mechanical interpretation	energy „landscape“
• stable		 <p>exists when an object is in its <u>lowest energy condition</u></p>
• metastable		 <p>exists when additional energy must be introduced before the object can reach true stability</p>
• unstable		 <p>unstable equilibrium exists when no additional energy is needed before reaching metastability or stability</p>

Although true stable equilibrium conditions seldom exist, the study of equilibrium systems is valuable, because it constitutes a limiting condition from which actual conditions can be estimated. Phase diagrams can help elucidating the path from metastable states to stable states.

Phase equilibrium

Phase rule

$$F + P = C + 2$$

For $p=\text{const.}$: $F+P=C+1$

F: degrees of freedom (e.g. temperature T , composition c , pressure p)

C: number of components (unary = 1, binary = 2, ternary = 3, ...)

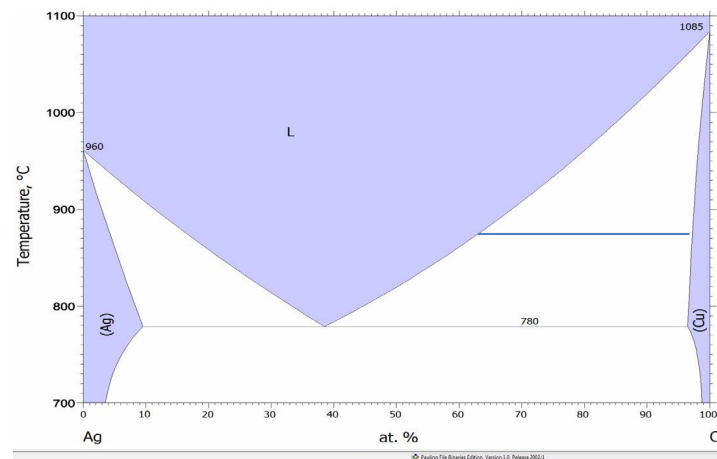
P: number of phases

Invariant equilibrium: 0 degrees of freedom

Univariant equilibrium: 1 degrees of freedom

Bivariant equilibrium: 2 degrees of freedom

Example of binary
eutectic system
($C=2$)
at constant
pressure



PAULING FILE, Binaries Edition, ASM International, 2002

$F+P=3$

Single phase field: $F=2$

Two-phase field: $F=1$

Three phases in equilibrium: $F=0$

Gibbs phase rule can be used to analyze nonequilibrium conditions. E.g., a microstructure for a binary alloy that developed over a range of temperatures and consists of three phases is a nonequilibrium one; under these circumstances, three phases exist only at a single temperature.

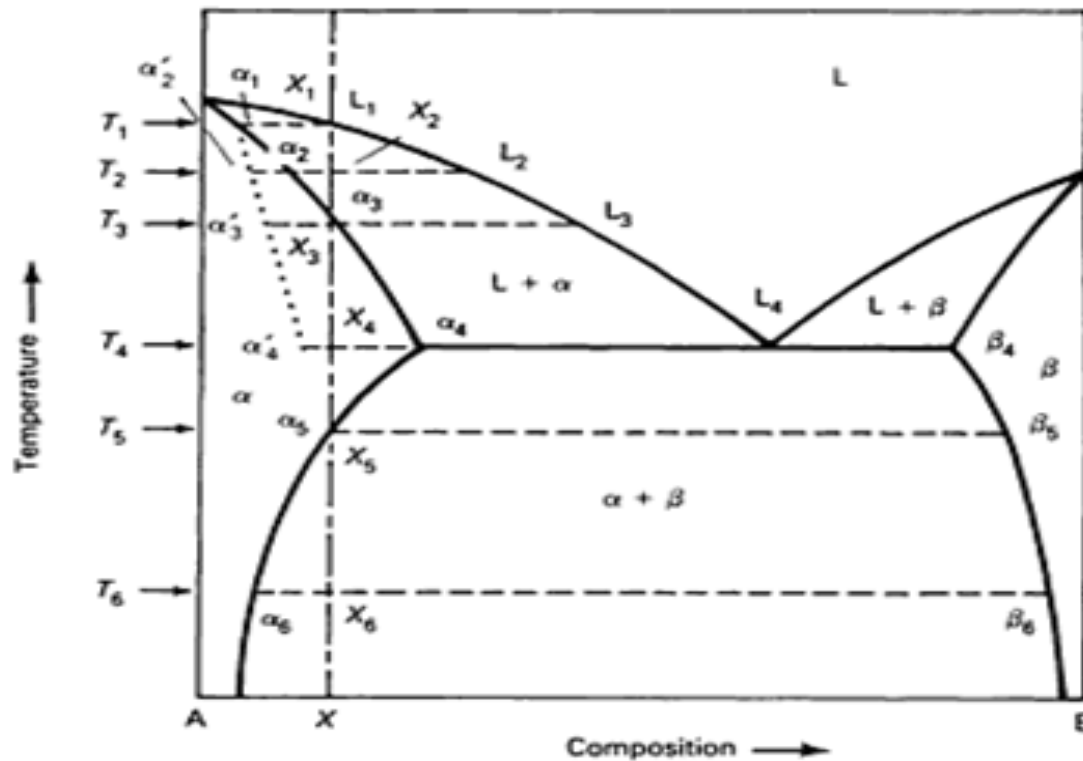
Phase equilibrium

Effects of heating/cooling rate

Binary phase diagram:

Illustration of the effect of cooling rate on an alloy lying outside the equilibrium eutectic-transformation line.

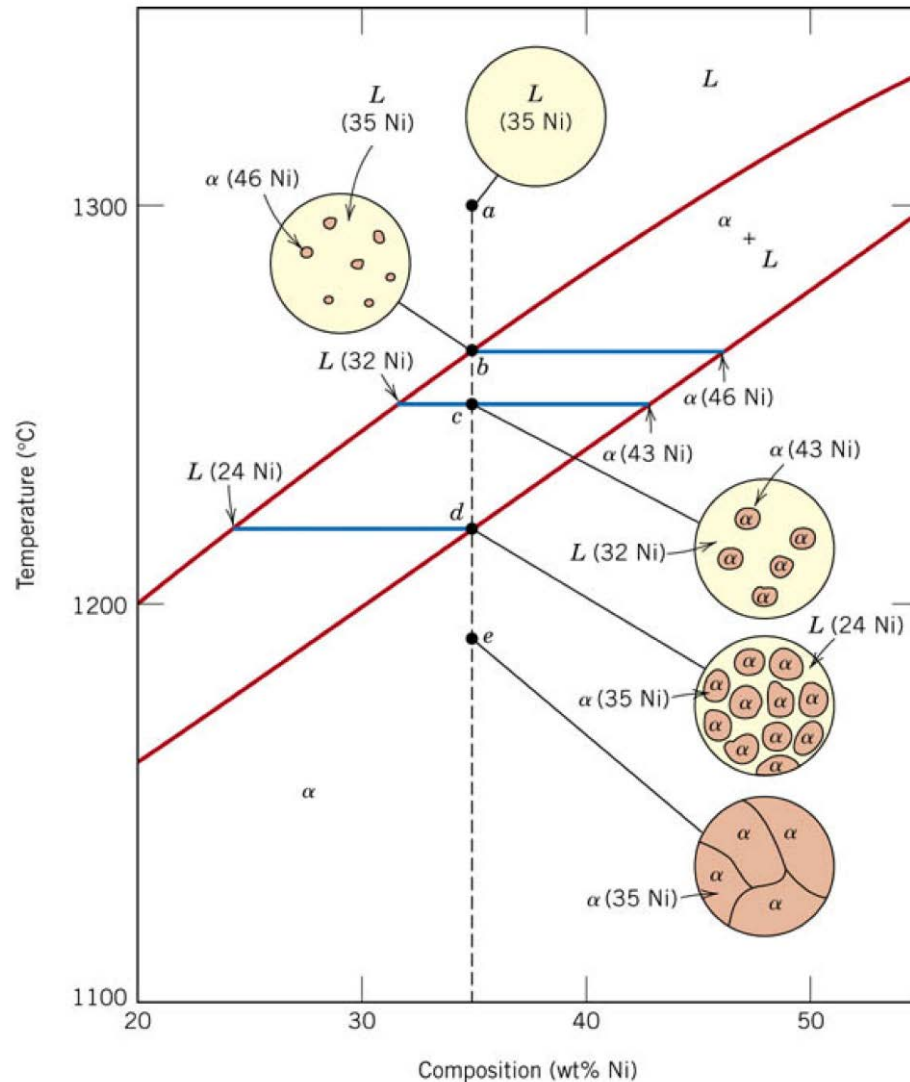
Rapid solidification into a terminal phase field can result in some eutectic structure being formed;
homogenization at temperatures in the single-phase field will eliminate the eutectic structure;
 β phase will precipitate out of solution upon slow cooling into the $\alpha + \beta$ field.



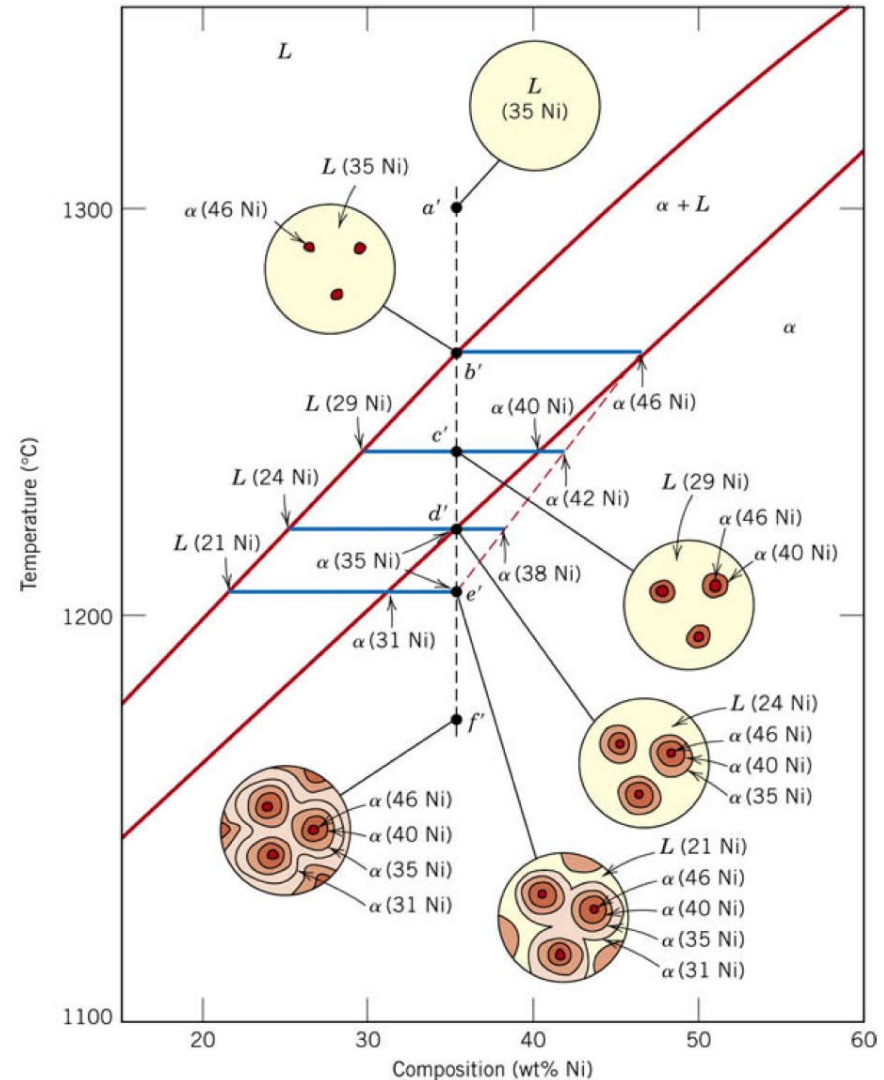
Phase equilibrium

Effects of heating/cooling rate

Equilibrium solidification



Non-equilibrium solidification

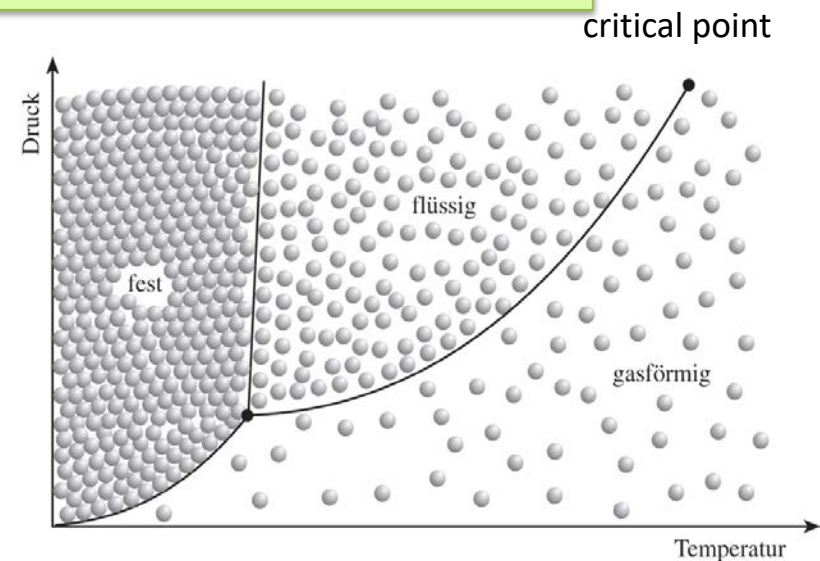
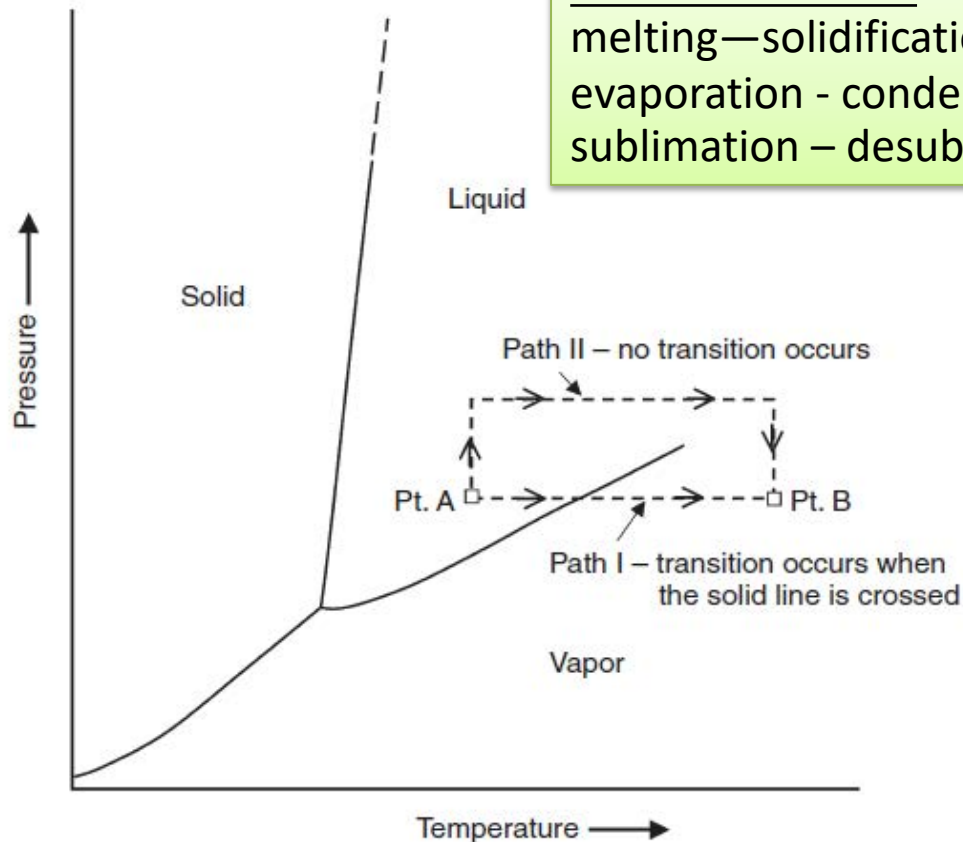


Coring due to non-equilibrium solidification

Unary phase diagrams (schematic)

Which phases and phase transitions can occur in a unary system?

Phase transitions:
melting—solidification
evaporation - condensation
sublimation – desublimation (deposition)

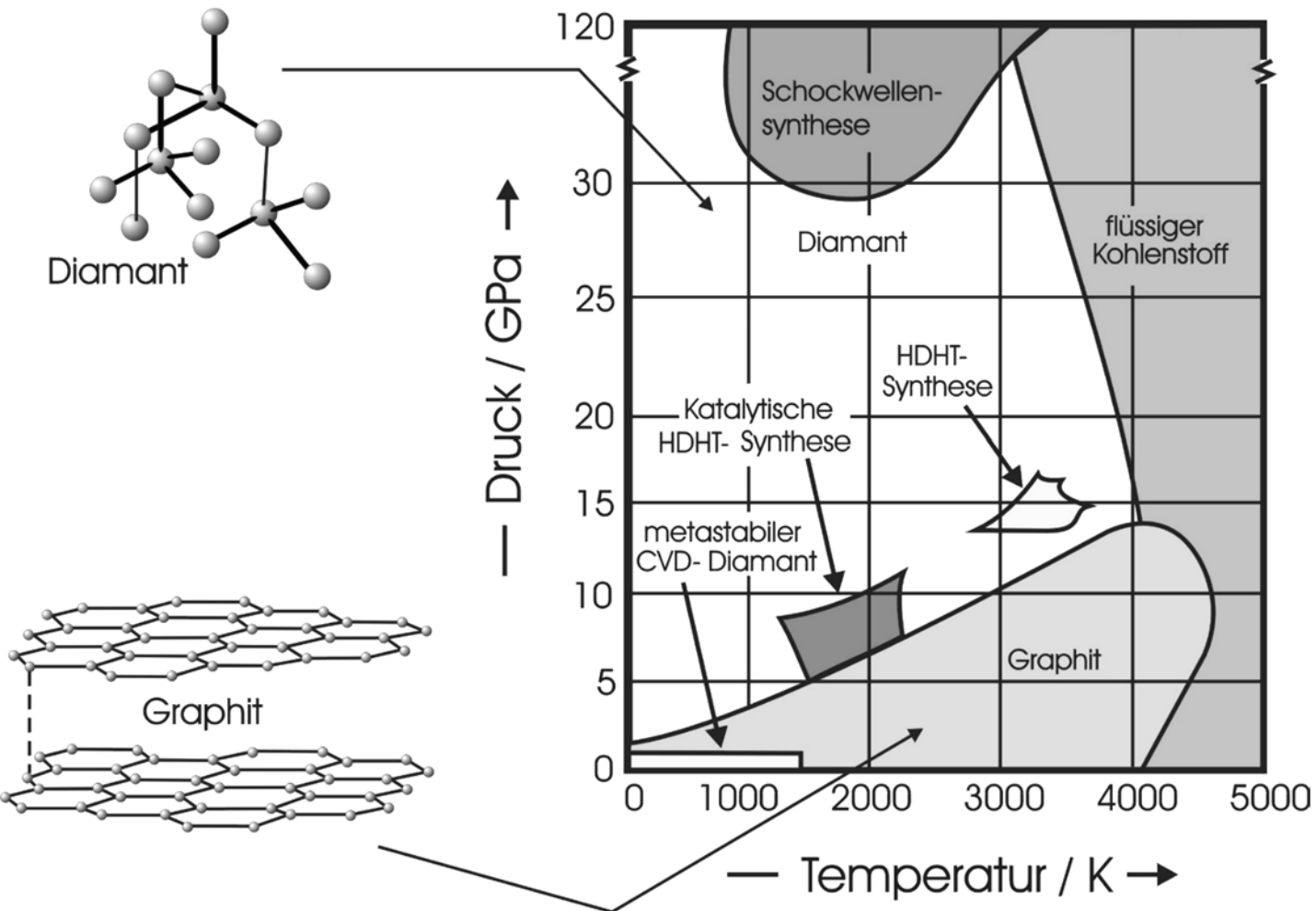


On phase boundaries (in the phase diagram), phases coexist

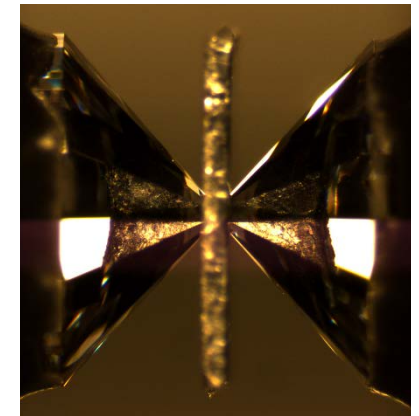
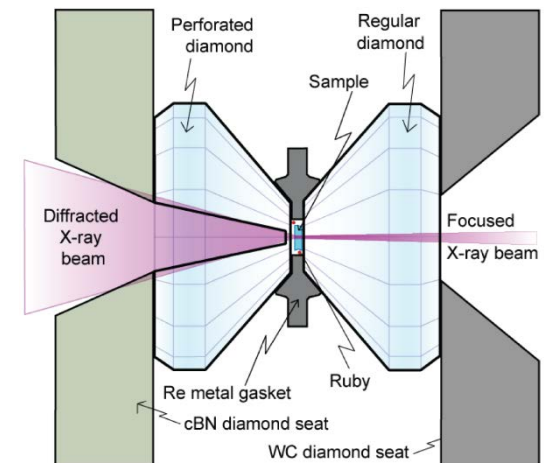
Phase diagram of molecules (H₂O)

Unary phase diagrams and allotropy of elements

Examples: Carbon (different allotropes)

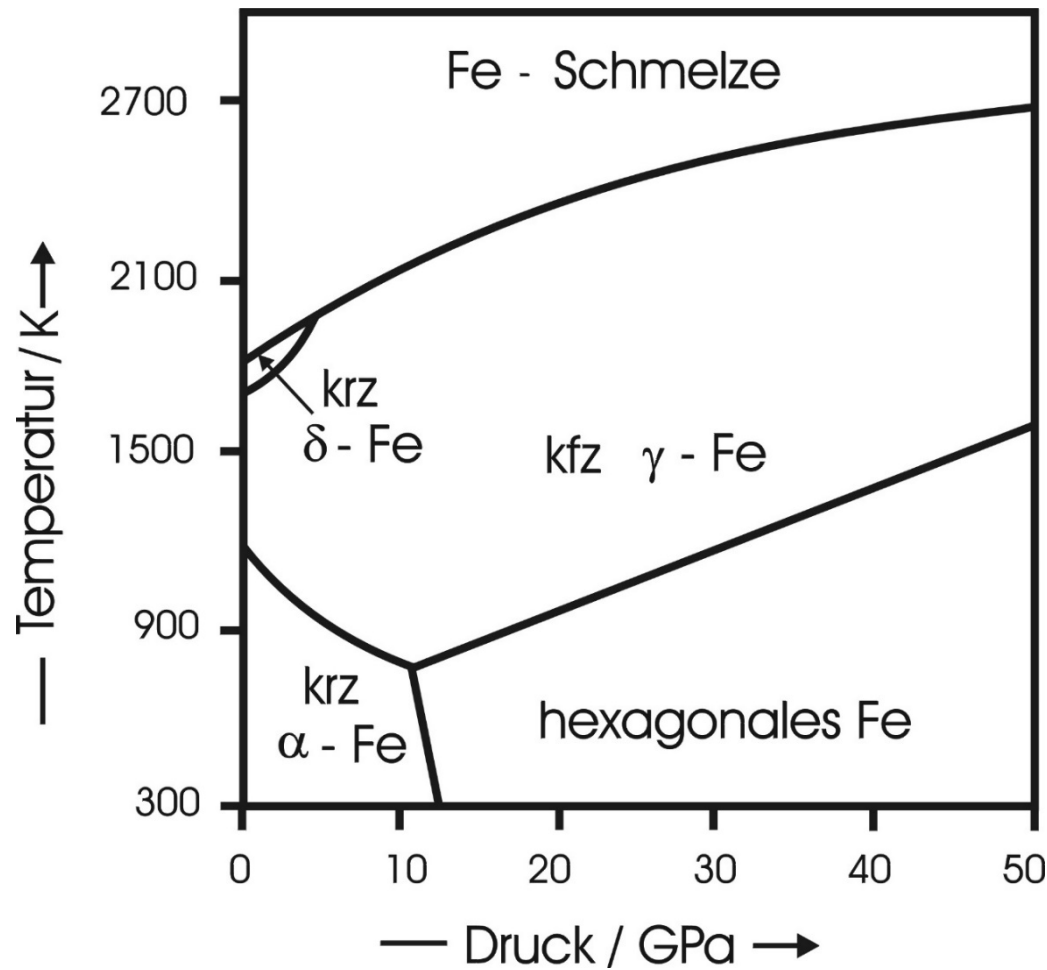


DAC:
diamond anvil cell



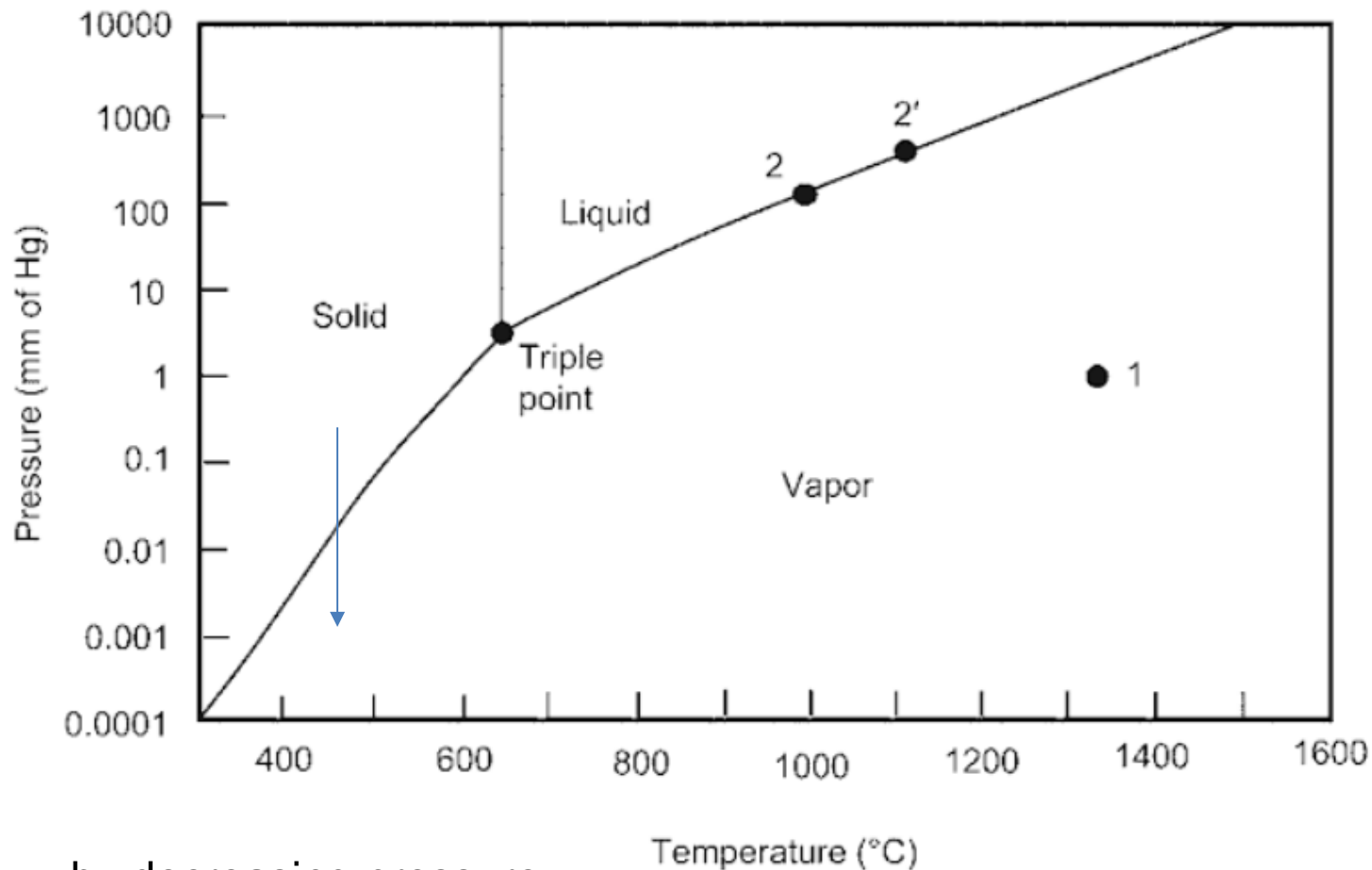
Unary phase diagrams and allotropy of elements

Examples: Fe



Unary phase diagrams

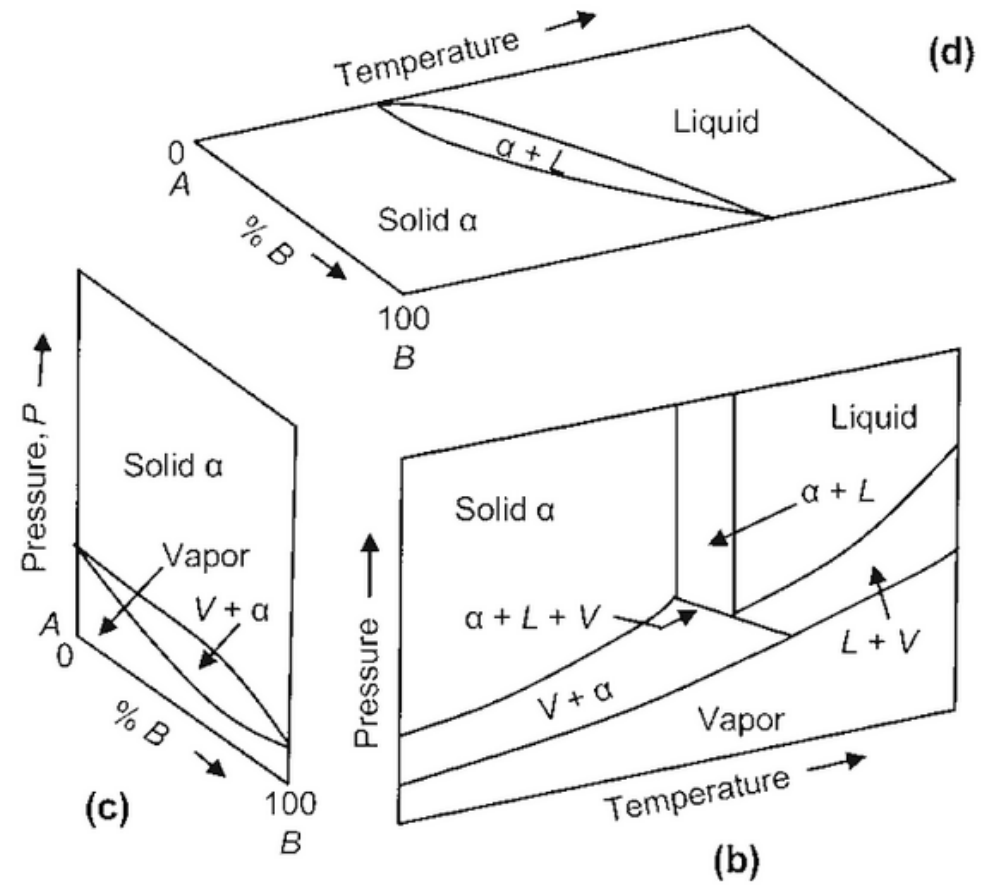
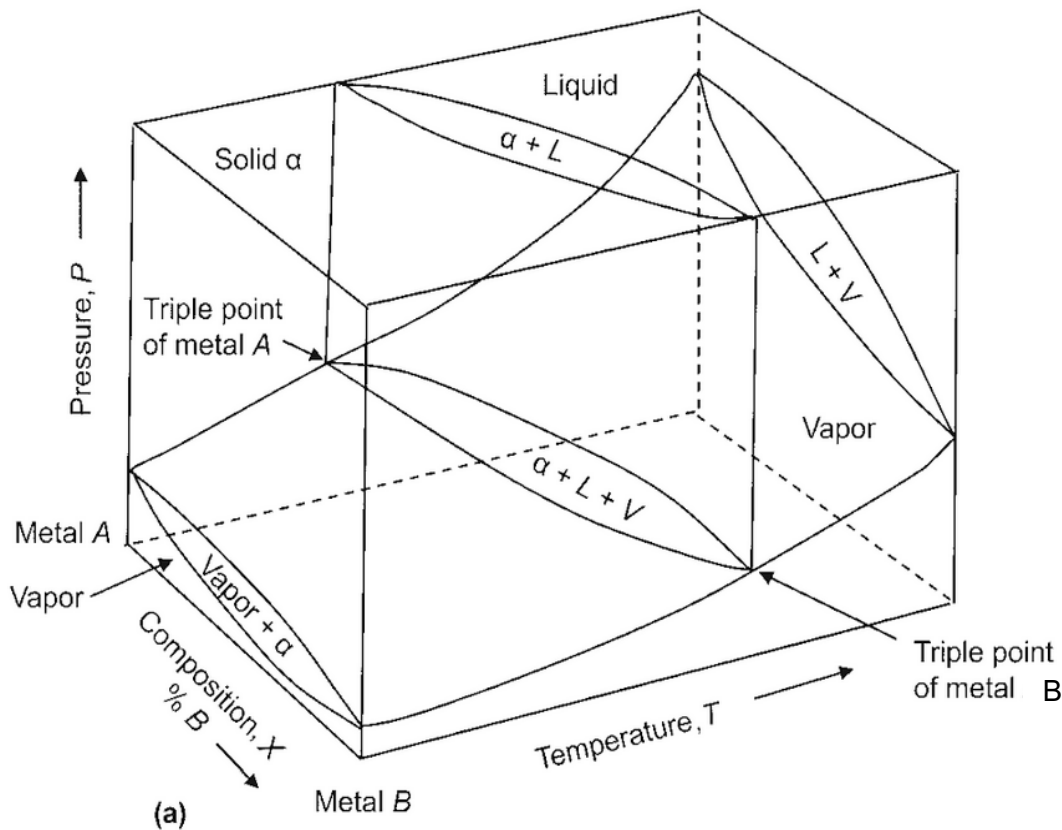
Examples: Mg (effect of low pressure)



by decreasing pressure,
a solid can sublime

General example of a phase diagram (P-T-X): binary system with complete miscibility

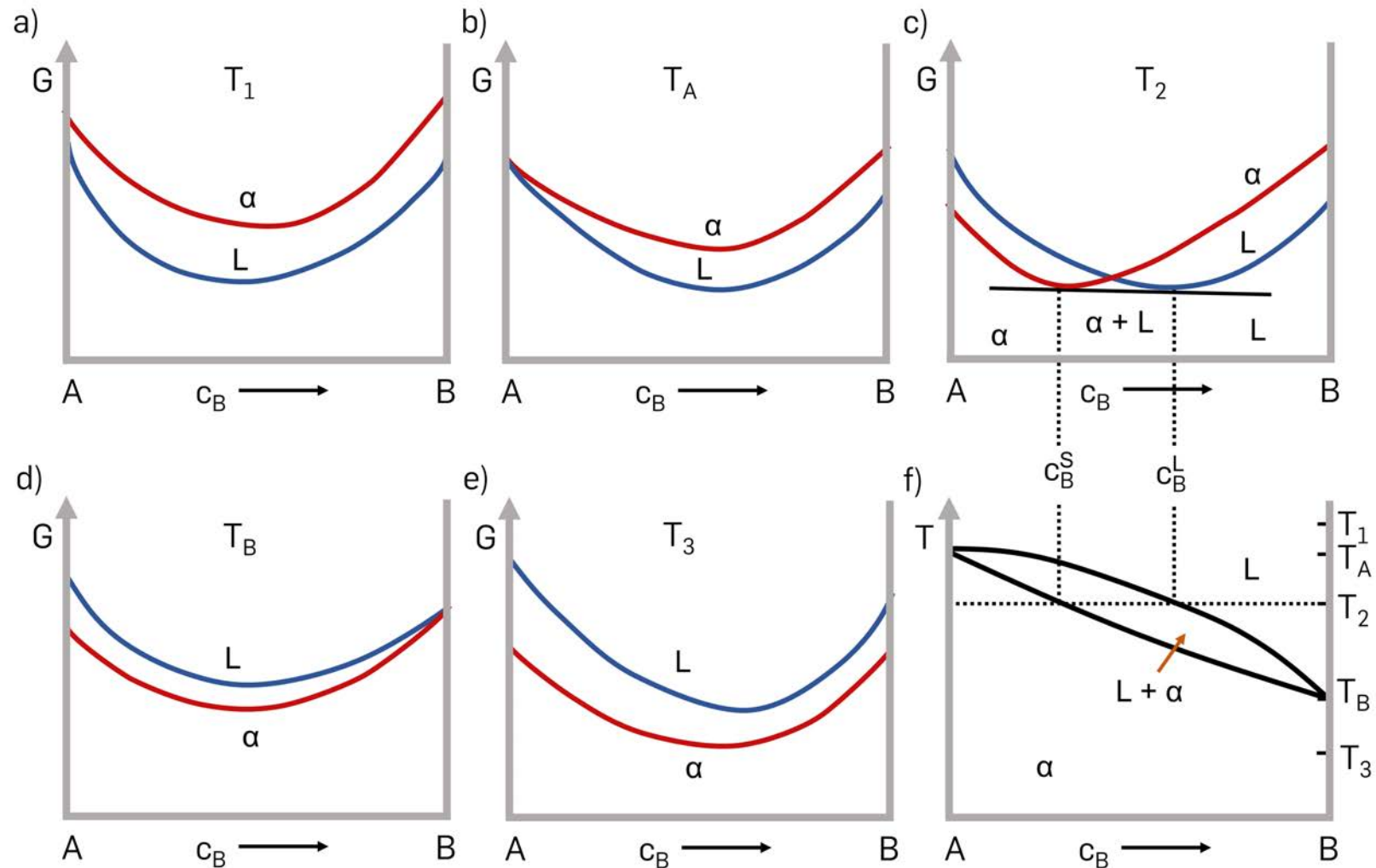
- materials science (especially metallurgy): isobaric phase diagrams (d) most common



Phase diagram determination

Construction based on thermodynamics

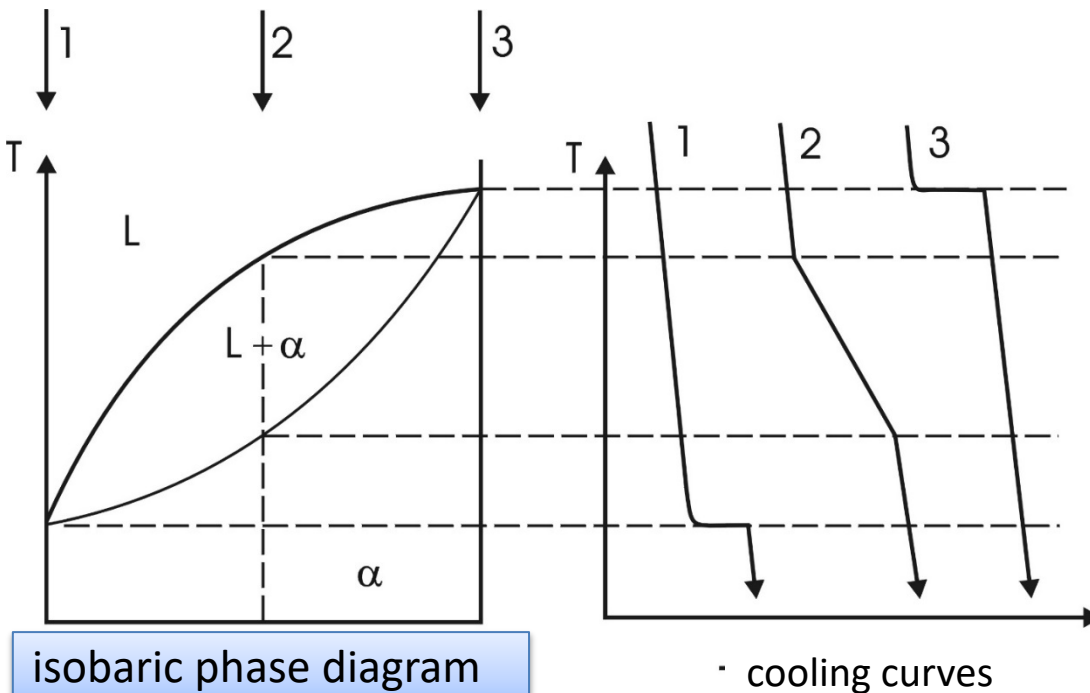
Use **Gibbs energy (G)** curves to construct a binary phase diagram that shows miscibility in both liquid and solid



Phase diagram determination

Complete miscibility (solid solutions)

a system bordered by two components (A, B)



$$F + P = C + 1,$$

p = e.g. atmospheric pressure

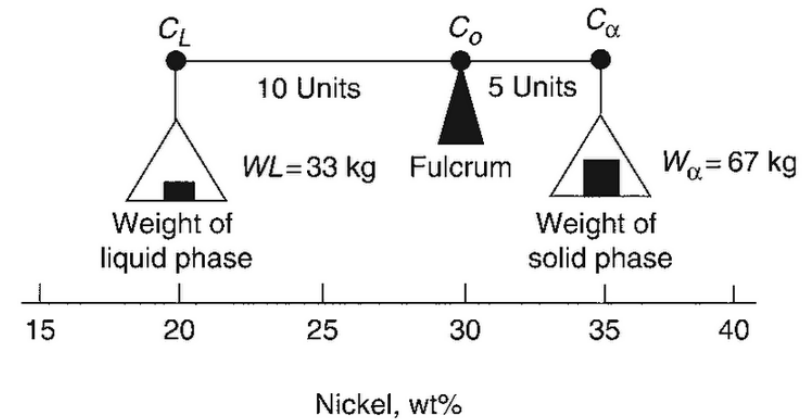
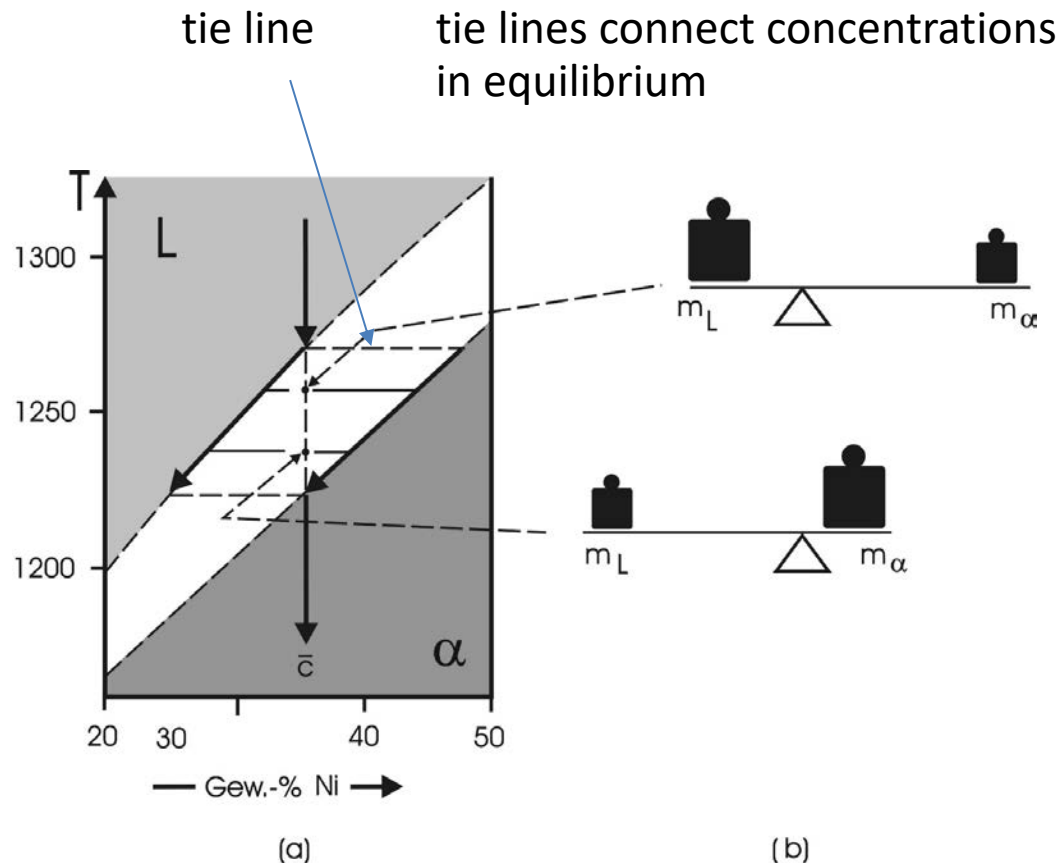
Examples of binaries with complete miscibility

Au-Ag	Co-Re	α -Fe-V	Ni-Pd
Ag-Pd	Co-Rh	γ -Fe-Co	Ni-Pt
As-Sb	Co-Ru	α -Fe-Ni	Pd-Rh
Au-Cu	Cr- α -Fe	α -Fe-Pd	Pd-Pt
Au-Ni	Cr-Mo	γ -Fe-Pt	Pt-Rh
Au-Pd	Cr-Ti	Hf-Zr	Se-Te
Au-Pt	Cr-W	Ir-Pt	Si-Ge
Bi-Sb	Cs-K	K-Rb	Ta- β -Ti
Ca-Sr	Cs-Rb	Mn-Ni	Ta-W
Co-Ir	Cu-Mn	Mo-Ta	Ti-Mo
Co-Ni	Cu-Ni	Mo-W	Ti-Nb
Co-Os	Cu-Pd	Nb-Ta	Ti-V
Co-Pd	Cu-Pt	Nb-Mo	Ti-Zr
Co-Pt	Cu-Rh	Nb-W	

Formation of solid solutions
(at least at higher temperatures)

Binary phase diagrams

Lever rule



$$\text{wt\% } L = \frac{C_{\alpha} - C_o}{C_{\alpha} - C_L} = \frac{35 - 30}{35 - 20} \approx 0.33, \text{ or } 33\%$$

$$\text{wt\% } S = \frac{C_o - C_L}{C_{\alpha} - C_L} = \frac{30 - 20}{35 - 20} \approx 0.67, \text{ or } 67\%$$

$$m_L \cdot (\bar{c} - c_L) = m_{\alpha} \cdot (c_{\alpha} - \bar{c})$$

Phase diagram determination

Vegard's law

- Vegard's law allows determination of lattice constants in a solid solution in dependence of composition

$$a_{AB} = x_A \cdot a_A + (1 - x_A) \cdot a_B$$

- a_A, a_B :
lattice constants
of elements A and B
- x_A : concentration of A
in solid solution

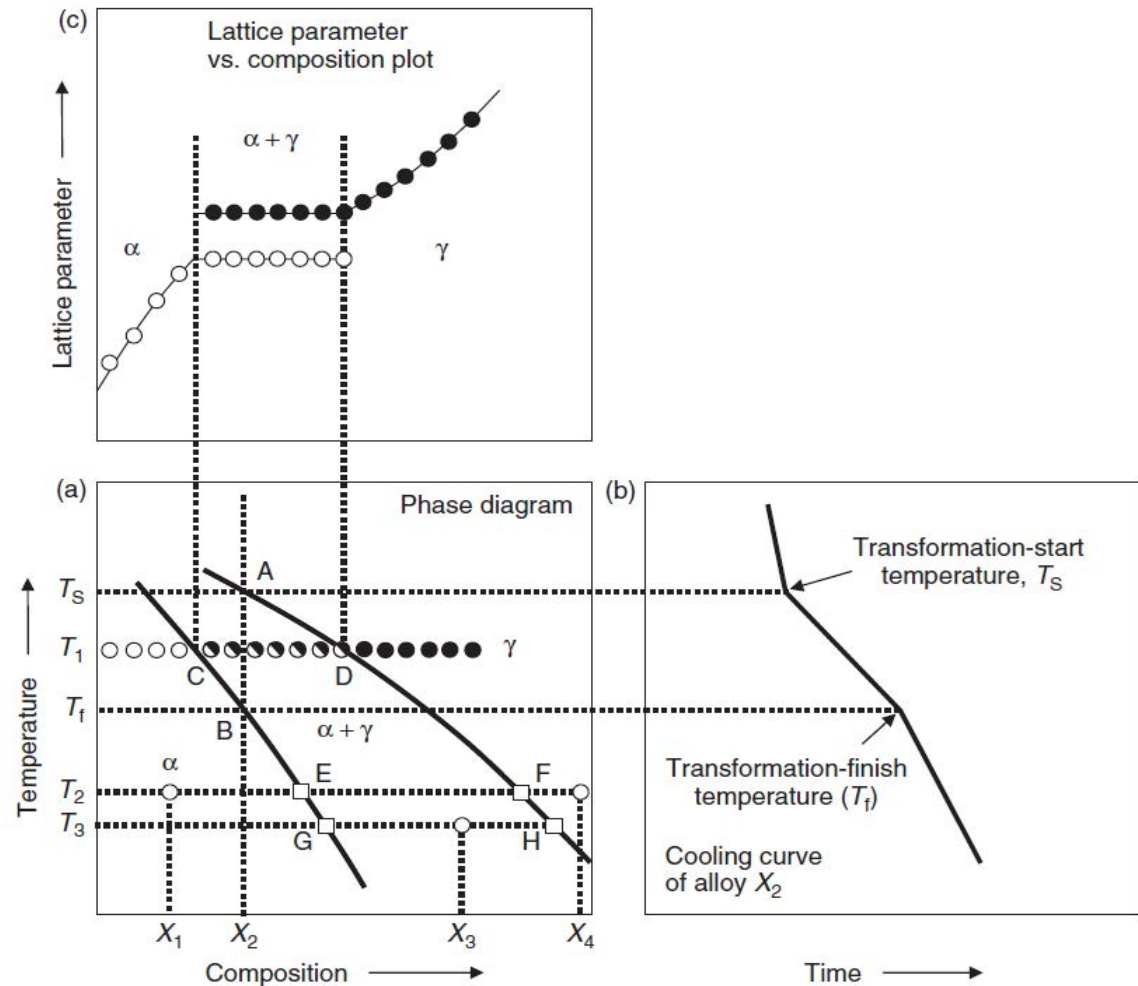


Figure 2.1 Schematic diagram showing two methods for the determination of phase diagrams: (a) schematic phase diagram; (b) the schematic cooling curve of a specific alloy X_2 ; and (c) schematic plot of lattice parameters vs. composition.

Binaries: Mixtures, solid solutions, alloys, compounds, intermediate phases, ...

What can happen if we bring two materials A and B together?

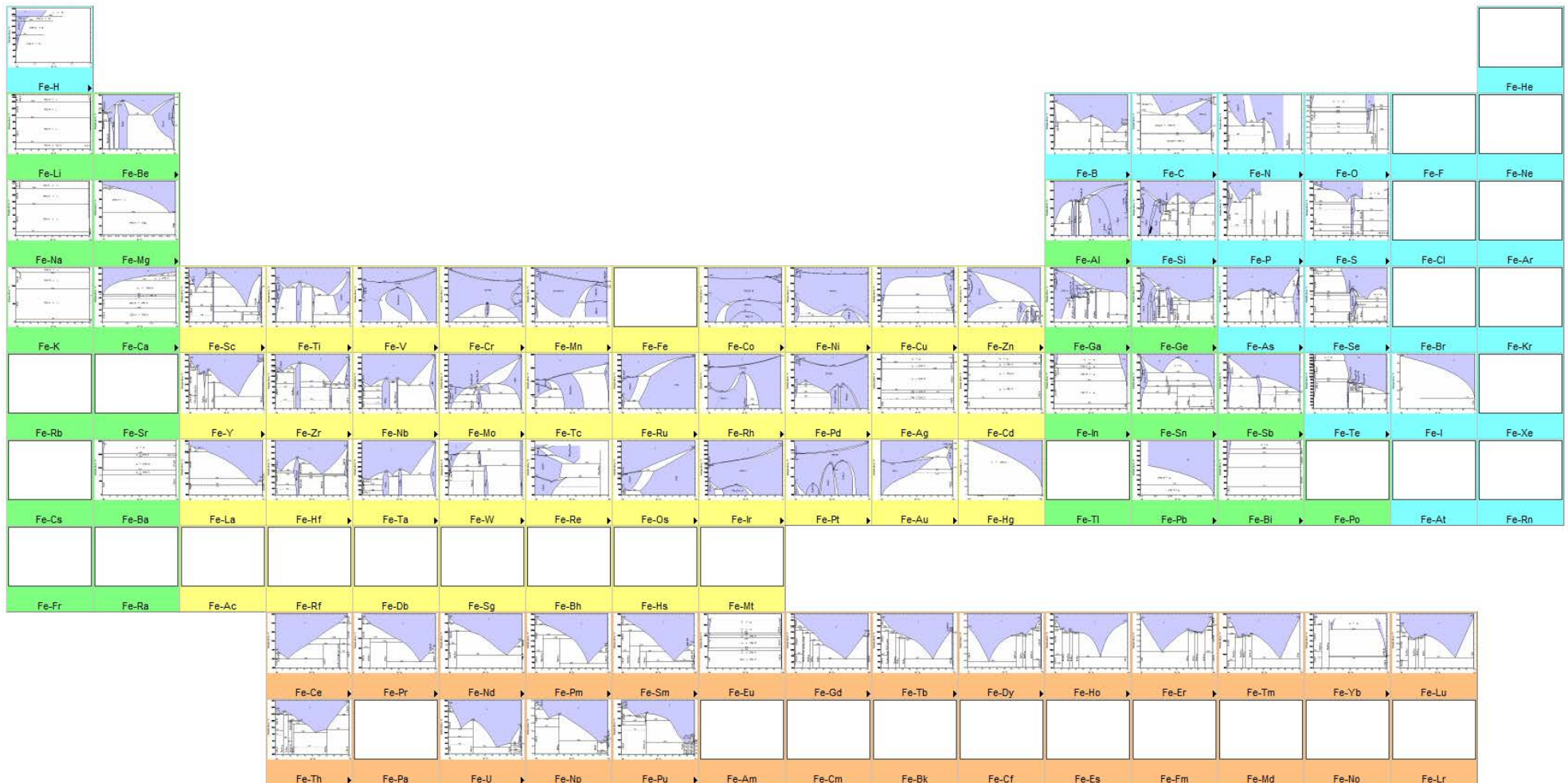
- Mixing of elements: Formation of mixtures, solid solutions,
- Reactions: Formation of alloys, compounds, intermediate phases
- A-B compositions: Influence on T_M
- A-B compositions: Influence on crystal structure

Doping

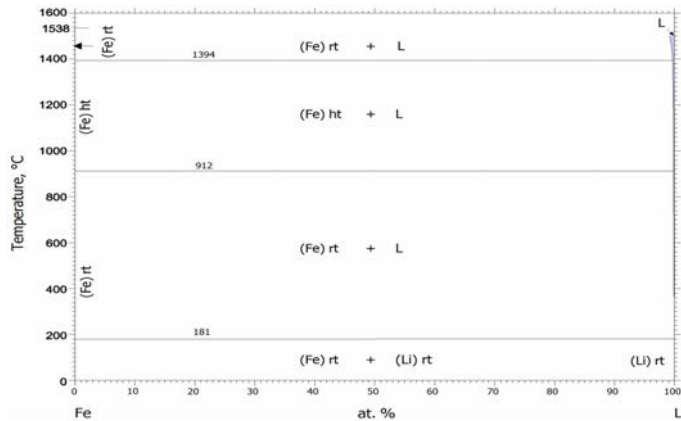
Doping is very important in semiconductor physics,
a very small amount of B in A, typical $\ll 1$ at.% (ppm)
e.g. B or P in Si

Binary phase diagrams

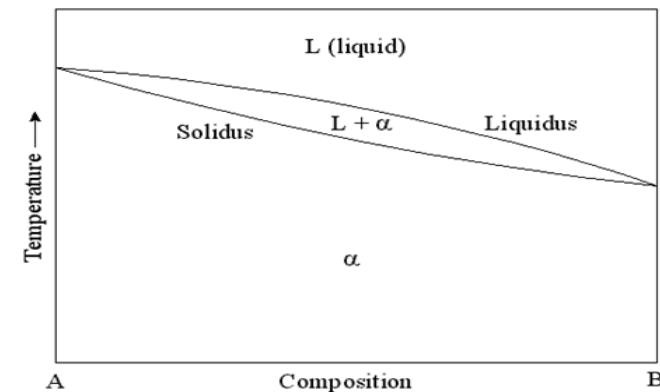
Overview of all Fe-X phase diagrams



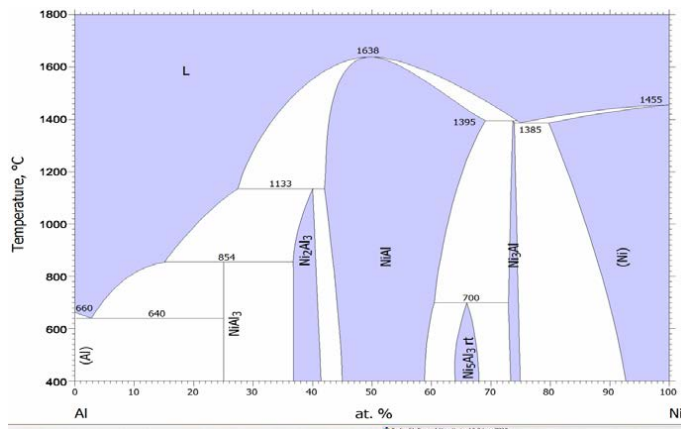
Types of binary materials diagrams



non-compound forming system

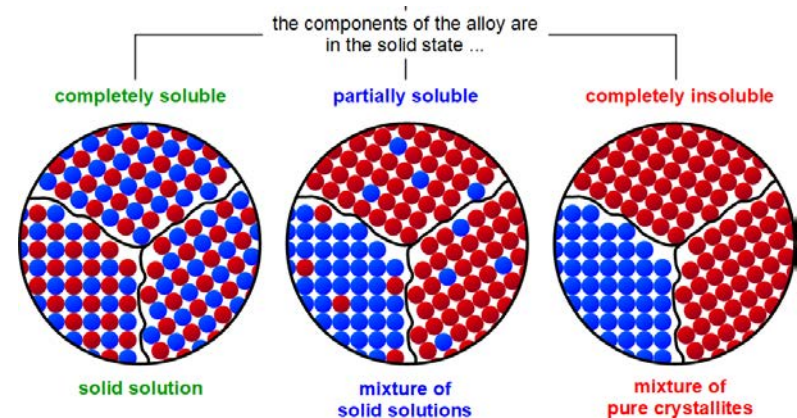


only solid solution(s)



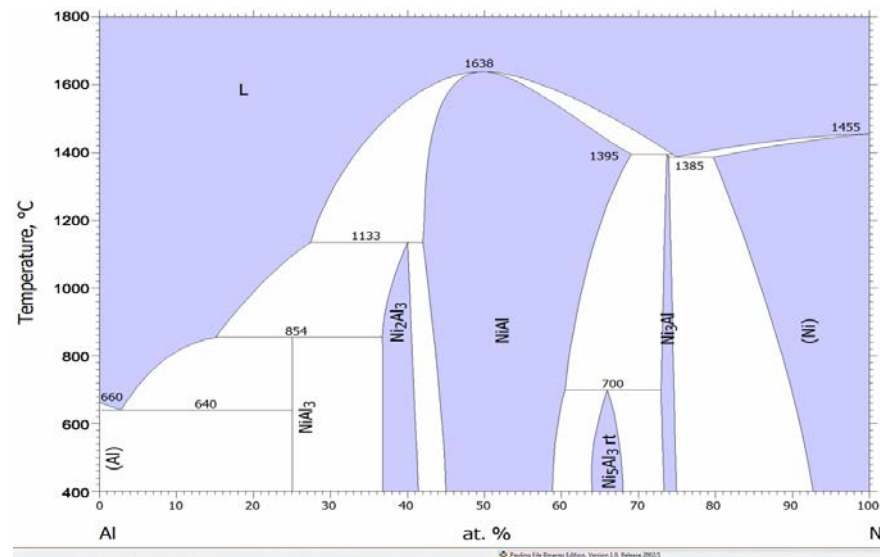
compound forming system

Types of alloys

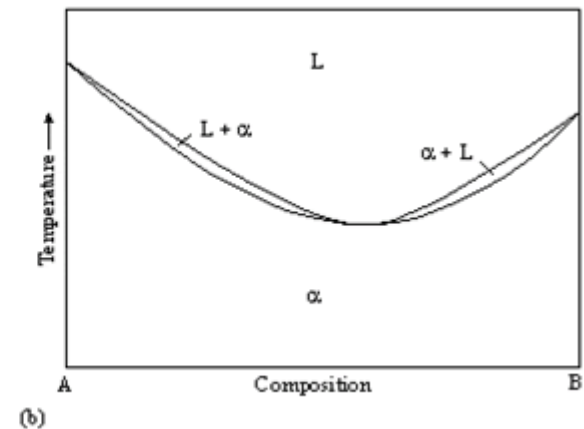
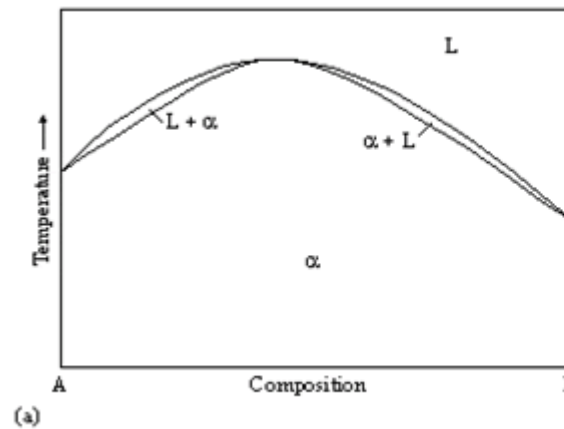
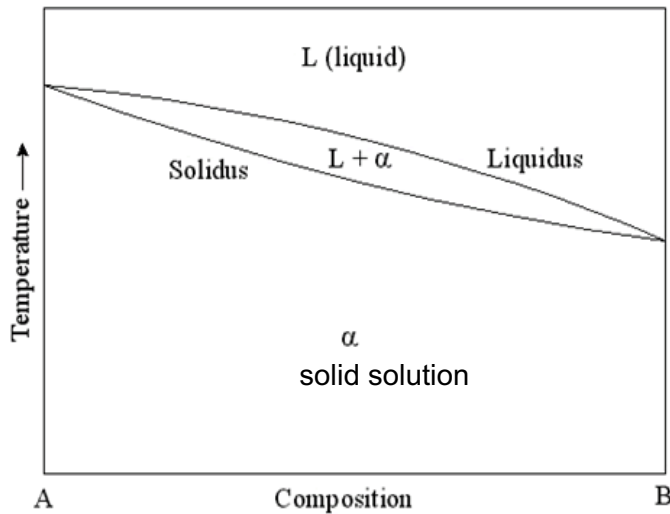


Definitions of alloys and compounds

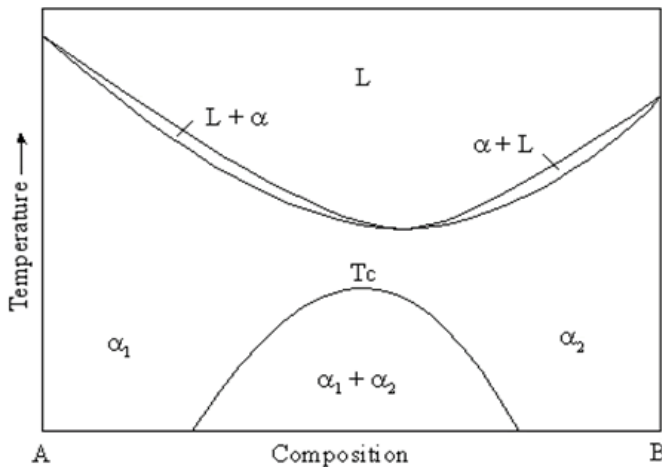
- **Alloy**: A substance having metallic properties and being composed of two or more chemical elements of which at least one is a metal
- **Intermediate phase**: In an alloy or a chemical system, a distinguishable homogeneous phase whose composition range does not extend to any of the pure components of the system
- **Intermetallic phase**: A compound or intermediate solid solution, containing two or more metals, which usually has a composition, characteristic properties, and crystal structure different from those of the pure components of the system
- **Intermetallic compound**: An intermediate phase in an alloy system, having a narrow range of homogeneity and relatively simple stoichiometric proportions; the nature of the atomic binding can be of various types, ranging from metallic to ionic



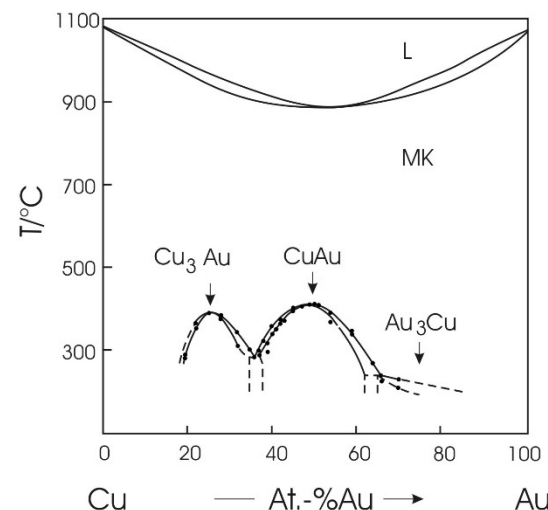
Binary phase diagrams



Schematic binary phase diagrams with solid state miscibility where the liquidus shows a maximum and a minimum



Schematic binary phase diagram with *miscibility gap* in a single-phase field

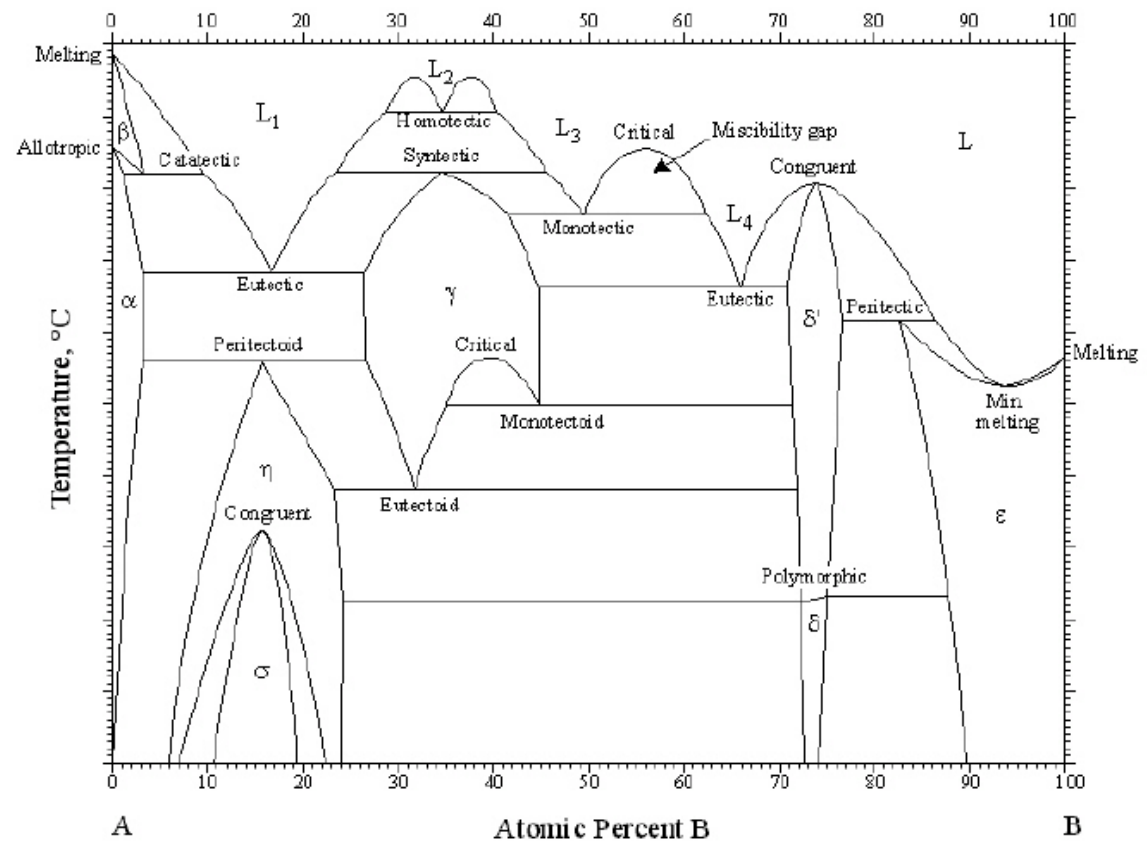


increase of T_m with increasing element B corresponds to stronger bonding, and tends to the formation of intermediate phases

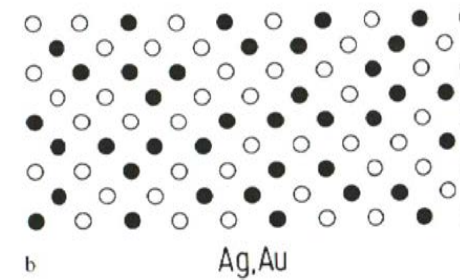
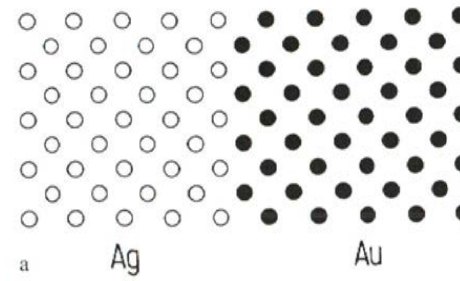
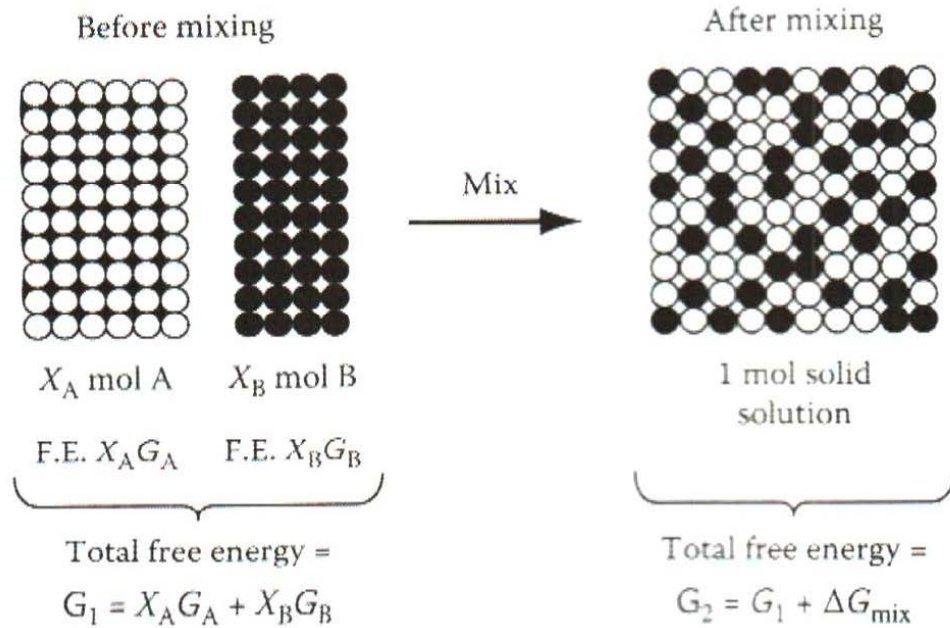
decrease of T_m with increasing element B corresponds to weaker bonding, and tends to decomposition

Phase equilibria in binary systems

Name of reaction	Phase equilibrium	Schematic representation
Eutectic	$L \leftrightarrow s_1 + s_2$	$s_1 > \frac{s_1 + L}{s_1 + s_2} \downarrow \frac{L}{s_1 + s_2} \downarrow \frac{L + s_2}{s_1 + s_2} < s_2$
Peritectic	$s_1 + L \leftrightarrow s_2$	$s_1 > \frac{s_1 + L}{s_1 + s_2} \downarrow \frac{s_1 + L}{s_2} \downarrow \frac{s_2 + L}{s_2 + L} < L$
Monotectic	$L_1 \leftrightarrow s_1 + L_2$	$s_1 > \frac{s_1 + L_1}{L_1 + L_2} \downarrow \frac{L_1}{L_1 + L_2} \downarrow \frac{L_1 + L_2}{L_1 + L_2} < L_2$
Eutectoid	$s_1 \leftrightarrow s_2 + s_3$	$s_2 > \frac{s_2 + s_1}{s_2 + s_3} \downarrow \frac{s_1}{s_2 + s_3} \downarrow \frac{s_1 + s_3}{s_2 + s_3} < s_3$
Peritectoid	$s_1 + s_2 \leftrightarrow s_3$	$s_1 > \frac{s_1 + s_2}{s_1 + s_3} \downarrow \frac{s_1 + s_2}{s_3} \downarrow \frac{s_3 + s_2}{s_3 + s_2} < s_2$
Monotectoid	$s_{1a} \leftrightarrow s_{1b} + s_2$	$s_{1b} > \frac{s_{1b} + s_{1a}}{s_{1b} + s_2} \downarrow \frac{s_{1a}}{s_{1b} + s_2} \downarrow \frac{s_{1a} + s_2}{s_{1b} + s_2} < s_2$
Metatectic	$s_1 \leftrightarrow s_2 + L$	$s_2 > \frac{s_2 + s_1}{s_2 + L} \downarrow \frac{s_1}{s_2 + L} \downarrow \frac{s_1 + L}{s_2 + L} < L$
Syntectic	$L_1 + L_2 \leftrightarrow s$	$L_1 > \frac{L_1 + L_2}{L_1 + s} \downarrow \frac{L_1 + L_2}{s} \downarrow \frac{s + L_2}{s + L_2} < L_2$

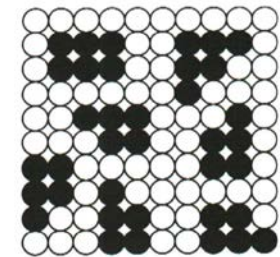


Solid solutions

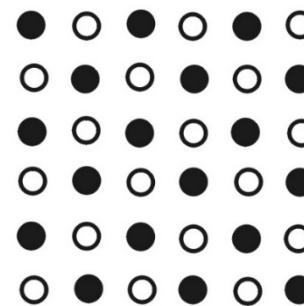
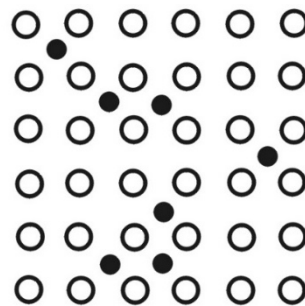


substitutional solid solutions

clustered solid solution



interstitial solid solutions (typical: H, B, C, N)



ordered solid solution

solid solutions can form,
if components have:

- similar size ($< 15\%$)
- same crystal structure
- similar electronegativity

34

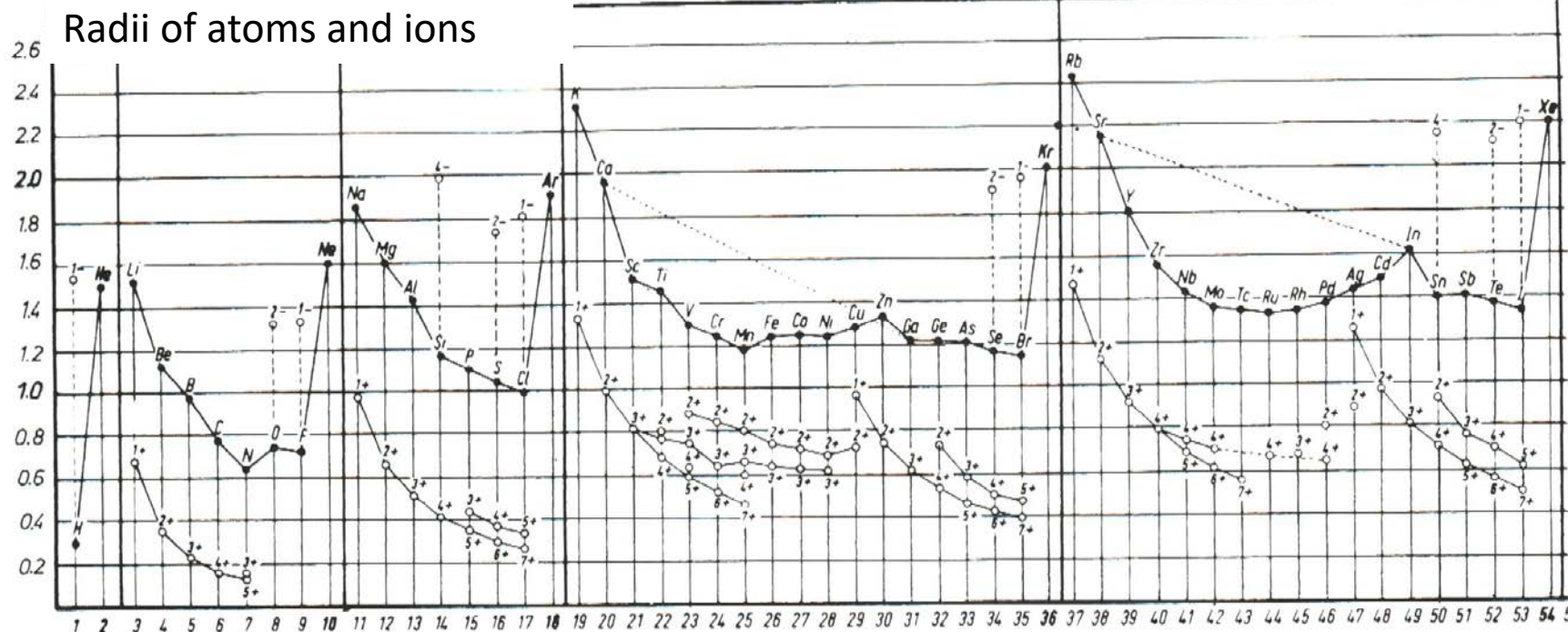
[illegible]

Ce fcc	Pr hcp	Nd hcp	Pm	Sm rhomb	Eu bcc	Gd hcp	Tb hcp	Dy hcp	Ho hcp	Er hcp	Tm hcp	Yb fcc	Lu hcp
Th fcc	Pa tetrag	U orthor	Np orthor	Pu monoc	Am hcp	Cm	Bk	Cf	Es	Fm	Md	No	Lr

2 Solid-State Chemistry



Figure 2.13. Preferred crystal structures of the elements. Shown are the ordinary forms of each element

**bb 12.7** Atom- und Ionenradien in Abhängigkeit von der Ordnungszahl, nach Ramdohr u. Strunz [35]

Fundamental Aspects of Materials Science and Engineering

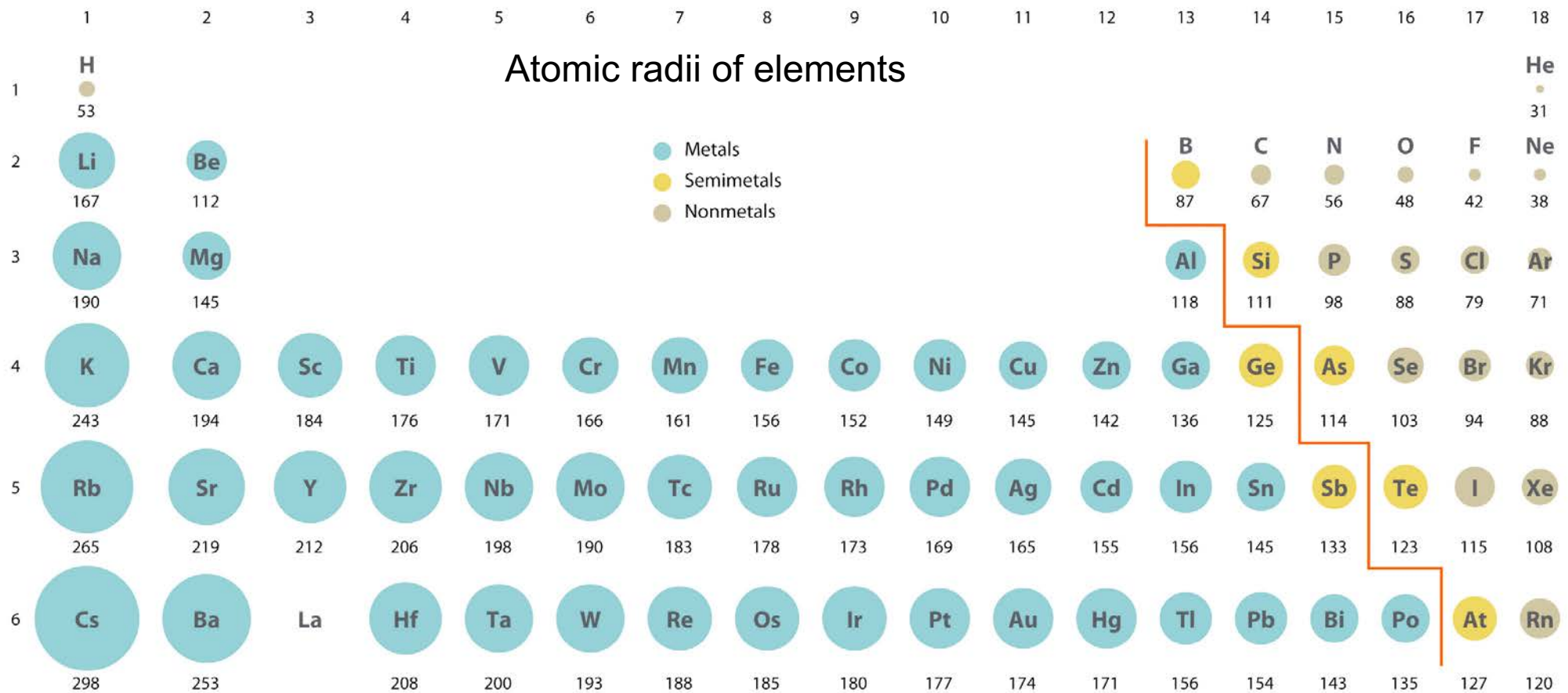
Prof. Dr.-Ing. Alfred Ludwig | Materials Discovery and Interfaces

Solid solutions

When do they form?

Solid solutions can form, if components have:

- similar size (< 15%)



Solid solutions

When do they form?

Solid solutions can form, if components have:

- similar electronegativity

Electronegativity of elements

1 H 2.20																	2 He no data
3 Li 0.98	4 Be 1.57											5 B 2.04	6 C 2.55	7 N 3.04	8 O 3.44	9 F 3.98	10 Ne no data
11 Na 0.93	12 Mg 1.31											13 Al 1.61	14 Si 1.90	15 P 2.19	16 S 2.58	17 Cl 3.16	18 Ar no data
19 K 0.82	20 Ca 1.00	21 Sc 1.36	22 Ti 1.54	23 V 1.63	24 Cr 1.66	25 Mn 1.55	26 Fe 1.83	27 Co 1.88	28 Ni 1.91	29 Cu 1.90	30 Zn 1.65	31 Ga 1.81	32 Ge 2.01	33 As 2.18	34 Se 2.55	35 Br 2.96	36 Kr 3.00
37 Rb 0.82	38 Sr 0.95	39 Y 1.22	40 Zr 1.33	41 Nb 1.6	42 Mo 2.16	43 Tc 1.9	44 Ru 2.2	45 Rh 2.28	46 Pd 2.20	47 Ag 1.93	48 Cd 1.69	49 In 1.78	50 Sn 1.96	51 Sb 2.05	52 Te 2.1	53 I 2.66	54 Xe 2.6
55 Cs 0.79	56 Ba 0.89	57-71	72 Hf 1.3	73 Ta 1.5	74 W 2.36	75 Re 1.9	76 Os 2.2	77 Ir 2.2	78 Pt 2.28	79 Au 2.54	80 Hg 2.00	81 Tl 1.62	82 Pb 2.33	83 Bi 2.02	84 Po 2.0	85 At 2.2	86 Rn no data
87 Fr 0.7	88 Ra 0.89	89-103	104 Rf no data	105 Db no data	106 Sg no data	107 Bh no data	108 Hs no data	109 Mt no data	110 Ds no data	111 Rg no data	112 Cn no data	113 Nh no data	114 Fl no data	115 Mc no data	116 Lv no data	117 Ts no data	118 Og no data

Low

High

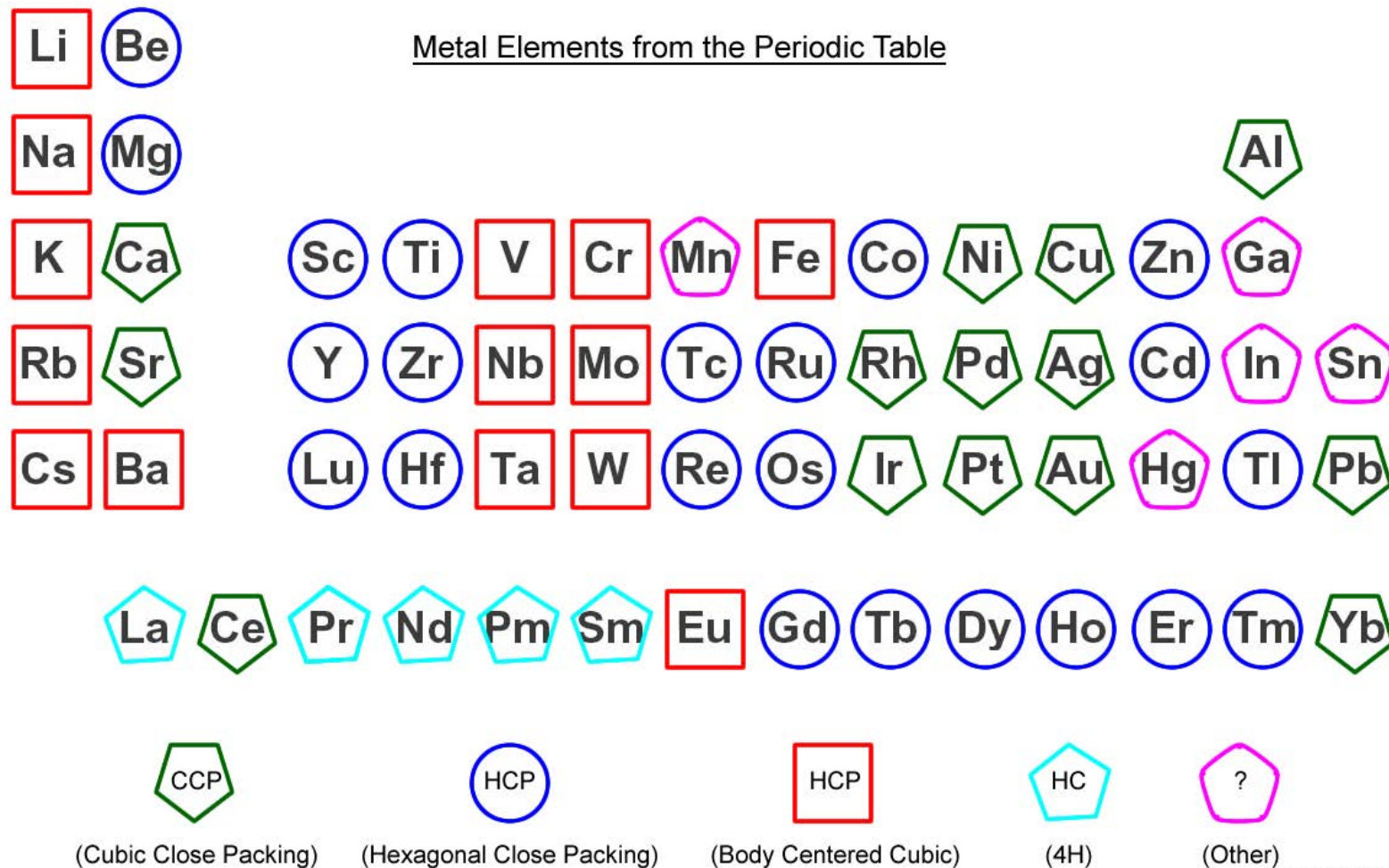
57 La 1.10	58 Ce 1.12	59 Pr 1.13	60 Nd 1.14	61 Pm 1.13	62 Sm 1.17	63 Eu 1.2	64 Gd 1.2	65 Tb 1.22	66 Dy 1.23	67 Ho 1.24	68 Er 1.24	69 Tm 1.25	70 Yb 1.1	71 Lu 1.27
89 Ac 1.1	90 Th 1.3	91 Pa 1.5	92 U 1.38	93 Np 1.36	94 Pu 1.28	95 Am 1.3	96 Cm 1.3	97 Bk 1.3	98 Cf 1.3	99 Es 1.3	100 Fm 1.3	101 Md 1.3	102 No 1.3	103 Lr no data

Solid solutions

When do they form?

Solid solutions can form, if components have:

- same crystal structure



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Binary phase diagrams

Comparison of the systems

Cu-Au, Cu-Ag, Cu-Ni, Ag-Au

Au, Ag, Cu, Ni:
fcc

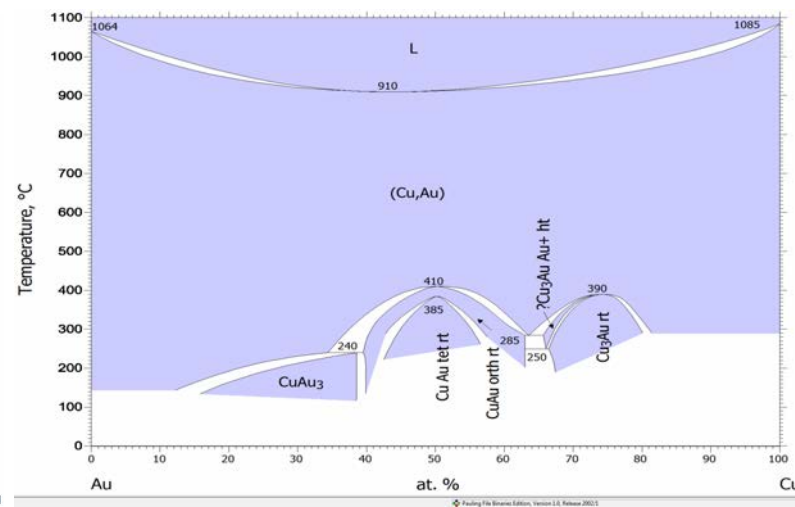
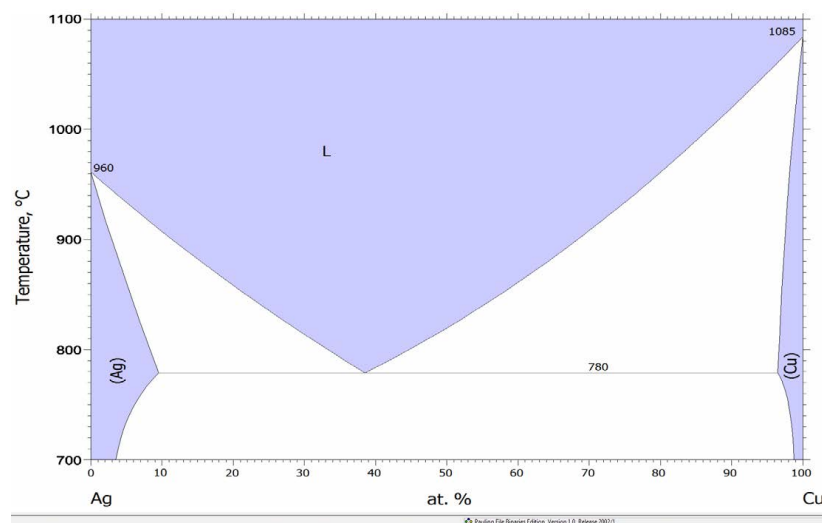
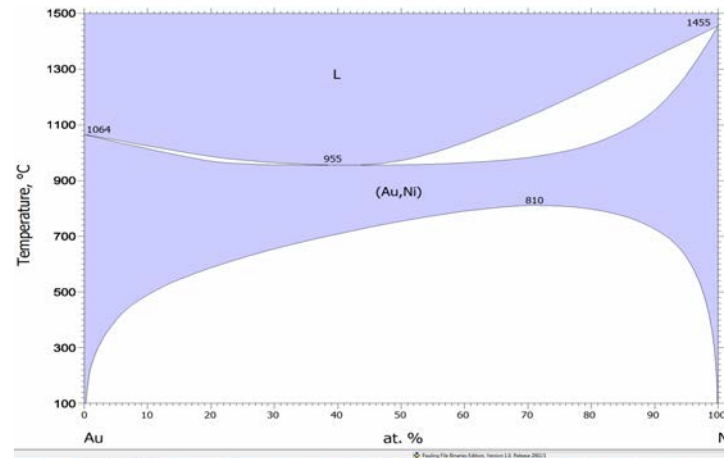
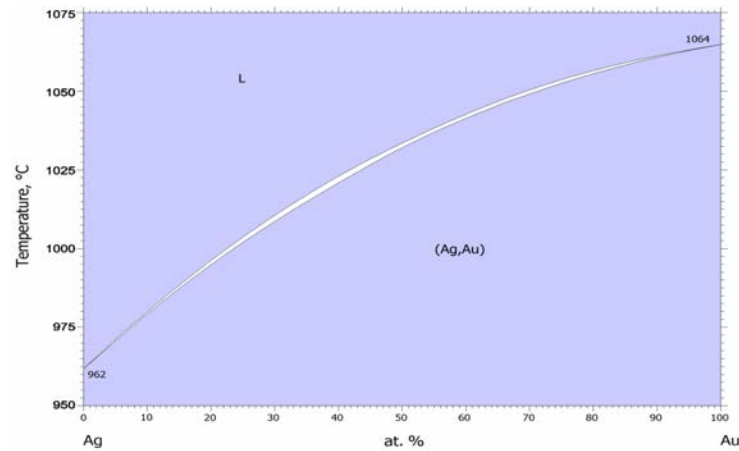
$a_{\text{Ag}} = 0.40863 \text{ nm}$

$a_{\text{Au}} = 0.40786 \text{ nm}$

$a_{\text{Ni}} = 0.38411 \text{ nm}$

$a_{\text{Cu}} = 0.36148 \text{ nm}$

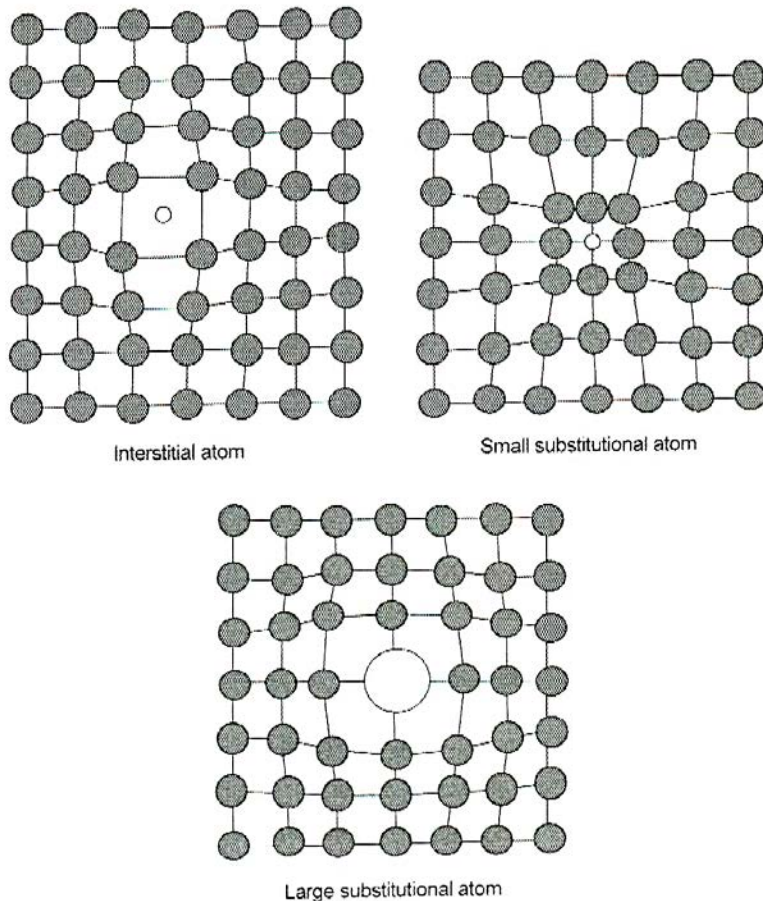
RUB



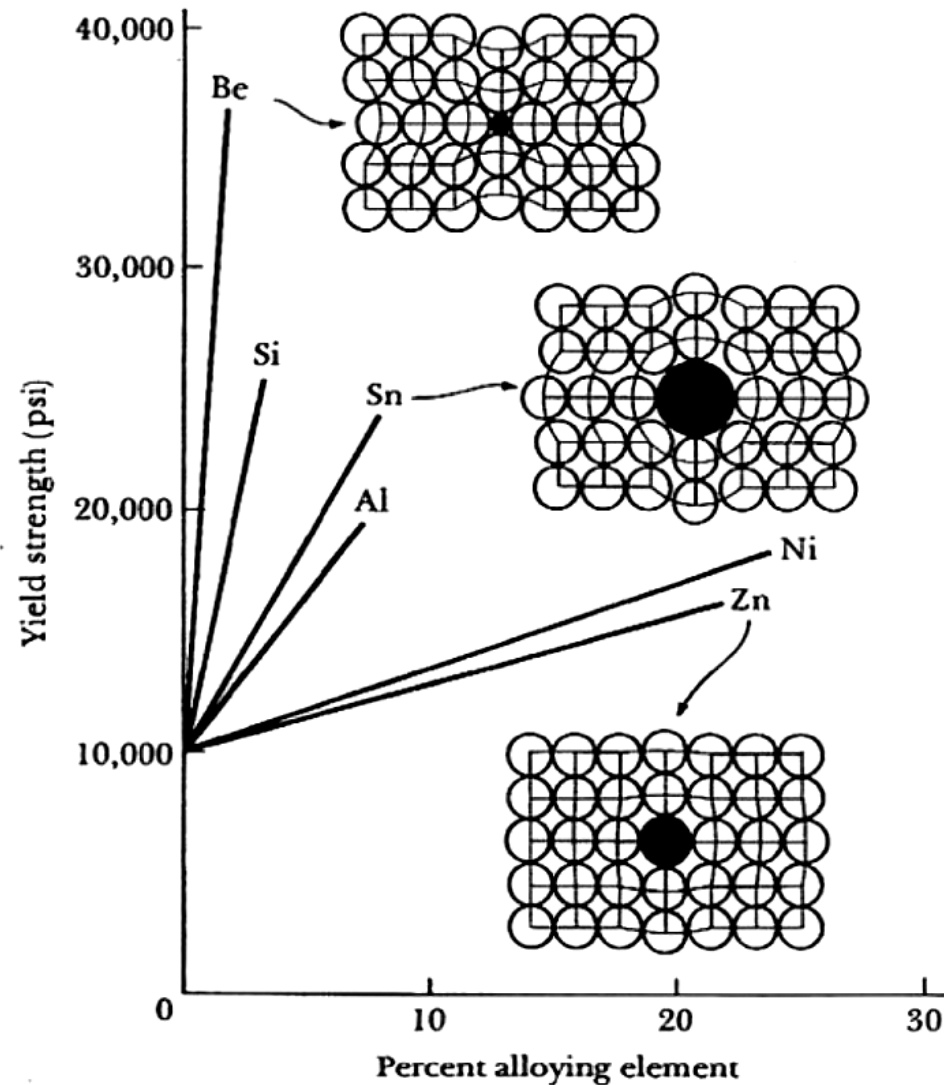
Solid solutions

Mechanical properties

- soluble atoms add lattice distortions depending on position in the lattice
- distortions affect mechanical properties

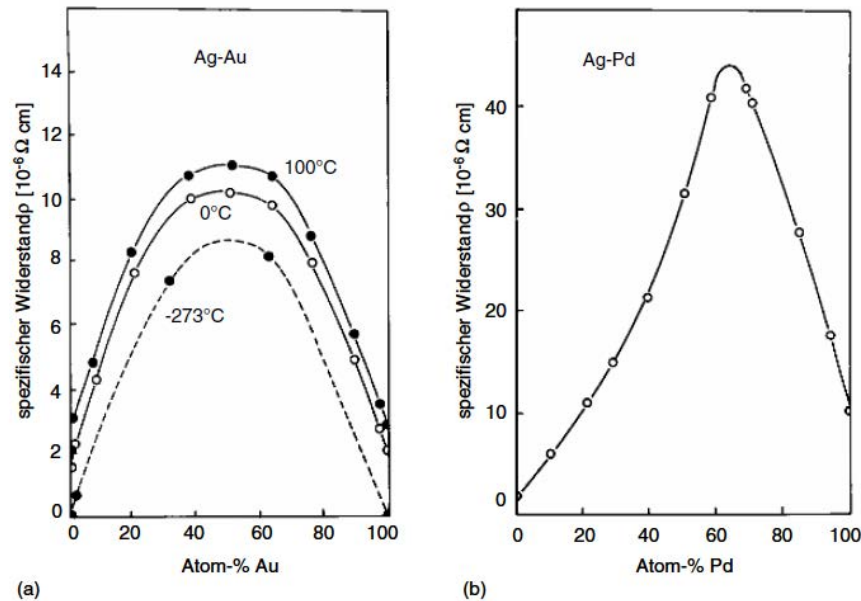


Effects of alloying elements on the yield strength of Cu



Dependence of physical properties on composition for binary systems with complete miscibility

electrical resistivity



mechanical properties

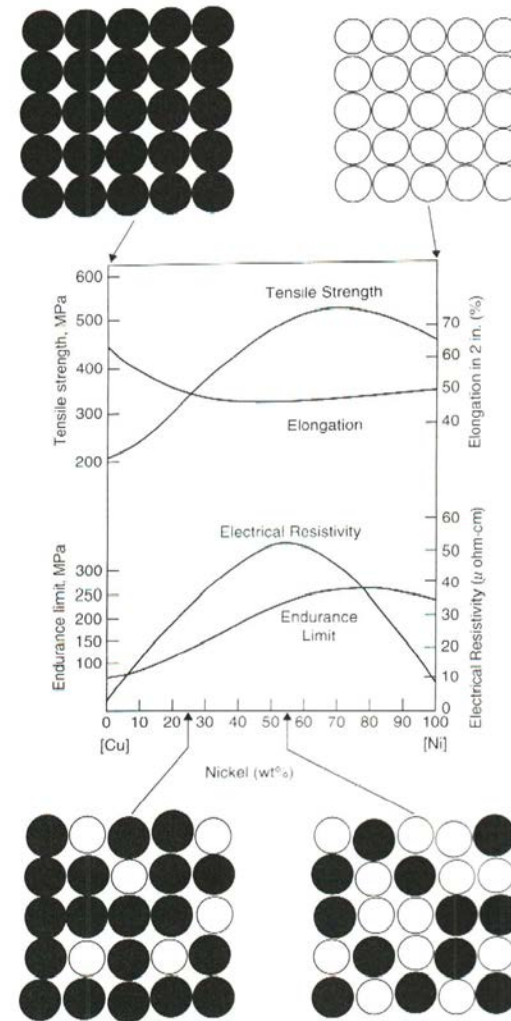
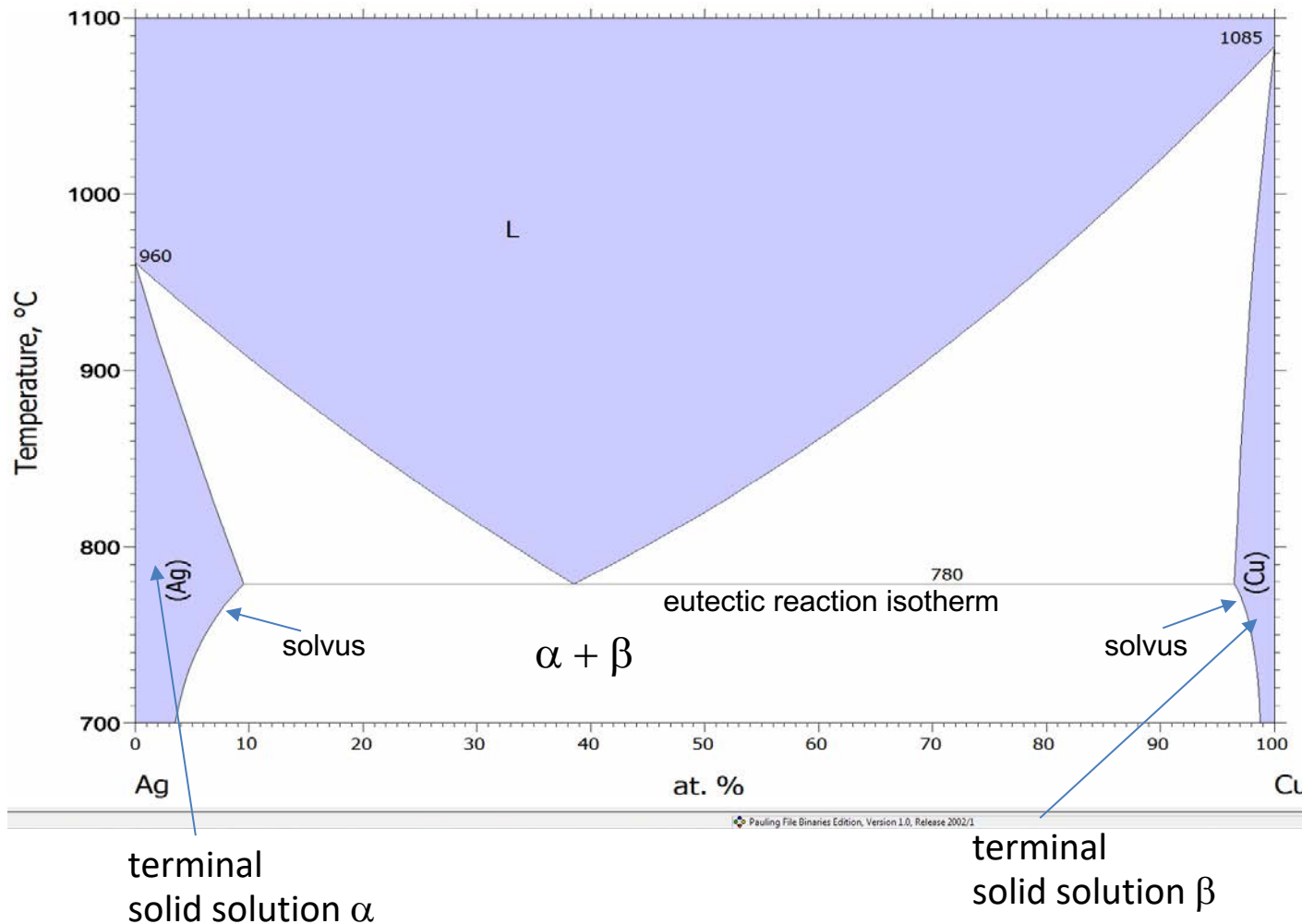


Fig. 4.6 Typical property variations in copper-nickel system. Source: Ref 4.2 as published in Ref 4.3

Abbildung 10.22. Spezifischer elektrischer Widerstand bei Raumtemperatur in Abhängigkeit von der Legierungskonzentration bei den lückenlos mischbaren Legierungen Ag-Au (a) und Ag-Pd (b) (nach [10.13]).

Eutectic phase diagrams

Systems with limited solubility



Indicate single-phase and two-phase field(s)

What is the solubility of Cu in Ag at 700°C?

What is the solubility of Ag in Cu at 700°C?

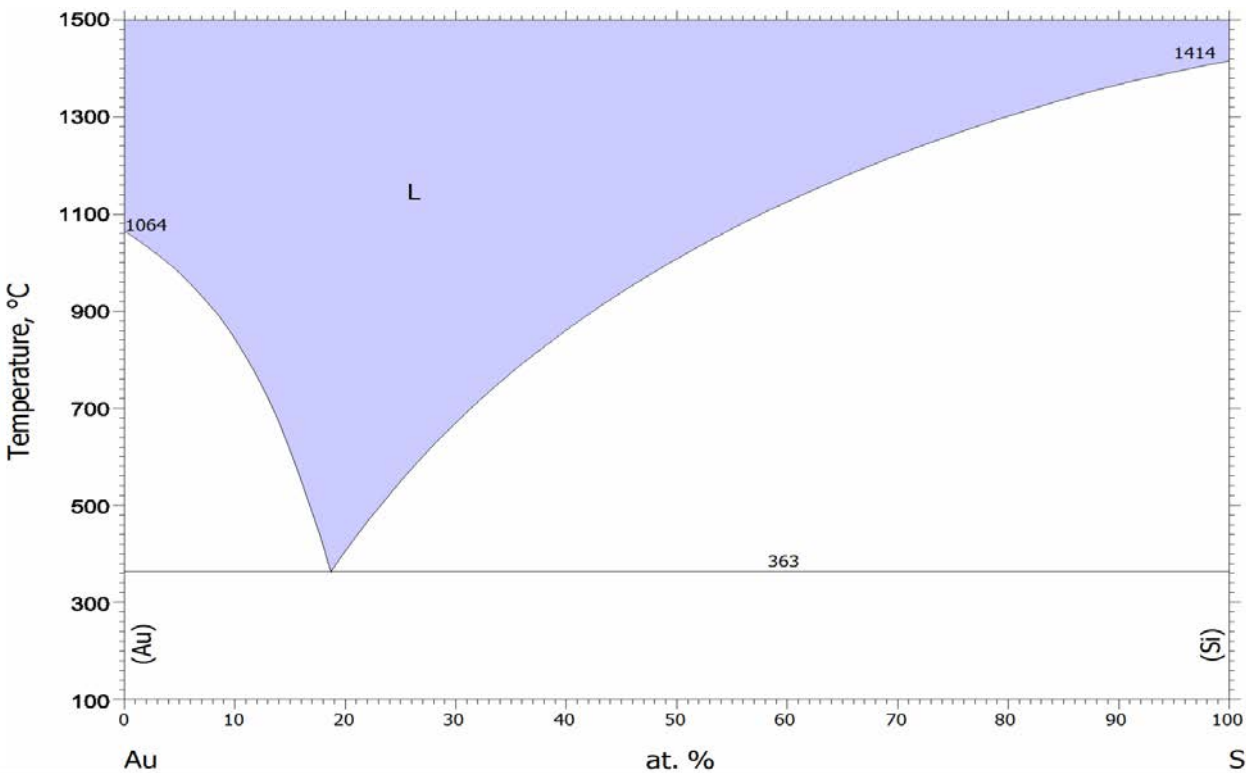
Write down the eutectic reaction.

Where do we find maximum solubility?

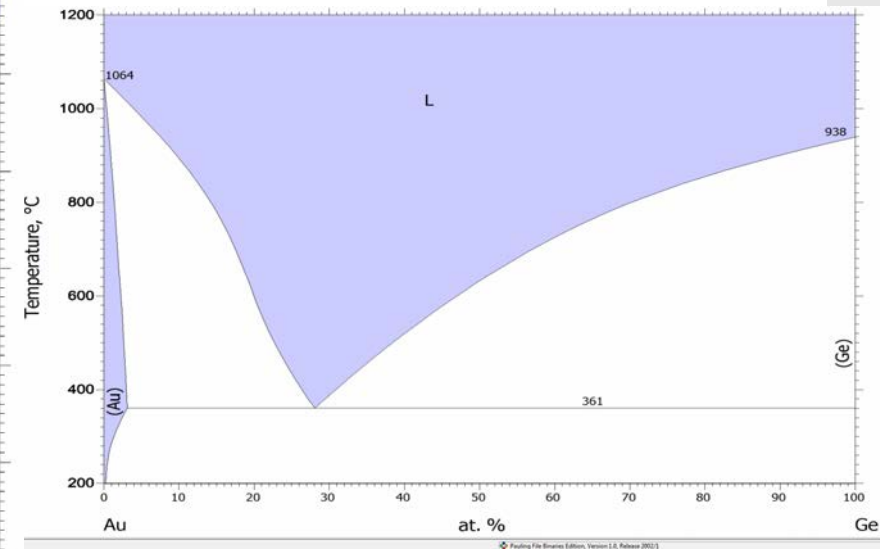
Eutectic phase diagrams

Examples

- Interesting phase diagrams:
high melting point of components, low eutectic temperature

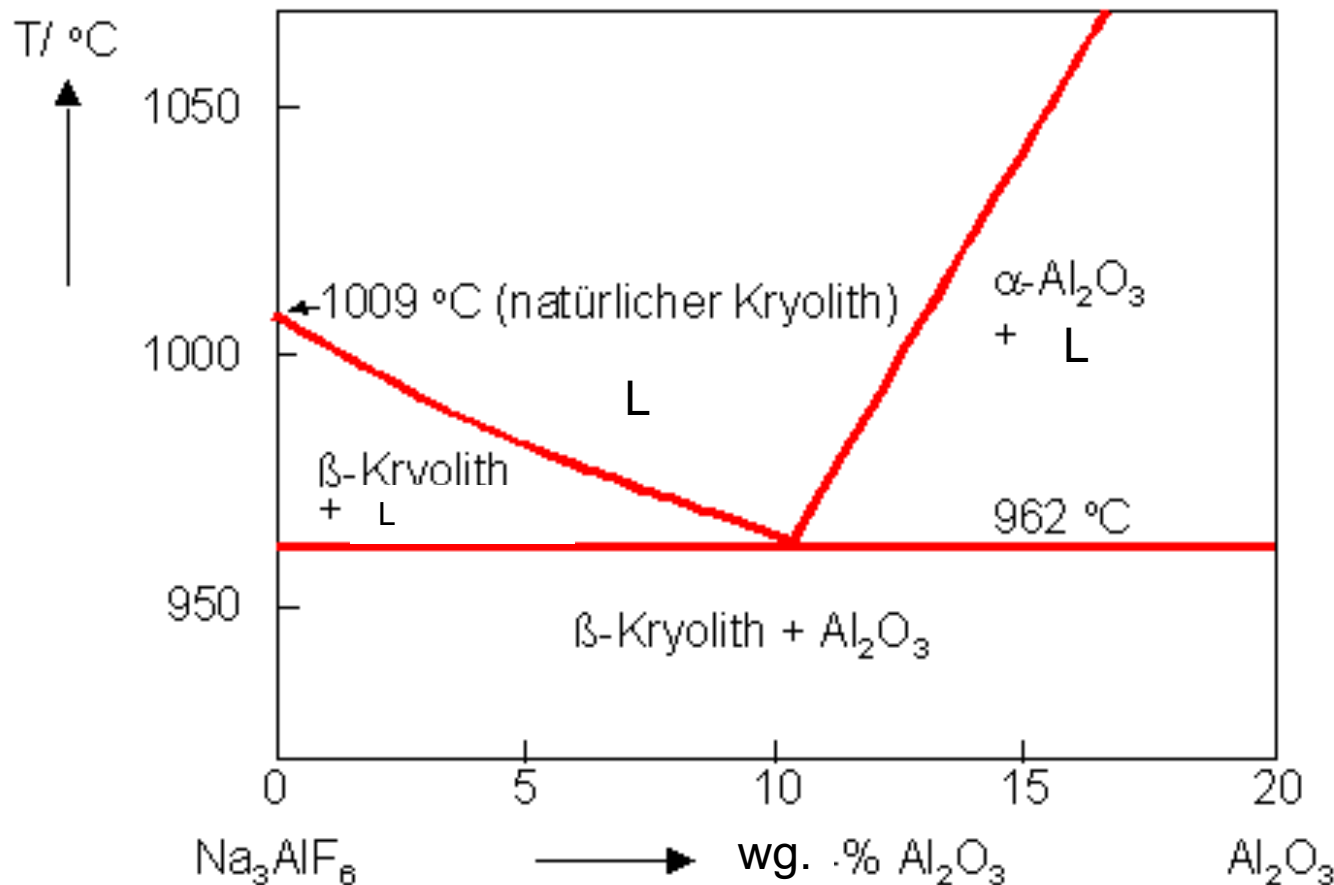


Au-Si eutecticum is used in MEMS
as bonding material



Eutectic phase diagrams

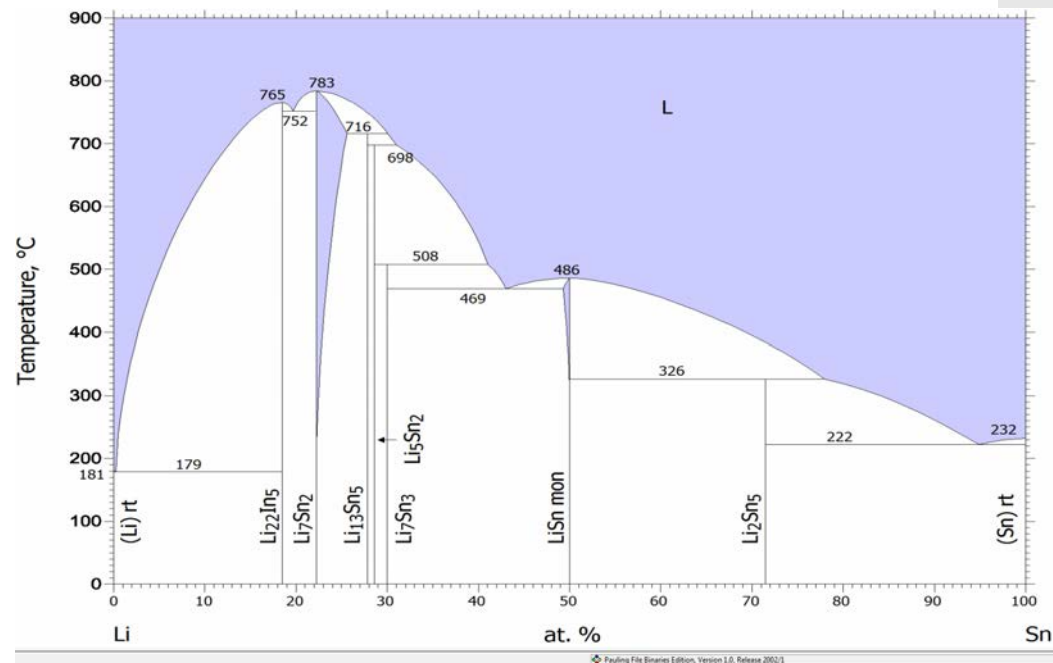
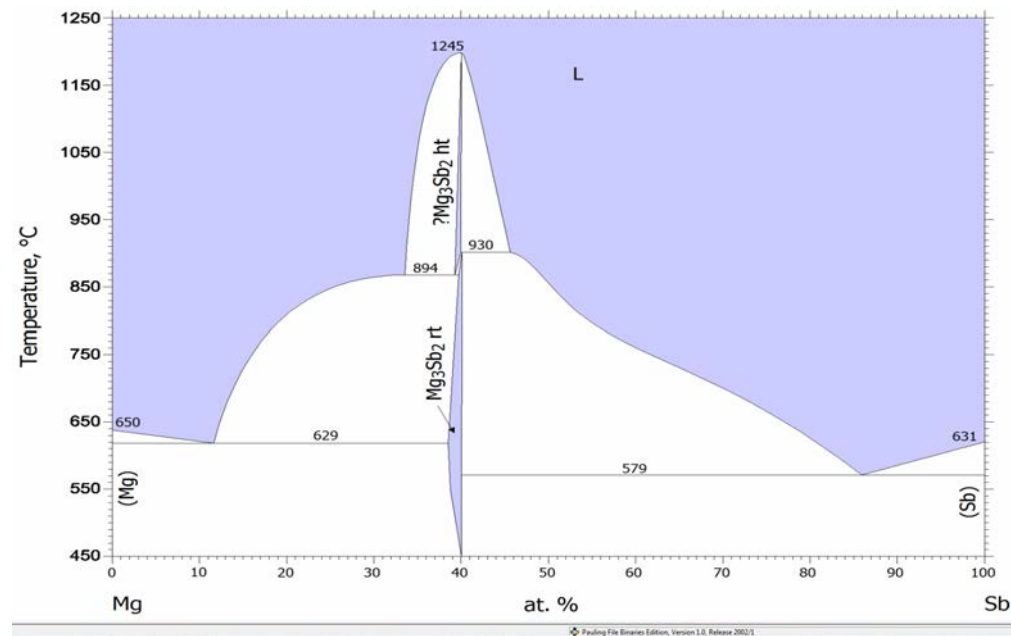
Application example Al production:
Lowering process temperatures by mixing
of two components



melt electrolysis process
Tm of Al_2O_3 : 2000 °C

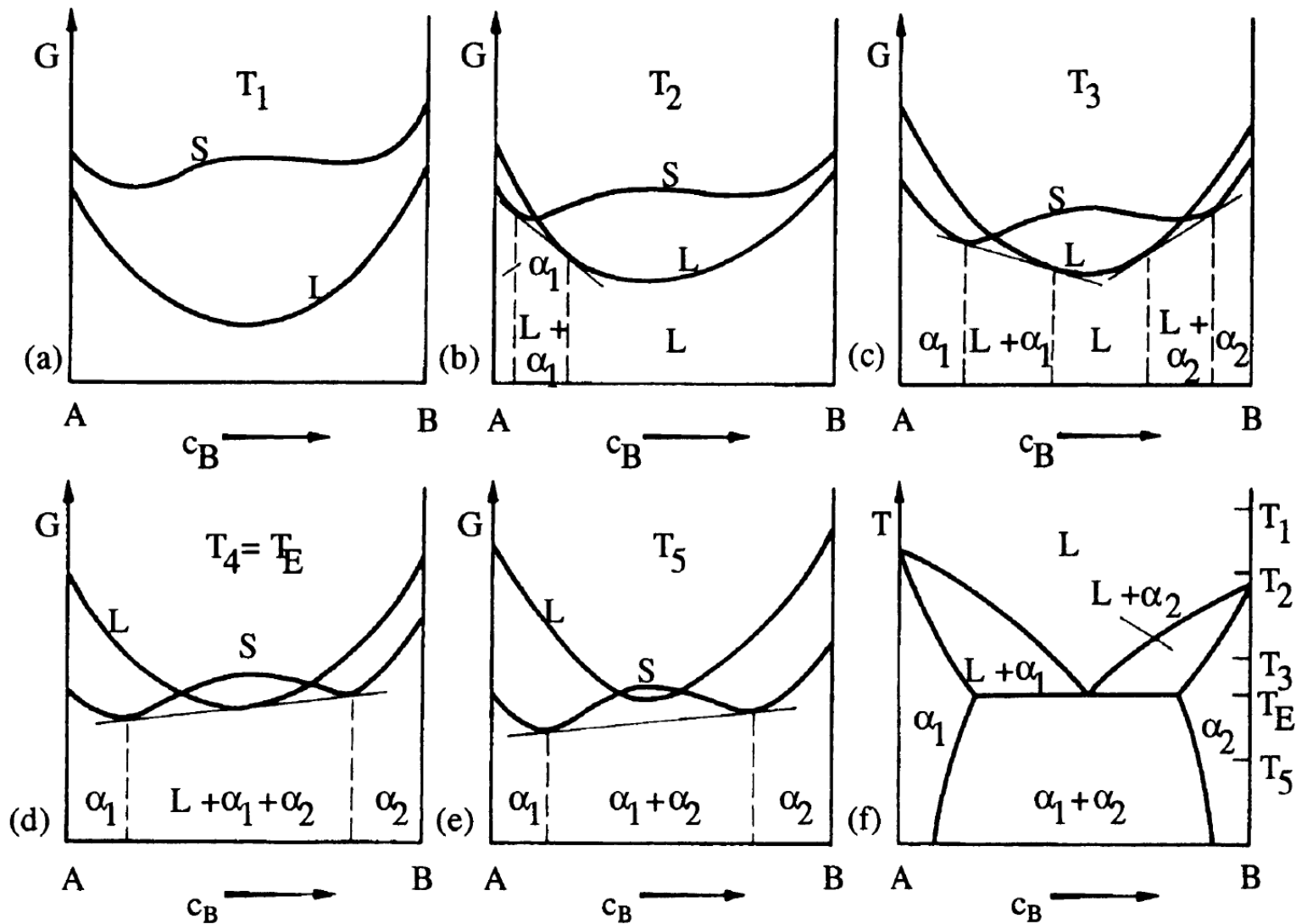
Examples of interesting phase diagrams

low melting point of components, high melting temperature of compound

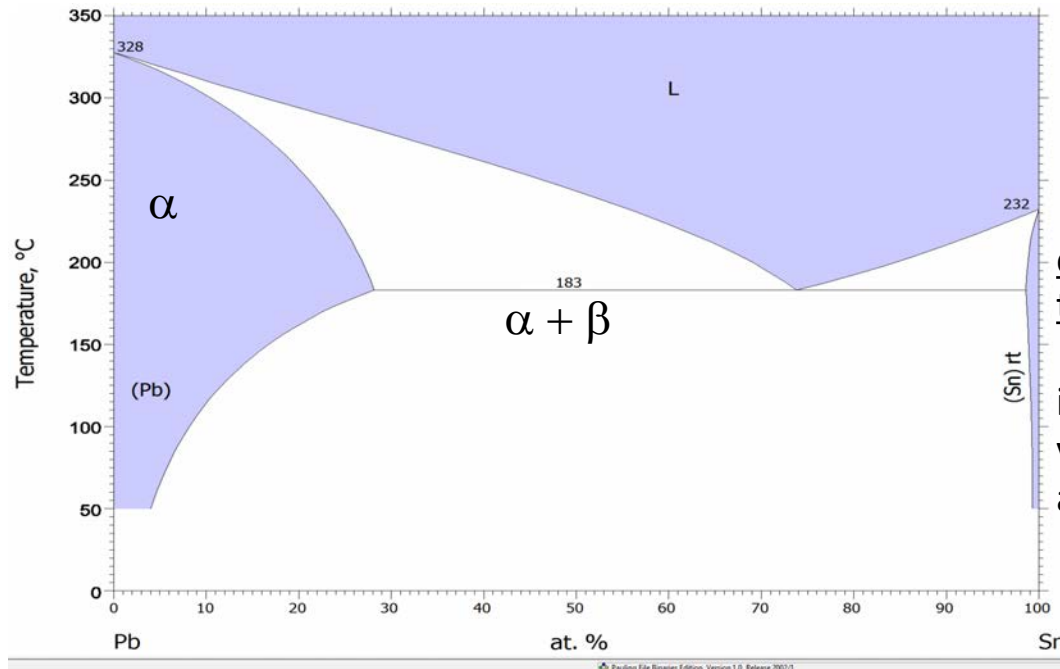


Eutectic phase diagrams

Thermodynamic evaluation



What are reasons for limited solid solubility?



size of atoms
crystal structure of components
electronegativity of components

extent of solid solution increases with temperature for both phases

increasing temperature leads to greater atomic vibrations, allowing more flexibility in accommodation of foreign atoms

β phase has same crystal structure as pure Sn, Pb atoms are distributed at random within the Sn crystal as defects.

Extent of solid solution in α phase is much greater than that in β phase, as smaller Sn atoms are more readily accommodated in the structure of the large Pb atoms, than are Pb atoms in the Sn structure.

Examples of the dependence of electrical properties of binary systems on the composition and type of phase diagram

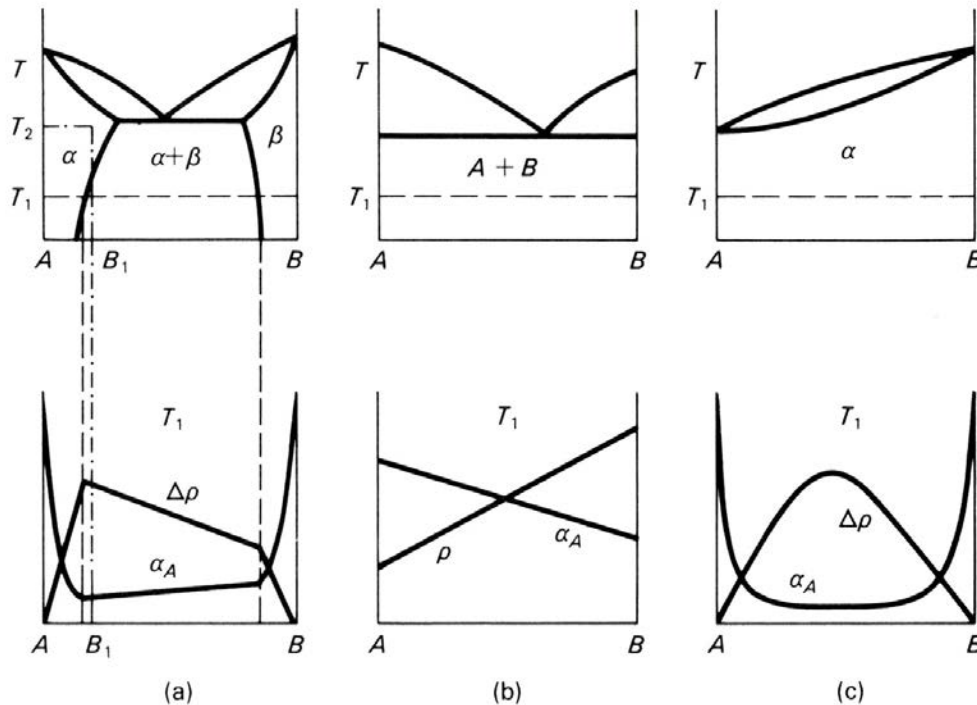


Figure 4.7 Variation of electrical conductivity with composition for (a) a continuous solid solution and (b) a binary eutectic system.

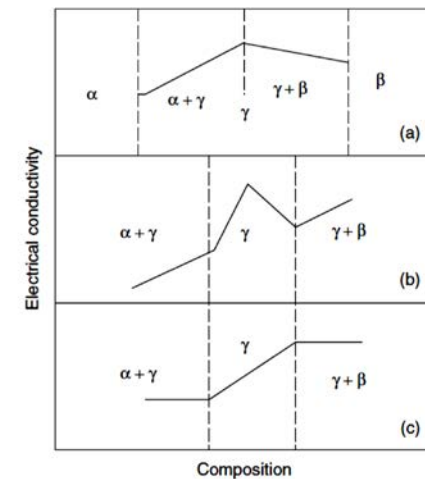
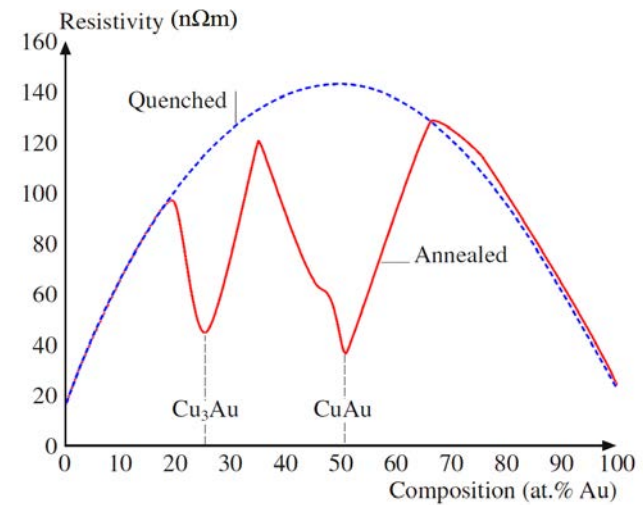


Figure 4.8 Different kinds of electrical conductivity vs. composition curve for binary systems with intermetallic phase γ [4]: (a) stoichiometric compound (γ), (b) ordered compound γ with wide homogeneity range, and (c) disordered compound with wide homogeneity range.

Temperature-dependent resistivity measurements indicate phase transformations

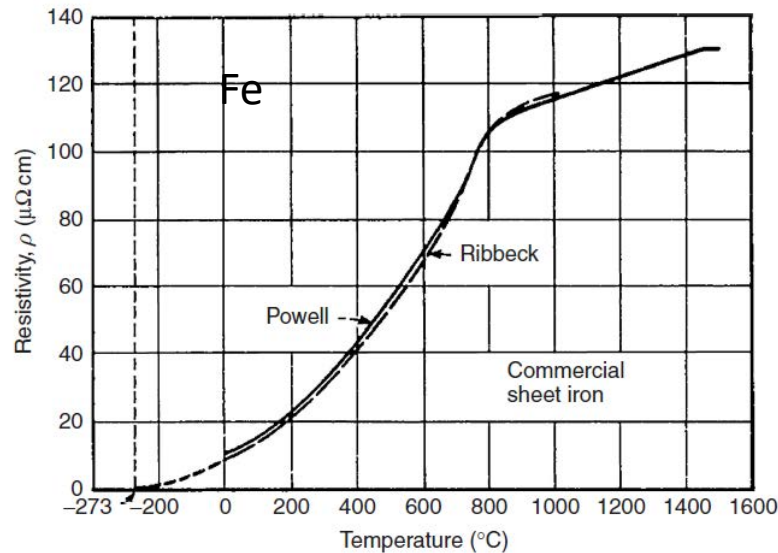


Figure 12.6 Temperature dependence of the resistivity of iron (Fe). From Ref. [7].

FeCo

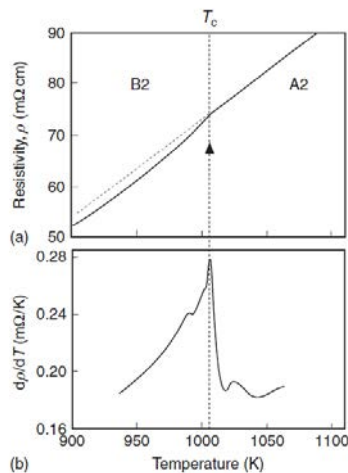


Figure 11.2 Electrical resistance curve (a) and its differential (b) showing a 2O-OD transition of a Fe-50 at.% Co alloy reported by Seehra and Silinsky [4]. The behavior of electrical resistance is similar to that in the 1O-OD transition.

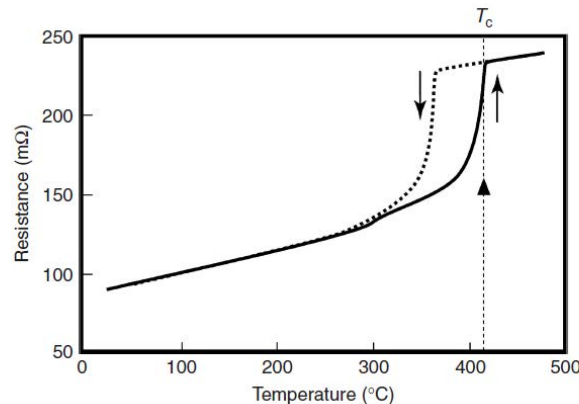


Figure 11.1 Electrical resistance curves showing the 1O-OD transition of a Au-50 at.% Cu alloy reported by Sprusil and Pfeiler [2]. The T_c is defined as the temperature with the maximal gradient in the heating curve.

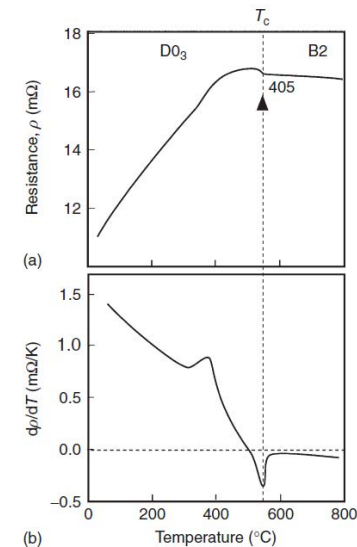
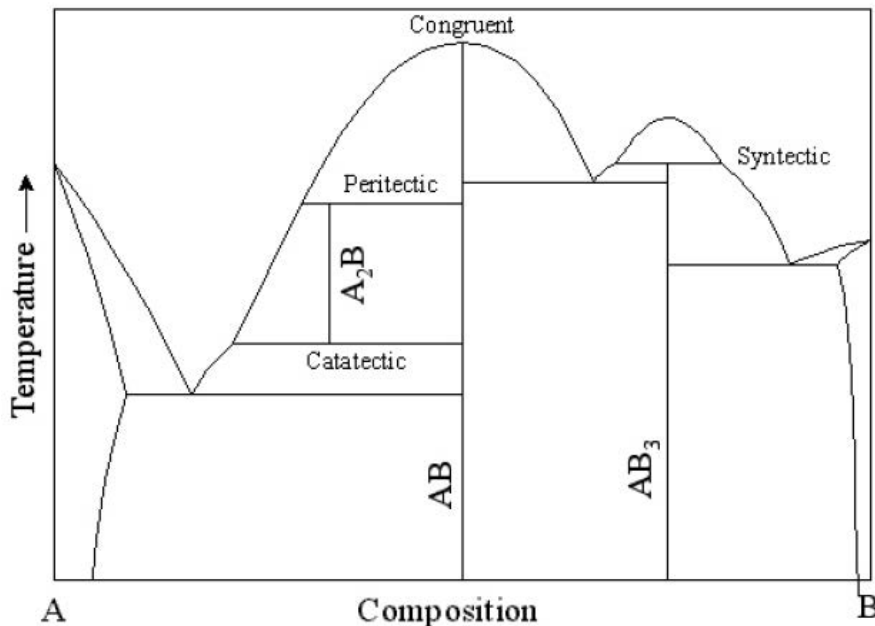


Figure 11.3 Electrical resistance curve (a) and its differential (b) showing a 2O-OD transition of Fe-27 at.% Al alloy reported by Sprusil and Pfeiler [5]. While a small and broad peak appears in the temperature region below T_c , the T_c is defined as the temperature at the minimum point of the negative peak in the differential curve.

Schematic binary phase diagram with intermetallic line compounds and different melting reactions



line compounds have stoichiometric compositions: AB , A_2B , AB_3 , ...

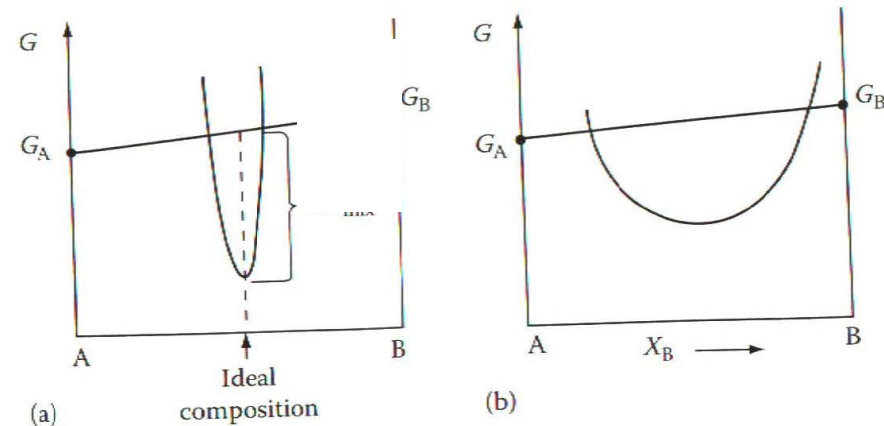


FIGURE 1.23

Free energy curves for intermediate phases: (a) for an intermetallic compound with a very narrow stability range, (b) for an intermediate phase with a wide stability range.

frequently intermediate phases have compositional existence ranges

Peritectic phase reactions

Not all intermediate compounds show congruent melting.

Many intermediate phases transform into a liquid at a peritectic point. On heating through a peritectic point, a solid transforms to a liquid plus another solid of a different composition.

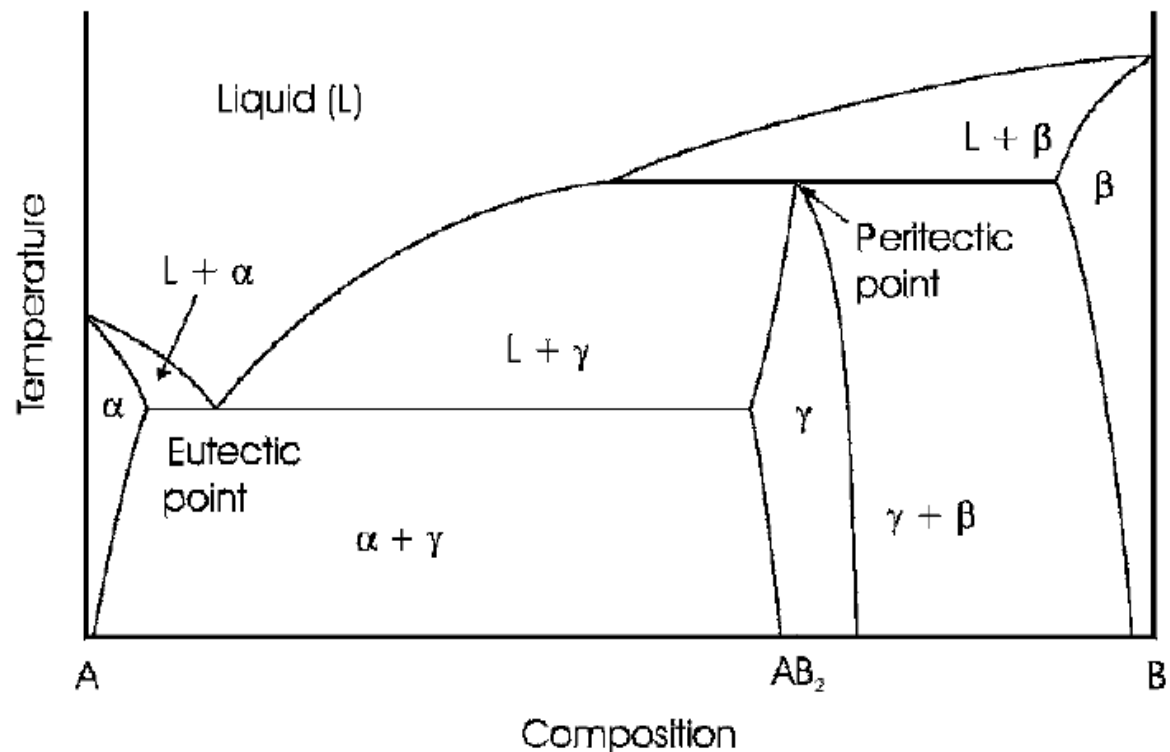
The solid melts “incongruently”.

Peritectic reaction:
usually for systems where the
components have very different T_M

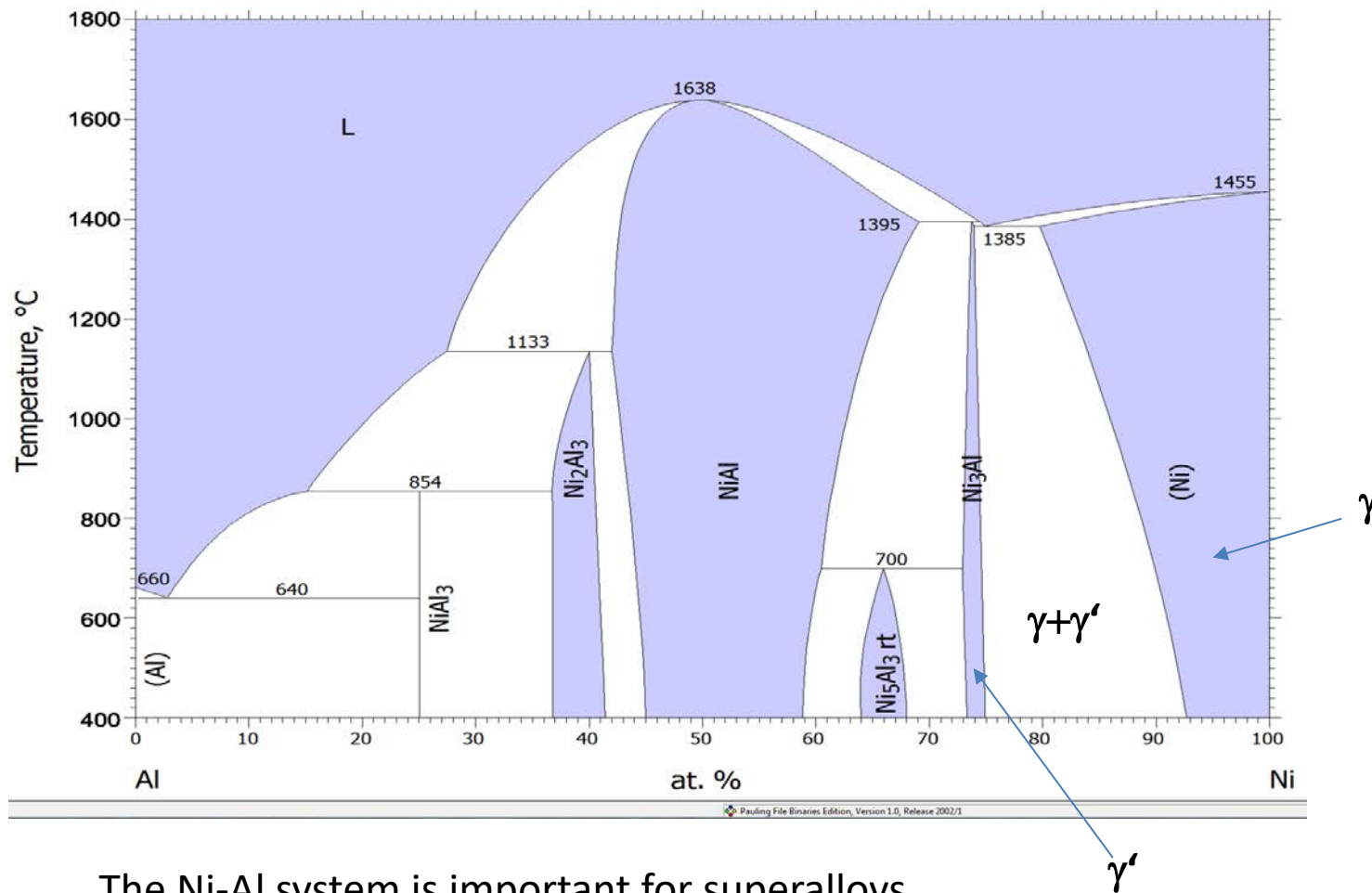
At the peritectic point:

On cooling: $L + \beta \rightarrow \gamma$

On heating: $\gamma \rightarrow L + \beta$

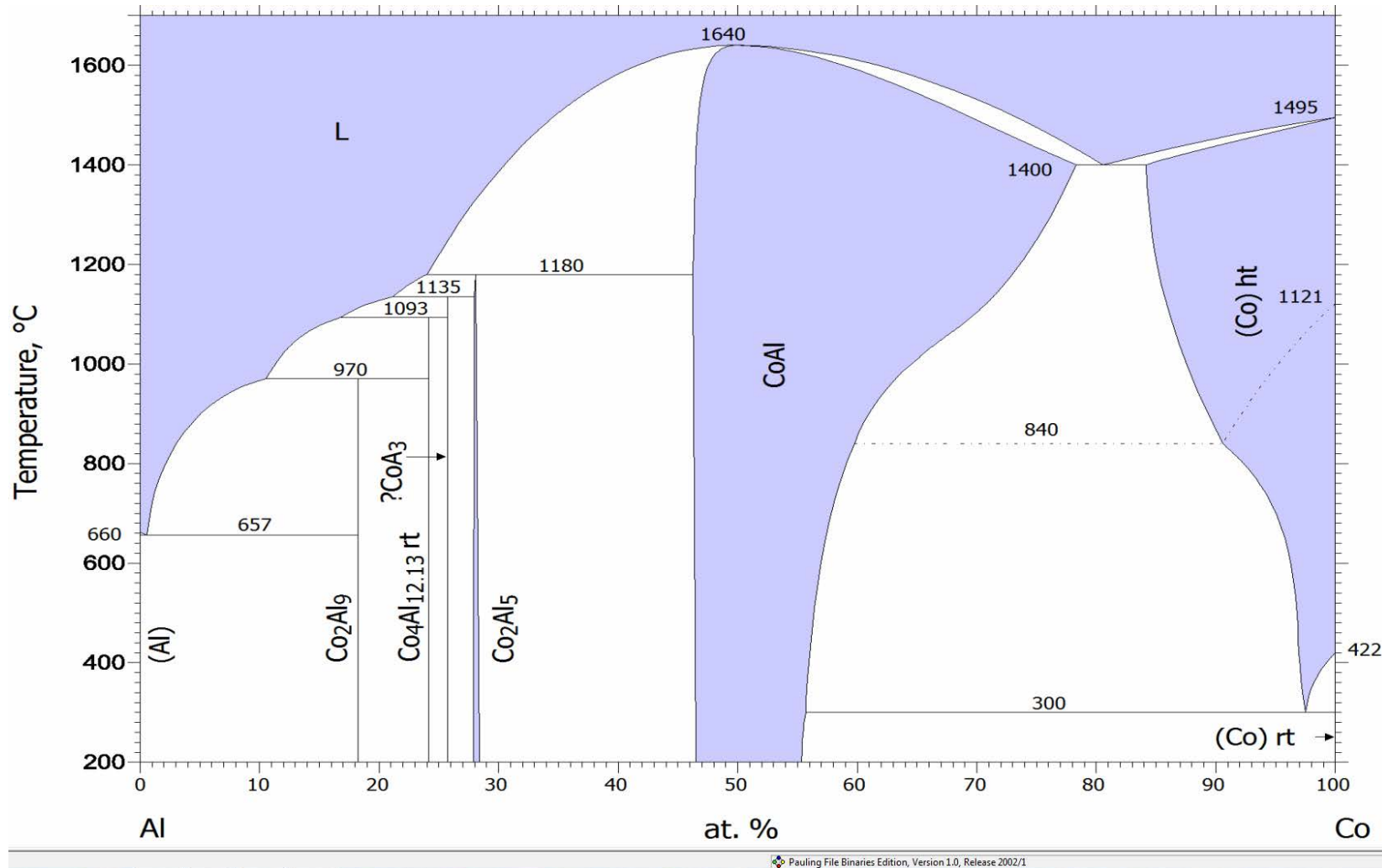


Binary system with limited mutual solubility, line compounds and alloys with existence range



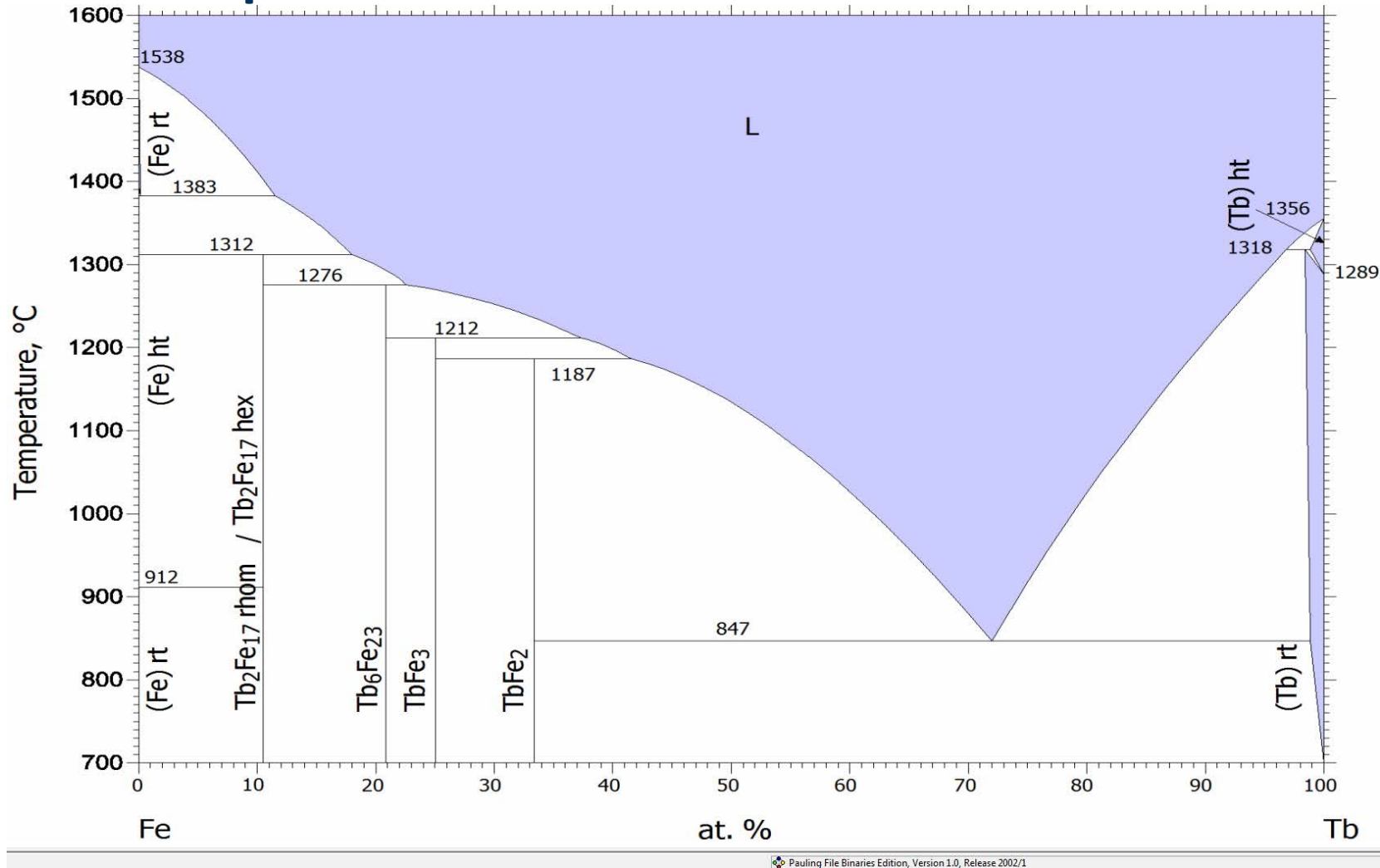
The Ni-Al system is important for superalloys

Binary system with limited mutual solubility, line compounds and alloys with existence range



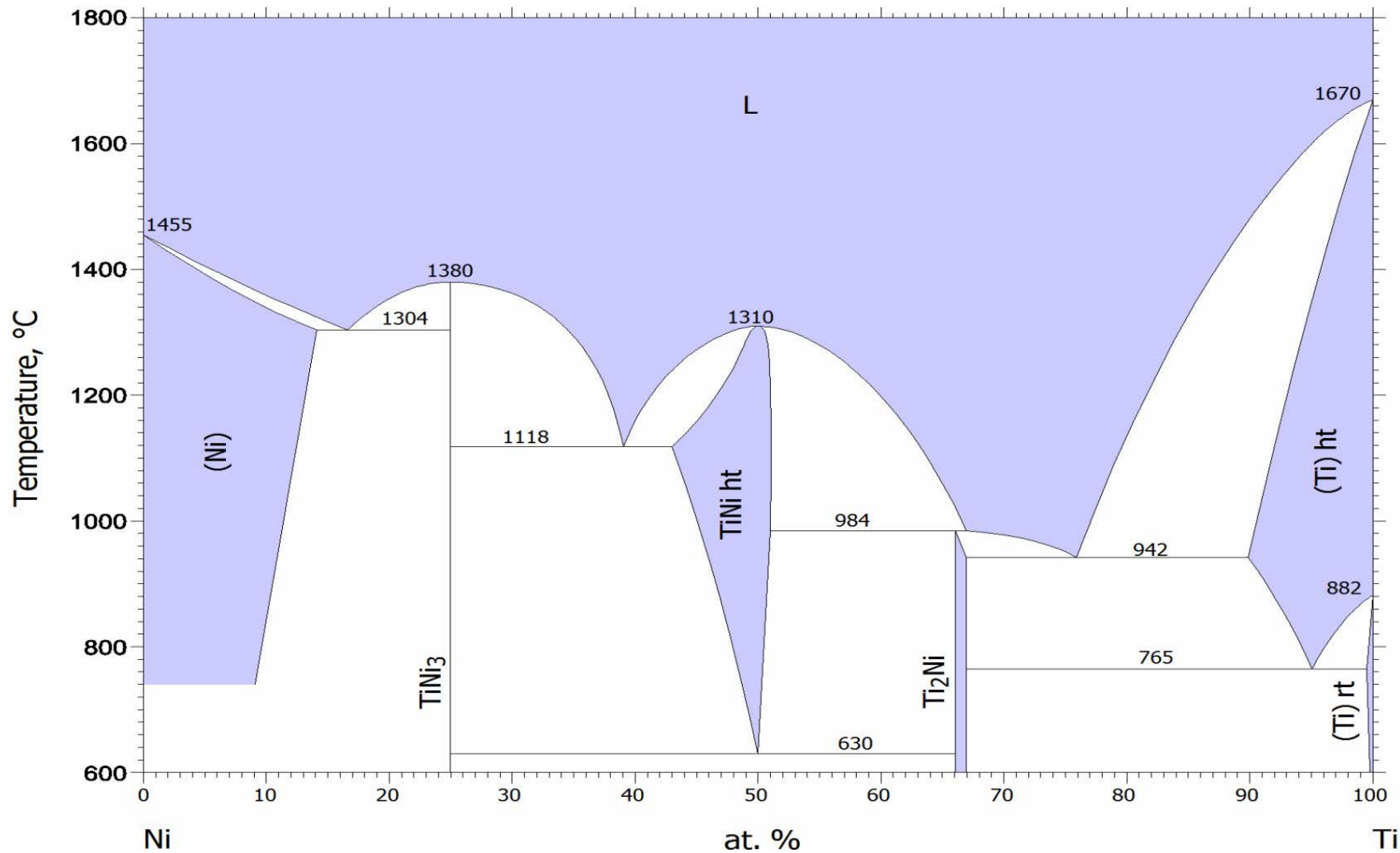
The Co-Al system is important for superalloys

Example of a binary system with intermetallic line compounds



Laves phase: TbFe₂ (magnetostriction)

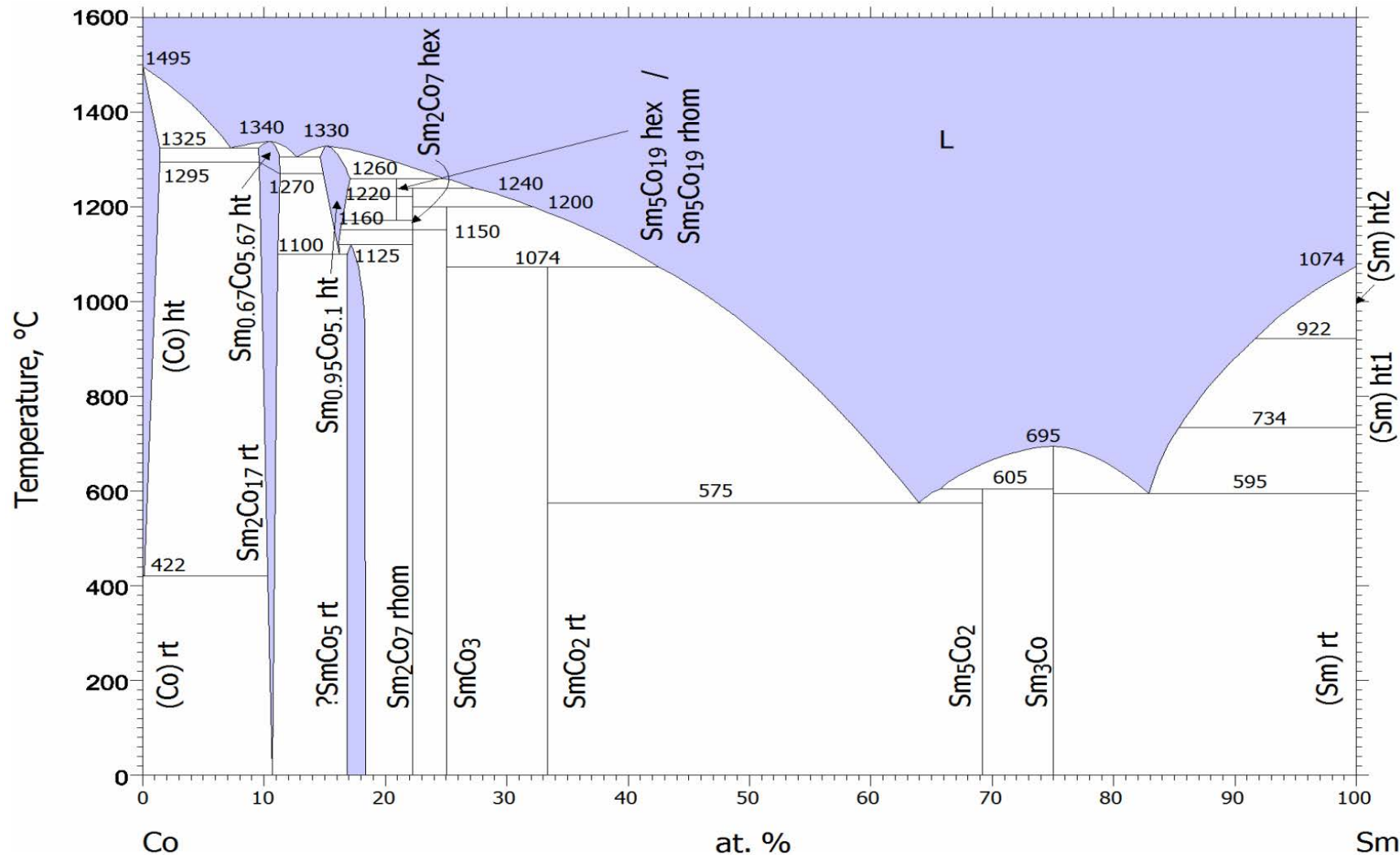
Example of a binary system with intermetallic phases



Pauling File Binaries Edition, Version 1.0, Release 2002/1

B2 phase: TiNi , shape memory alloys

Example of a binary system with intermetallic phases: hard magnets



Pauling File Binaries Edition, Version 1.0, Release 2002/1

The phases $SmCo_5$ and Sm_2Co_{17} are important for hard magnets

Example of a binary system with intermetallic phases: hard magnets

SmCo₅: hexagonal

Sm₂Co₁₇: hexagonal, rhomboedric,
Sm(Co_{0.67}Fe_{0.23}Cu_{0.08}Zr_{0.02})_{8.35}

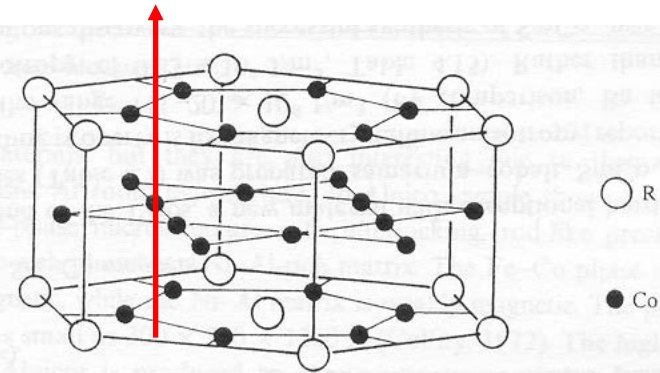


Fig. 6.28. The hexagonal unit cell of SmCo₅ (CaCu₅ type).

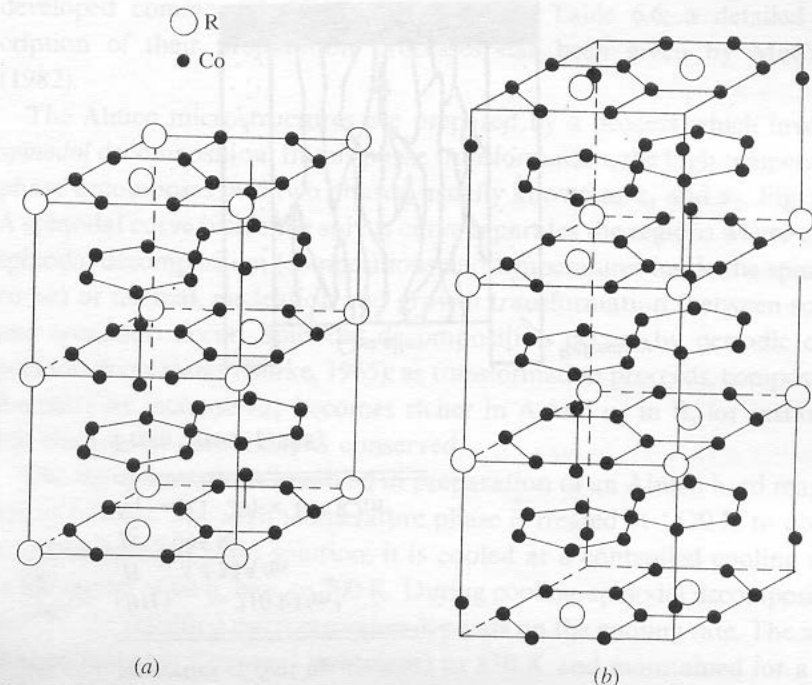
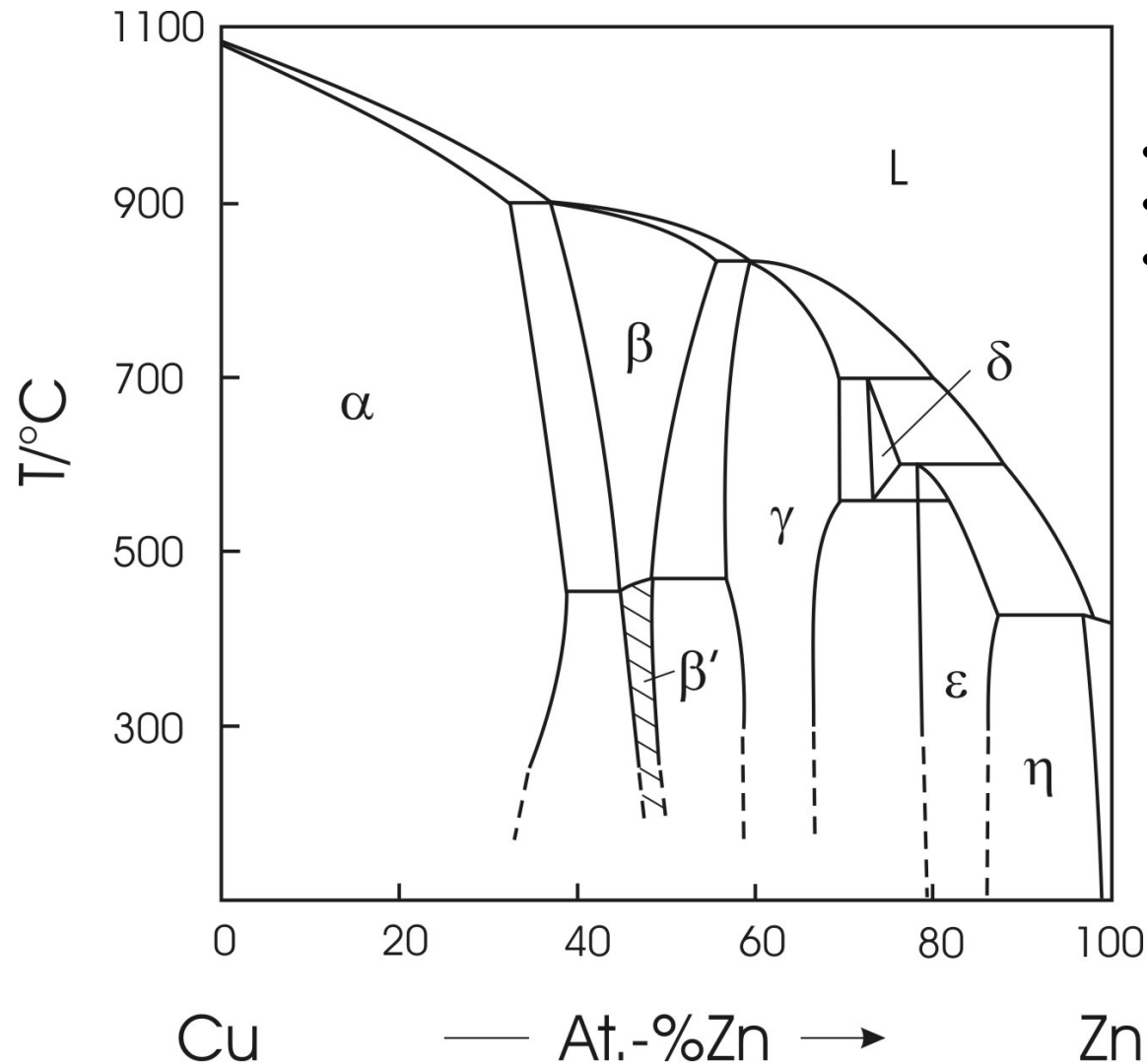


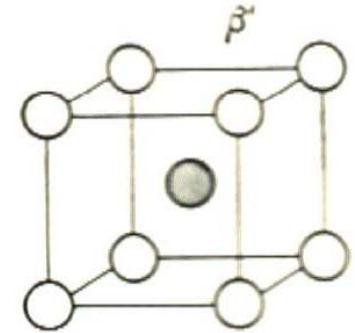
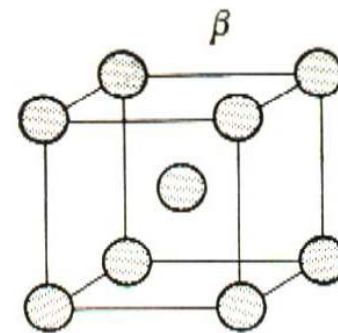
Fig. 6.29. (a) Hexagonal and (b) rhombohedral structures of Sm₂Co₁₇ compounds.

Hume-Rothery phases in the Cu-Zn system

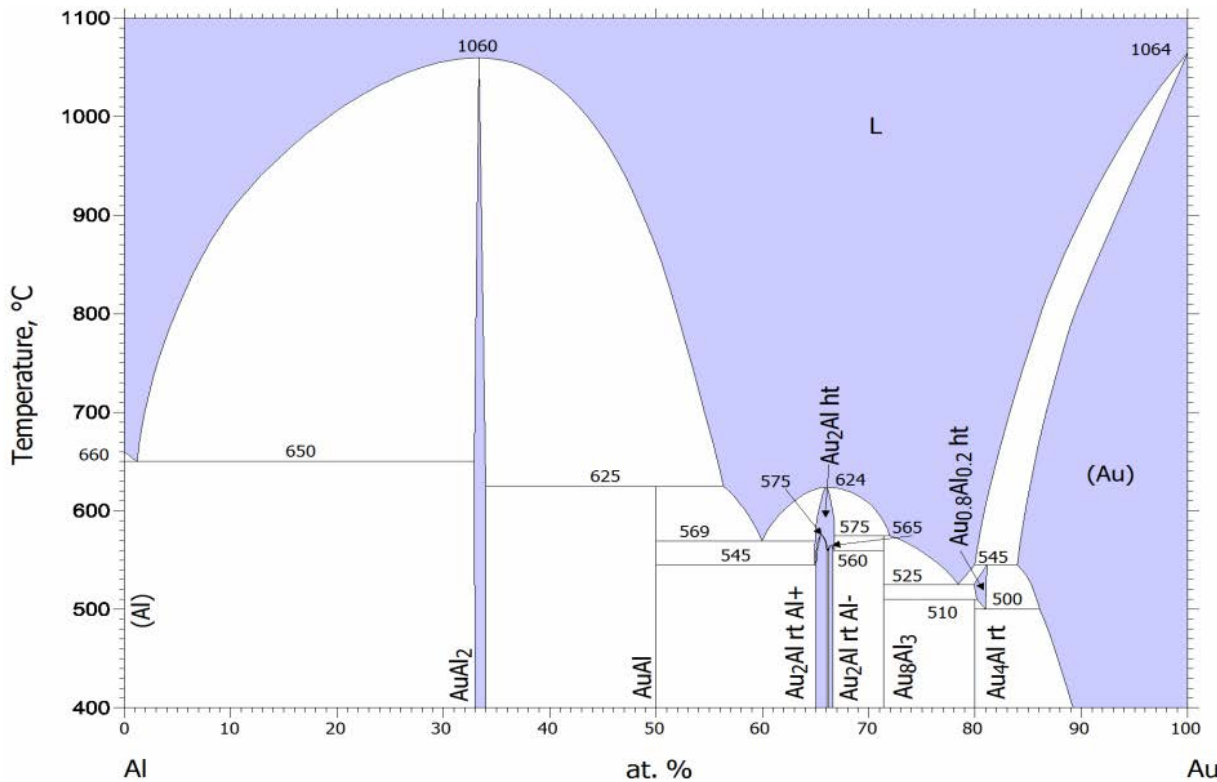
(valence electron phases)



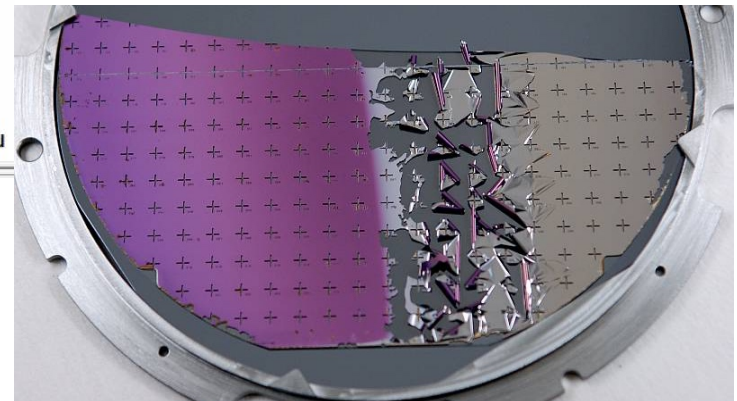
- a series of peritectic reactions
- phases are named consecutively
- order transformation (β , β')



“Purple plague” in the Au-Al system



- „purple plague“ AuAl₂
- “white plague” Au₅Al₂
- further phases
AuAl, Au₂Al, Au₄Al



Gas metal systems

Gas metal systems:

- during processing
- during service
- high-temperature oxidation
- hot-gas corrosion

Metal hydrogen systems

(e.g. Pd-H, Mg-H)

Nitrides

Oxides

Determination of Phase Diagrams for Hydrogen-Containing Systems

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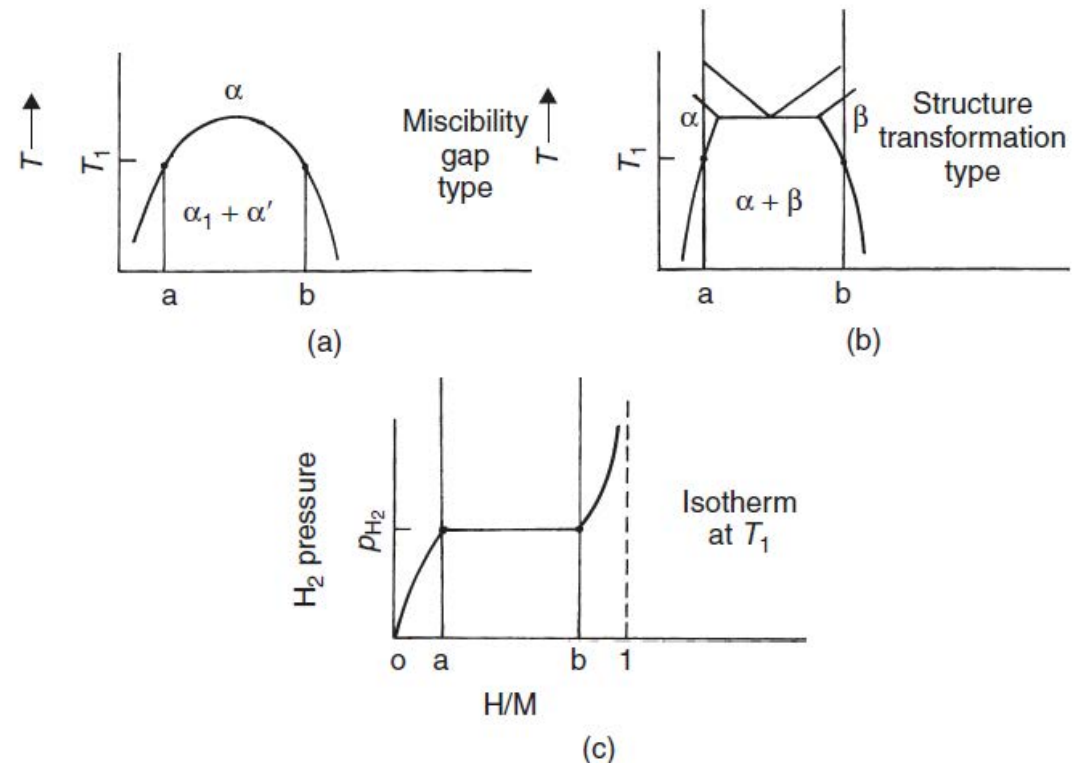
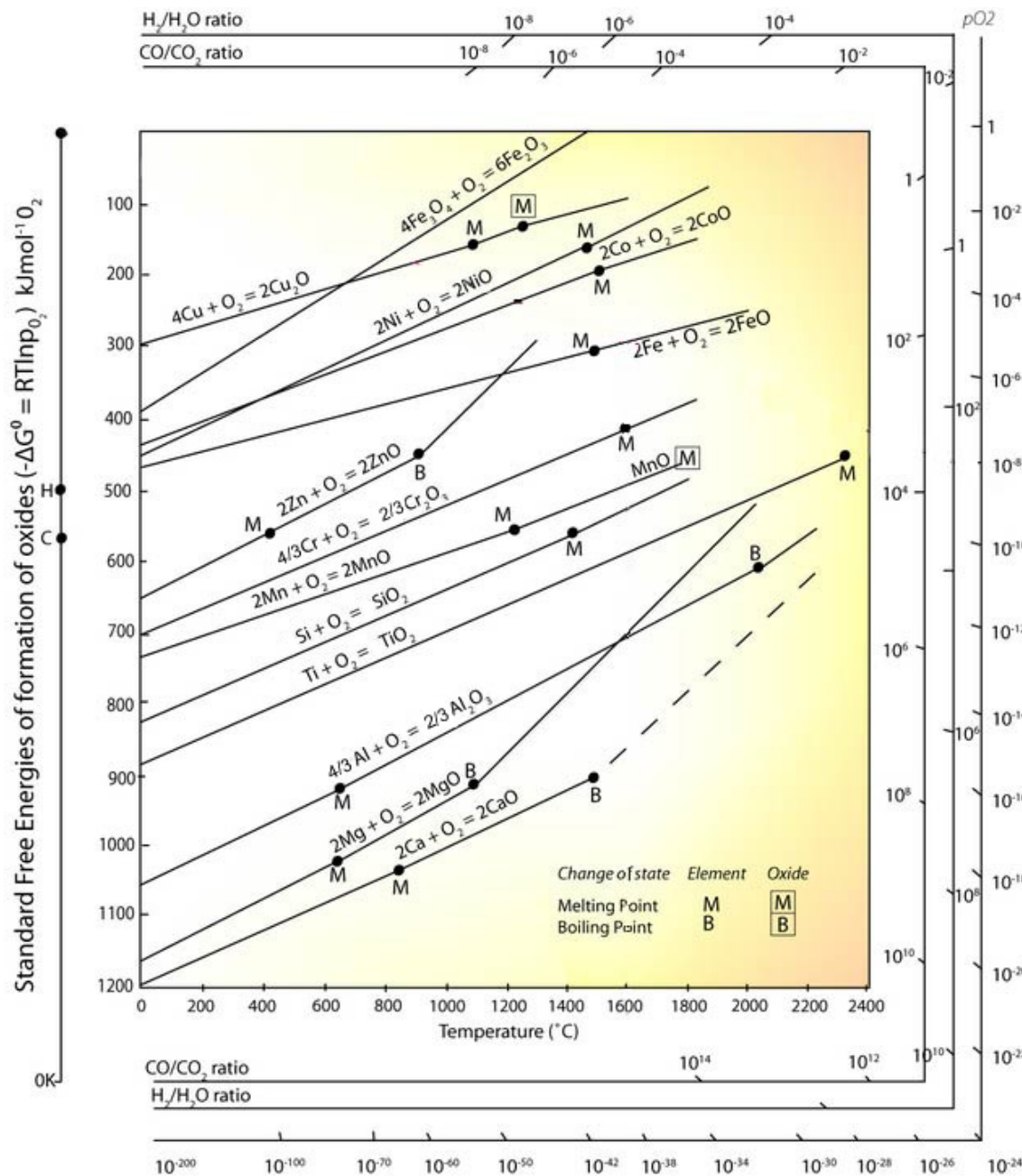
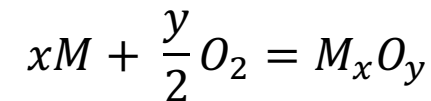


Figure 15.1 Schematic diagrams showing (a) a miscibility gap system, (b) a structural transformation system and (c) an isotherm corresponding either to (a) or (b) at temperature T_1 [8].

Gas metal systems: Ellingham diagram for oxidation of metals



Gibbs energy ΔG – temperature T diagram for metals in equilibrium with their oxides



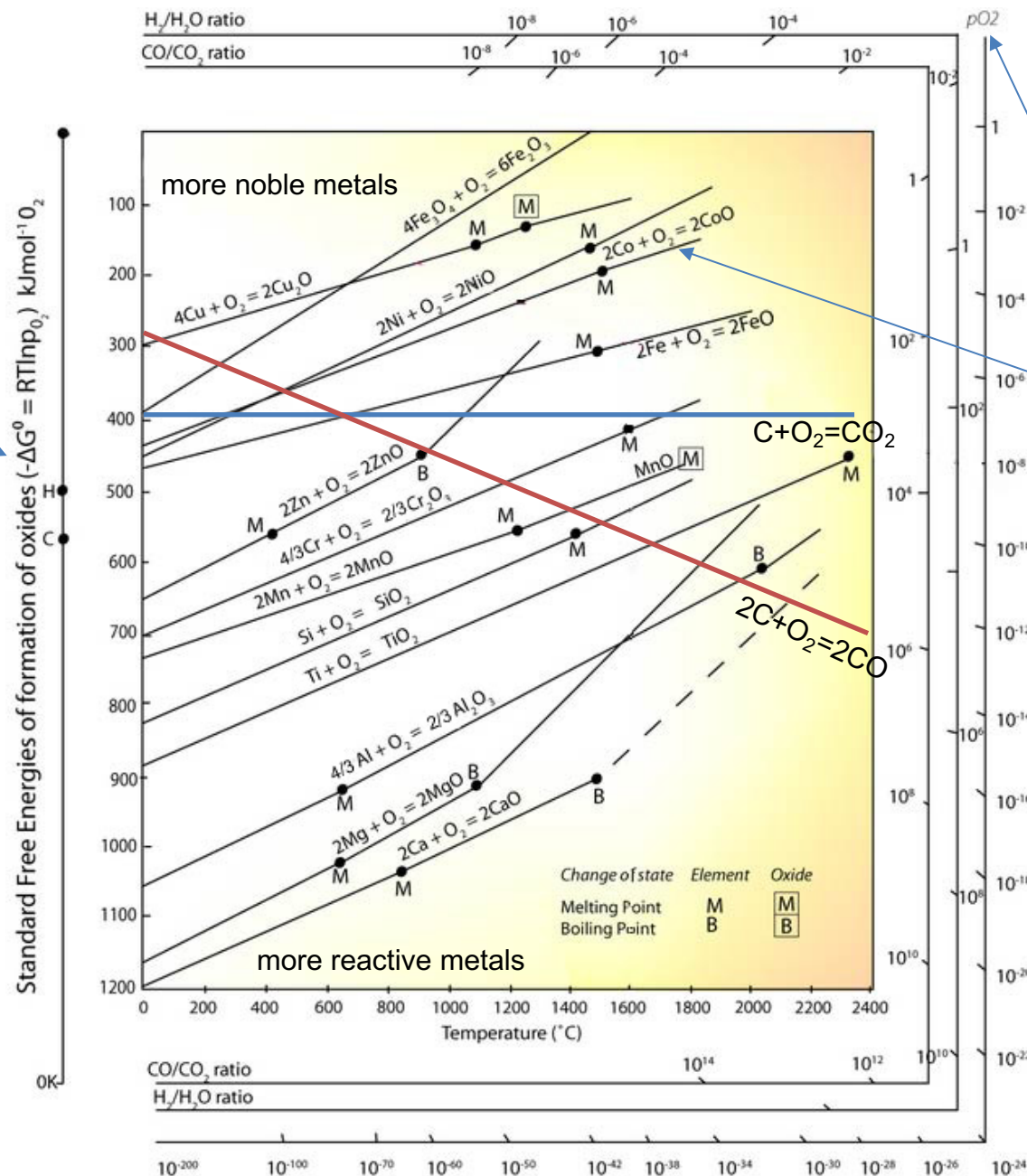
$$\Delta G^0 = \frac{y}{2} RT \ln p_{O_2}$$

Used for:

- „ease“ of oxidation/reduction (but only thermodynamics, no kinetics)
- Determine partial pressure of oxygen in equilibrium with metal oxide at given T
- Determine ratio of reducing gas mixtures (CO , H_2)
- Determine which metal can reduce which metal oxide(s)
 - E.g. Mg can reduce TiO_2

http://web.mit.edu/2.813/www/readings/Ellingham_diagrams.pdf

Gas metal systems: Ellingham diagram for oxidation of metals



Free energy of formation is negative for most metal oxides

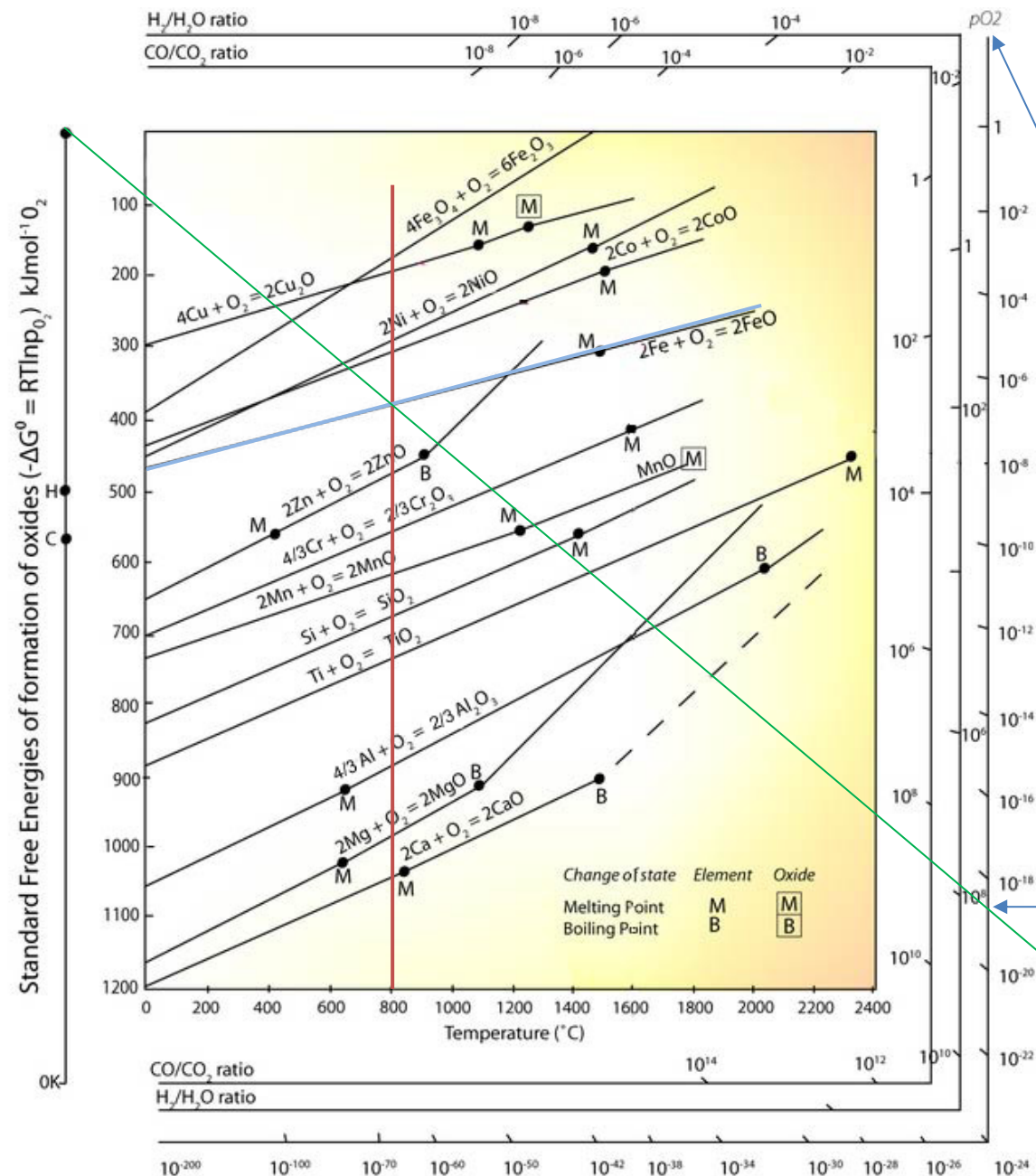
The lower the position of a metal's line, the greater is the stability of its oxide.

Which partial pressure of oxygen is in equilibrium? Higher p_{O_2} than equilibrium leads to oxidation, lower to reduction.

Positive slopes for metal + gas = solid as entropy is reduced

Carbon is an important reducing agent

Gas metal systems: Ellingham diagram for oxidation of metals

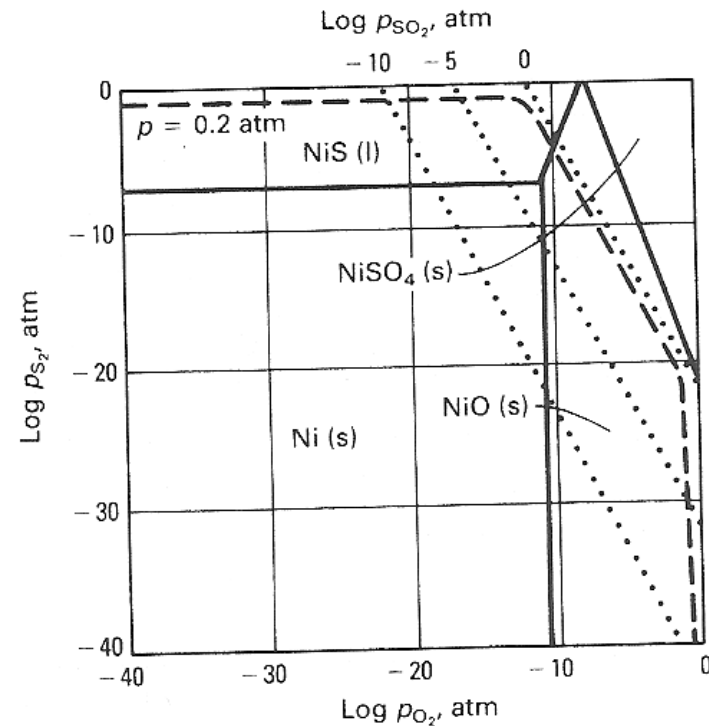


Which partial pressure of oxygen is in equilibrium? Higher p_{O_2} than equilibrium leads to oxidation, lower to reduction.

equilibrium partial pressure

Gas metal systems: isothermal stability diagrams

a metal and two gases



alloys and gas(es)

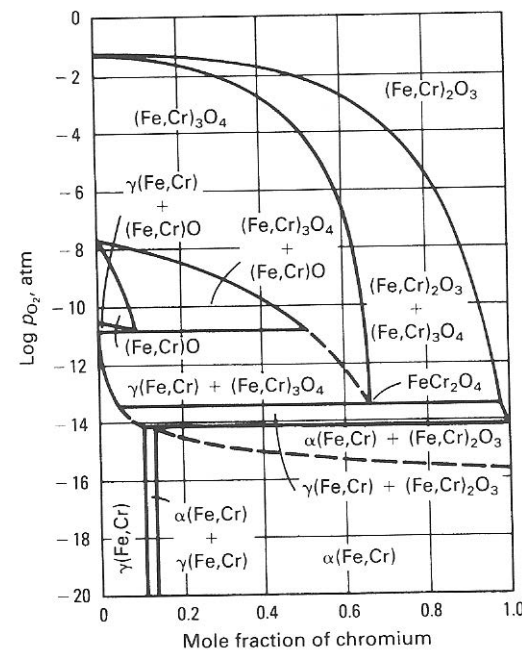
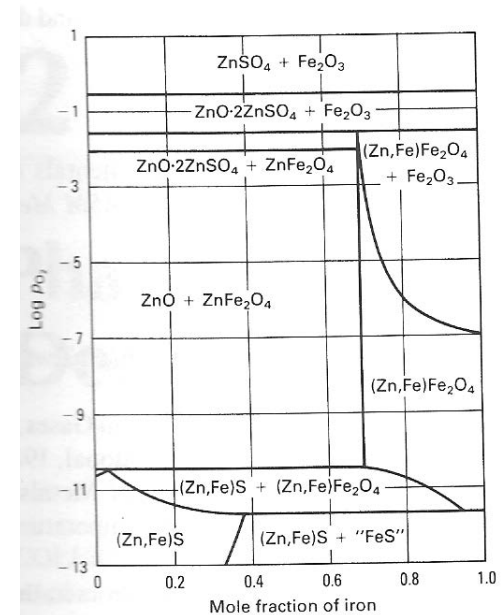


Fig. 11.3 Stability diagram for the Fe-Cr-O system at 1300 °C
Source: Ref 11.2



The Fe-Zn-S-O system for $p = 1$ atm at 890 °C (1635 °F). Source: Ref 11.4 as published in Ref 11.2

$p_{SO_2} =$
const.

The Ni-O-S system at 977 °C (1790 °F). Source: Ref 11.3 as published in Ref 11.2

also: Kellogg diagram

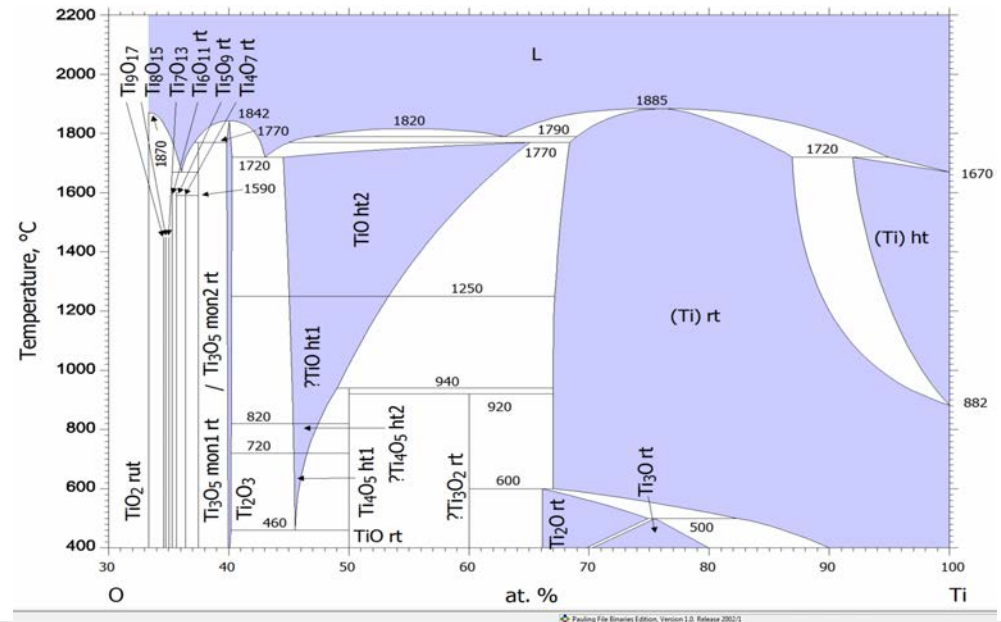
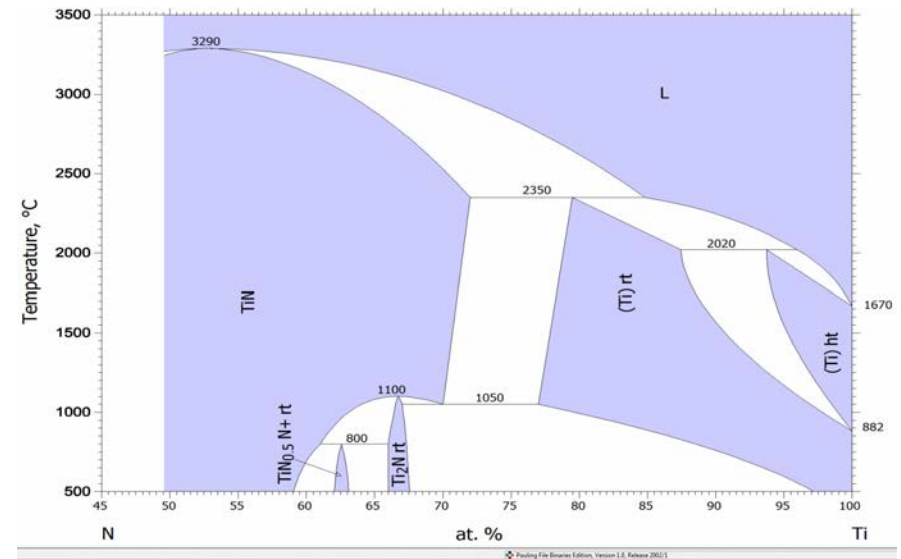
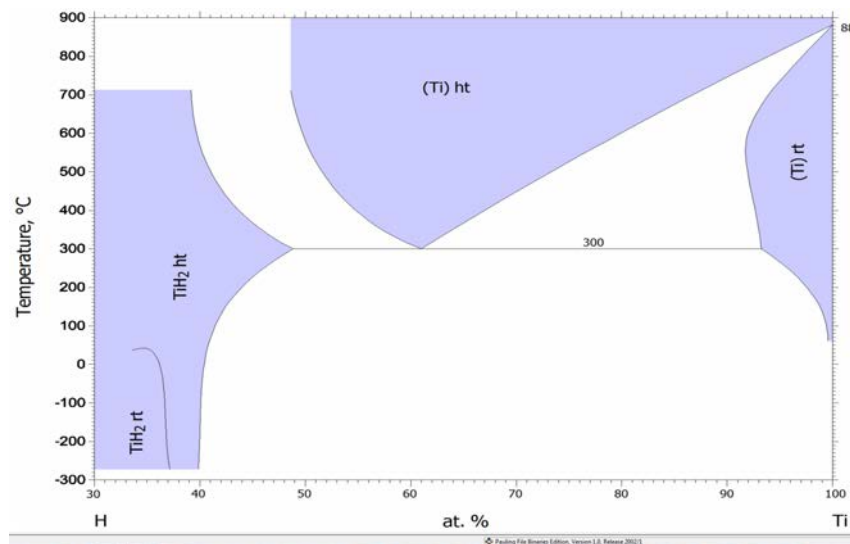
predominance diagram: phase fields show predominant phase,
other phases may also be present

Gas metal systems: Ti systems

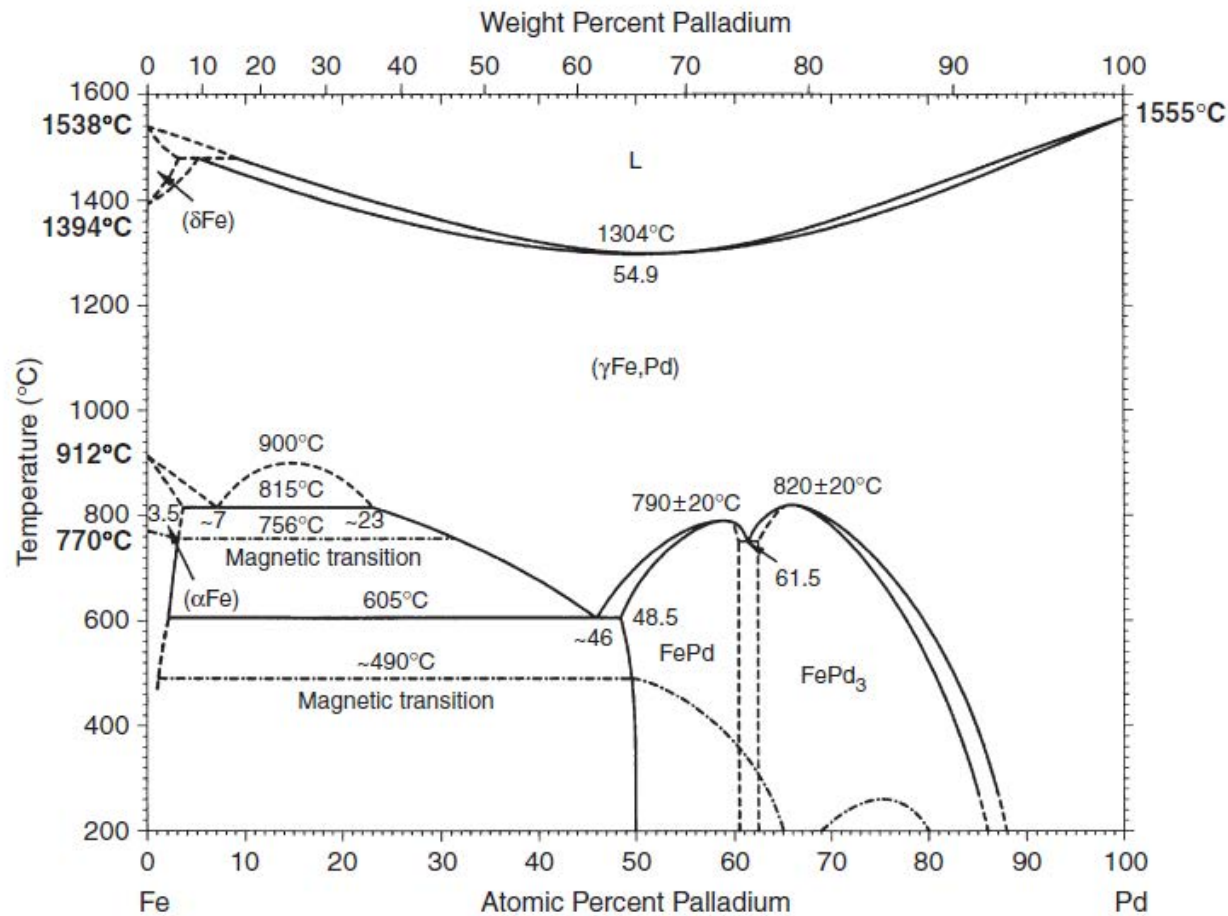
TiN: hard coating material

TiO₂: numerous applications, e.g. catalyst

TiH₂: hydrogen storage

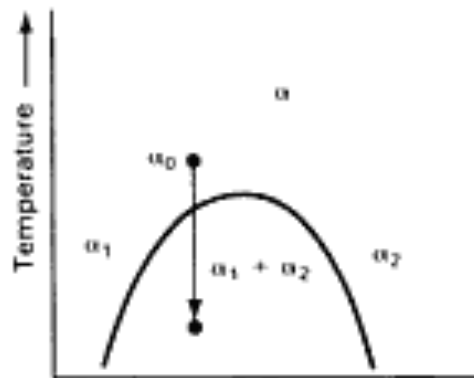


Further information in phase diagrams: magnetic transitions

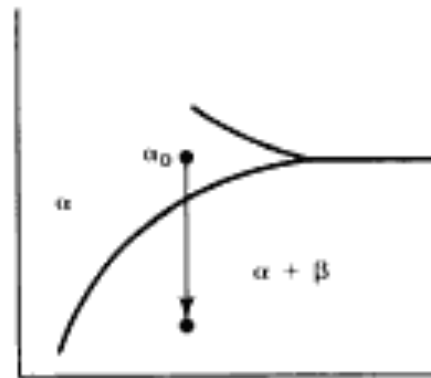
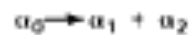


metastable Fe₇₀Pd₃₀:
ferromagnetic shape
memory alloy

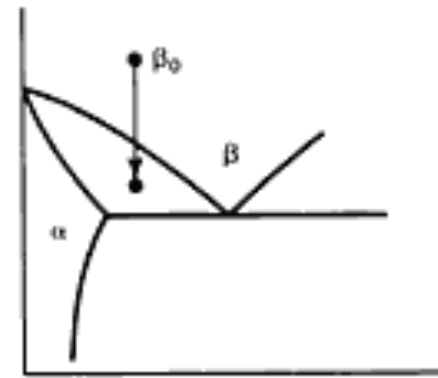
Examples of binary phase diagrams that give rise to precipitation reactions



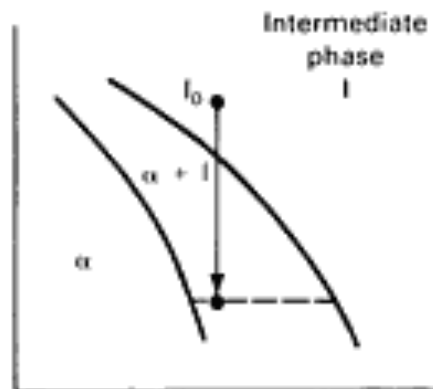
Miscibility gap



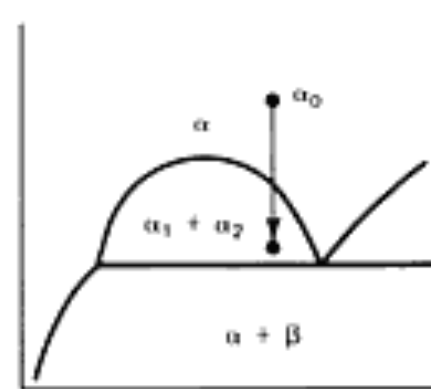
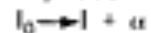
Sloping solvus:
decreasing solid
solubility with
decreasing temperature



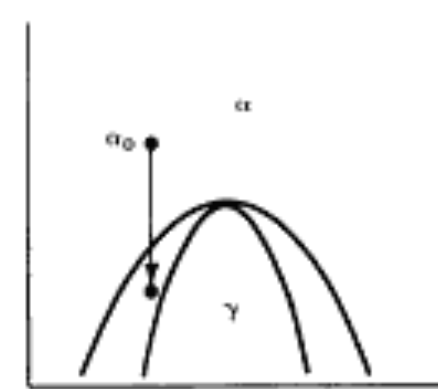
Proeutectoid
reaction



Intermediate
phase



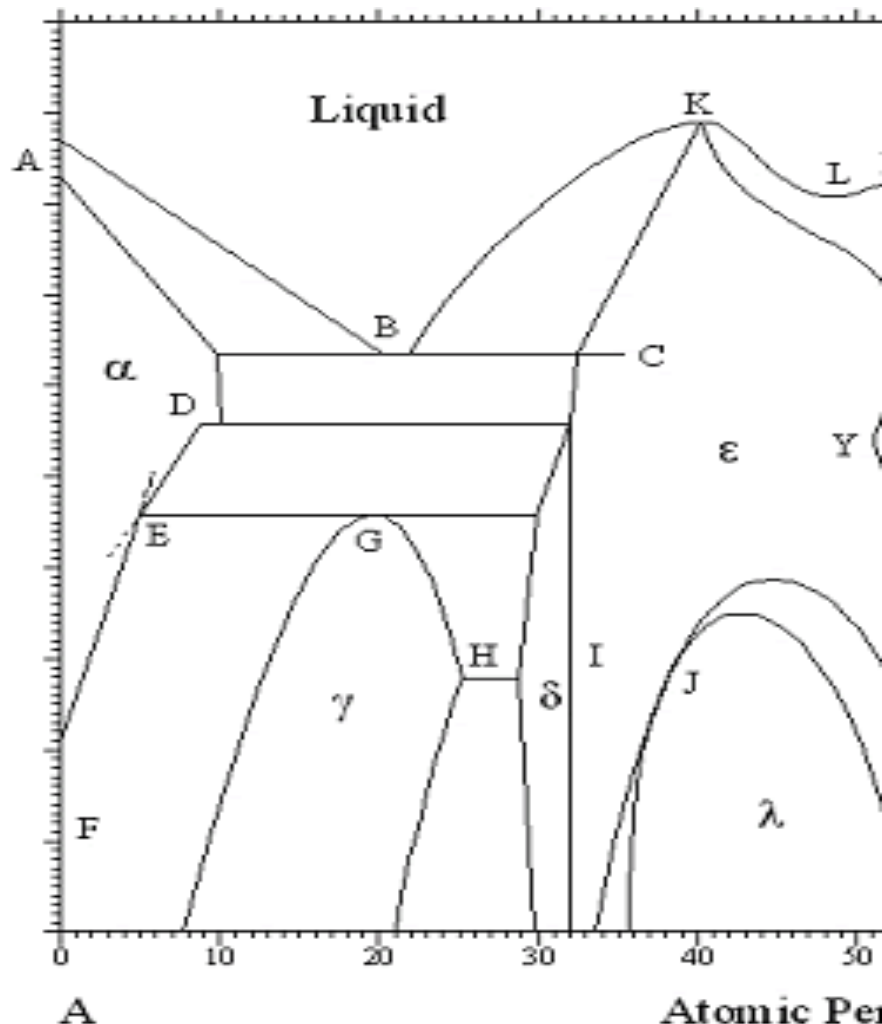
Promonotectoid; similar
to miscibility gap



Heterogeneous
ordering; γ is
an ordered phase



Summary of impossible phase relationships in published phase diagrams



A: The liquidus and solidus must meet at the melting point of the pure element.

B: Two liquidus curves must meet at one composition at a eutectic temperature.

C: A tie line must terminate at a phase boundary.

D: Two solvus boundaries (or two liquidus, or two solidus, or a solidus and a solvus) of the same phase must intersect at one composition at an invariant temperature.

E: A phase boundary must extrapolate into a two-phase field after crossing an invariant point.

F: A two-phase field cannot be extended to a pure element end.

G: Two boundaries of γ must not be continuous at the invariant temperature. They must cross one another.

H: An invariant temperature line should involve equilibrium among three phases.

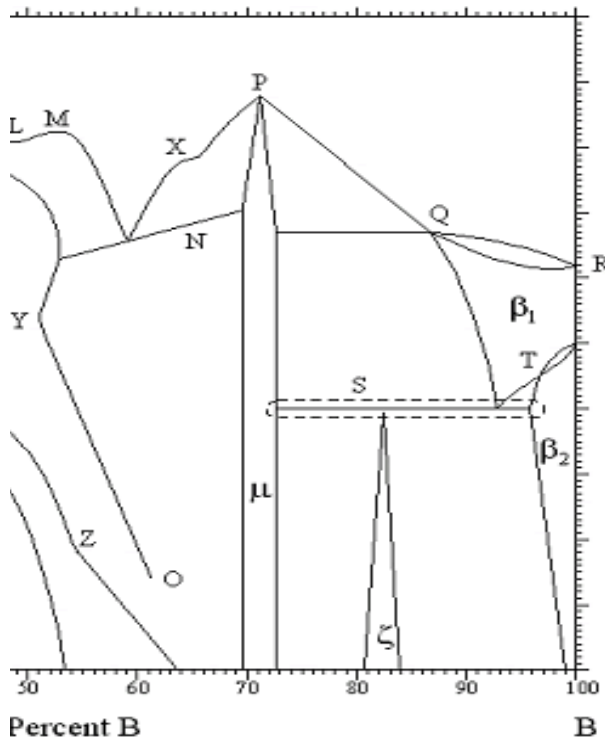
I: There should be a two-phase field between two single phase fields.

J: When two phase boundaries touch at a point, they should touch at an extremity of temperature.

K: A touching liquidus and solidus (or any two touching boundaries) must have a horizontal common tangent at the congruent point. In this case, the slope of the solidus appears to be discontinuous at the melting point.

L: A local minimum point in the lower part of a single-phase field cannot be drawn without an additional boundary in contact with it (minimum congruent point or monotectic reaction in this case).

Summary of impossible phase relationships in published phase diagrams



M: A local maximum point in the lower part of a single-phase field cannot be drawn without a monotectic, monotectoid, syntactic, and syntectoid reaction occurring at a lower temperature. Alternatively, a solidus curve must be drawn to touch the liquidus at point M. (If the maximum is not local, as in a miscibility gap, this is not a phase rule violation.)

N: The temperature of an invariant reaction must be constant. (The reaction line must be horizontal.)

O: A phase boundary cannot terminate within a phase field (except the case when the boundary is unknown beyond this point).

P: The liquidus should not have a discontinuous sharp peak at the melting point of a compound. (See exceptions below.)

Q: The compositions of all three phases at an invariant reaction must be different.

R: Temperatures of liquidus and solidus (or any two boundaries) must either increase or decrease together from one point on the pure element line as the content of a second element increases.

S: A four-phase equilibrium is not allowed in a binary system. (See exceptions below.)

T: Two separate phase boundaries that create a two-phase field between two phases in equilibrium should not cross one another.

Although phase rules are not violated, three additional unusual situations (X, Y, and Z) are also included in Fig. 27. These unlikely situations are discussed in the next section.

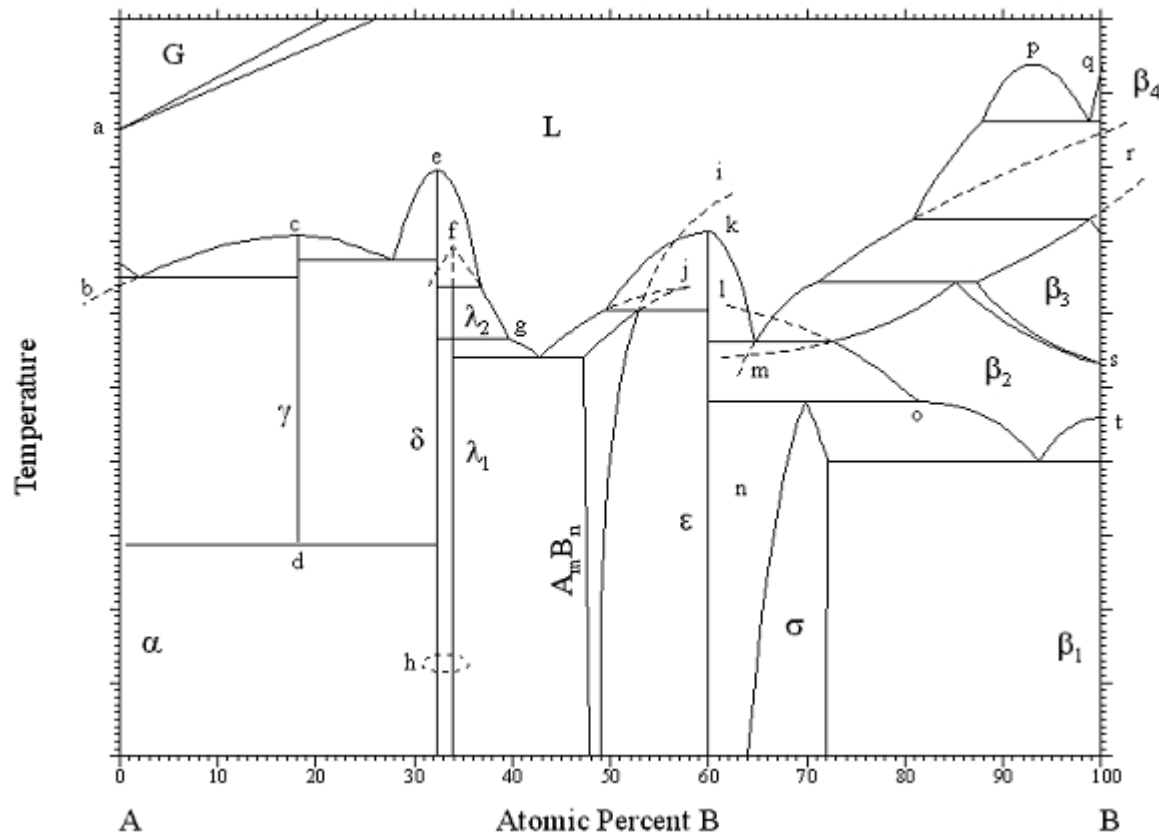
An additional problem, not shown in Fig. 27: would be a continuous solid solution phase between two phases with different crystal structures. For example, a fcc phase and a bcc phase cannot form a continuous phase. There must be a two-phase field between them.

Exceptions

A four-phase reaction in appearance, as S in Fig. 27, may occur if temperatures of two invariant reactions with an overlapping composition range are very close to one another.

A sharp peak as P may occur if this phase exists in the same molecular state in the liquid phase as it does in the solid state. The apparent sharpness of the peak varies depending on the degree of association of the liquid molecules.

Types of improbable phase boundaries



k: The liquidus is too asymmetric. According to the author's criterion, a liquidus is already too asymmetric if the liquidus width ratio to the left and right of a compound exceeds 2 to 3.

l: The transformation temperature of ϵ to β_2 should be higher than the melting point of ϵ . Otherwise, the β_2 phase is stable above point j.

m: Extrapolation of two boundaries of $L + \beta_2$ should not cross. Problem T of Fig. 27 occurs.

n: A two-phase field must be narrower at higher temperatures.

o: The slope is too flat to have a maximum point at the composition of ϕ .

p: The liquid miscibility gap is too close to the edge of a phase diagram.

q: The liquidus slope is too steep. The initial slope of a liquidus must conform to the van't Hoff relationship. If no solubility can be assumed for the solid phase, extrapolation of the initial liquidus should go through the horizontal axis at 0 K near approximately 110 at.%.
r: Extrapolation of two boundaries of $L + \beta_3$ should cross at the 100 at.% line, not at some composition exceeding 100 at.%. Problem A of Fig. 27.

s: Two phase boundaries should have different initial slopes.
t: The slopes of two phase boundaries are too far apart.

a: G + L two-phase field is too narrow. The opening angle of G + L at 0 at.% must be much larger because the heat of vaporization of an element is usually much greater than the heat of fusion.

b: Extrapolation of the liquidus should not cross the 0 at.% line. Otherwise, problem F of Fig. 27 occurs.

c: The liquidus of δ at point c is too flat in comparison with the liquidus of δ at point e. Problems c, d, and e are related. Because entropy of fusion of elements and compounds cannot differ much, curvatures of liquidus curves for compounds in a binary system must be similar. A phase with a sharper liquidus tends to decompose into two neighboring phases at low temperatures.

d: A compound with a flat liquidus is stable and will not decompose at low temperatures.

e: Liquidus at point e is too sharp in comparison with the liquidus at point c.

f: Extrapolation of the liquidus of λ_2 must have a peak at the composition of λ_2 . Otherwise, problem P of Fig. 27 occurs.

g: Change of liquidus slope associated with an allotropic transformation must be small.

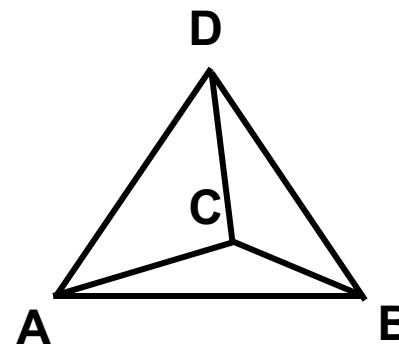
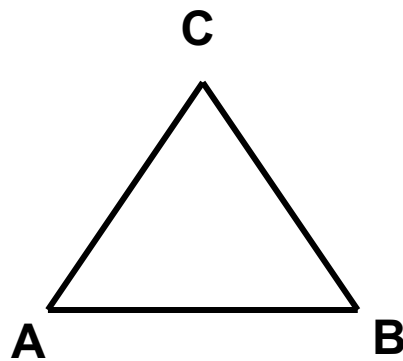
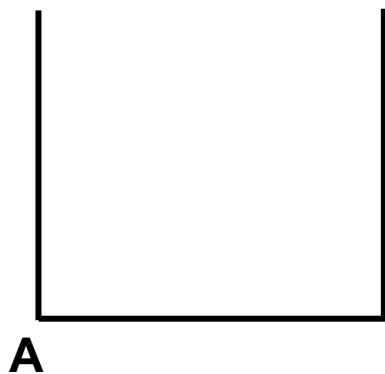
h: Two compounds having similar compositions cannot be stable over a wide temperature range.

i: A phase field of a compound cannot extend over a neighboring phase. Problem T of Fig. 27 occurs.

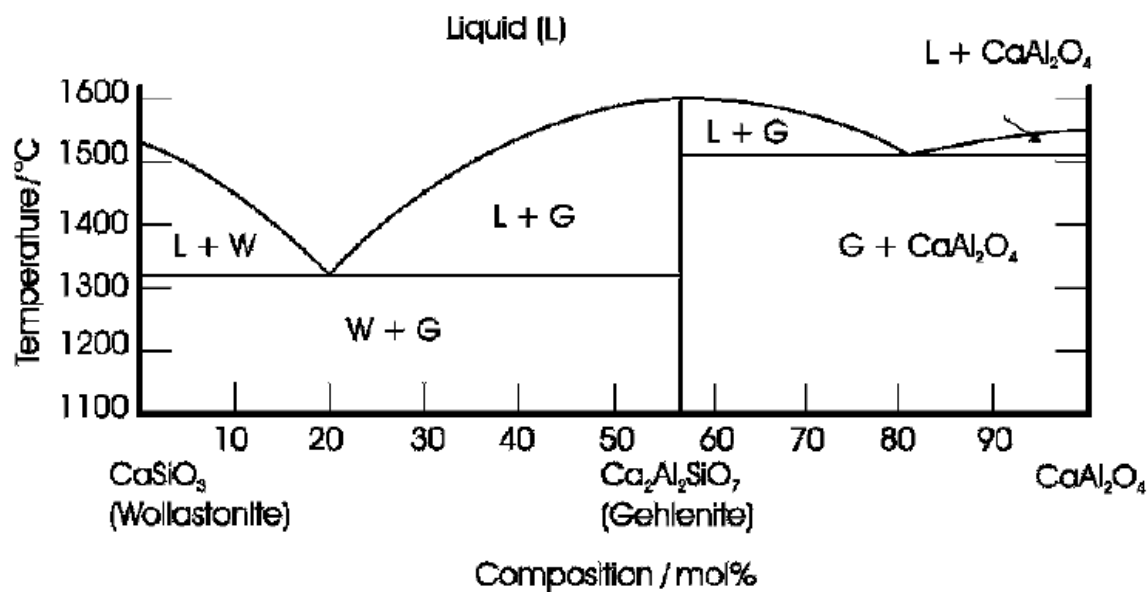
j: The congruent melting point of $A_m B_n$ compound is too far away from its stoichiometric composition.

Multinary systems

<u>Number of components</u>	<u>system</u>
1	unary
2	binary
3	ternary
4	quaternary
5	quinary



Example of a phase diagram of ceramics with a quaternary line compound



congruent
melting

Figure 4.14 The wollastonite–calcium aluminate (Ca–SiO₃–CaAl₂O₄) phase diagram showing the intermediate phase gehlenite, Ca₂Al₂SiO₇

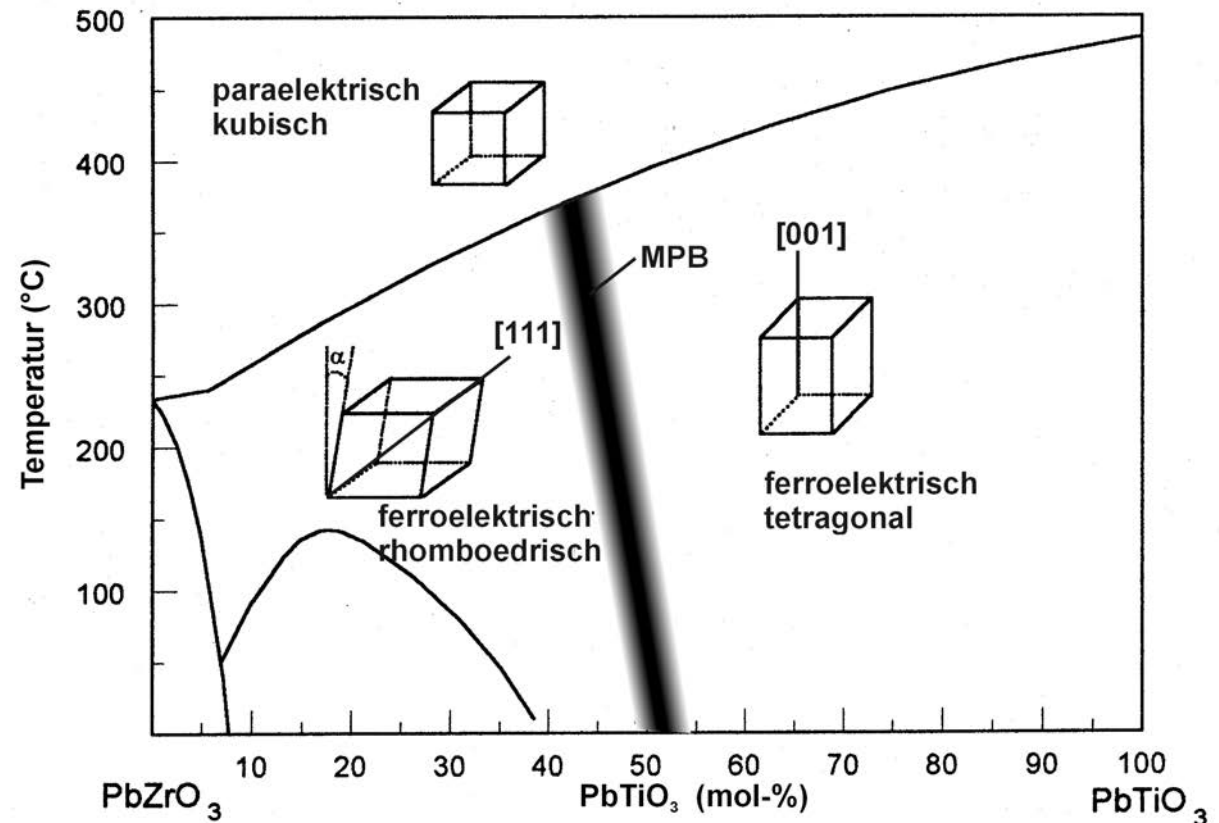
Phase diagram of functional ceramic PZT

solid solution of lead
titanate and lead zirconate

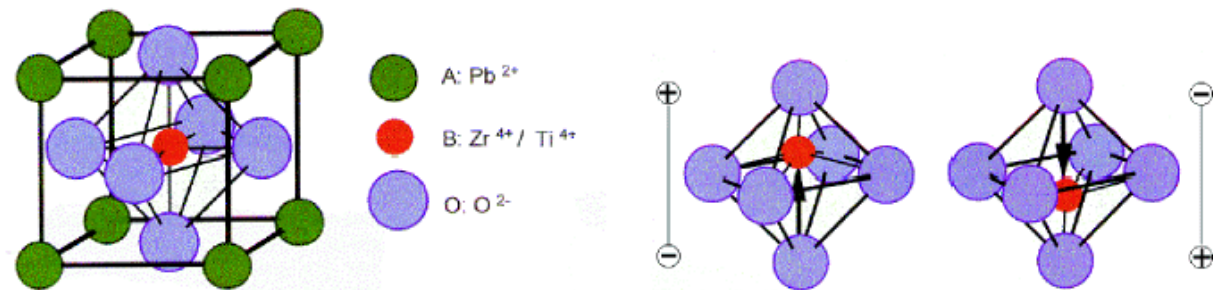
MPB:

morphotropic phase boundary

ferroelectric (piezoelectric)
sensors and actuators
(transducers)



phase
transformation of
perovskite structure



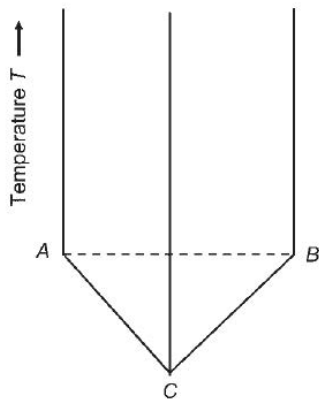
1.3 a: unpolar ($T > T_C$)

1.3 b: polar ($T < T_C$)

Ternary phase diagrams

Isothermal and isopleth sections

ternary: $C=3$,
shown in **equilateral triangle**
What about T and pressure?
most often: isobaric
equilateral prism



Space model for ternary phase diagrams

vertical section (isopleth)

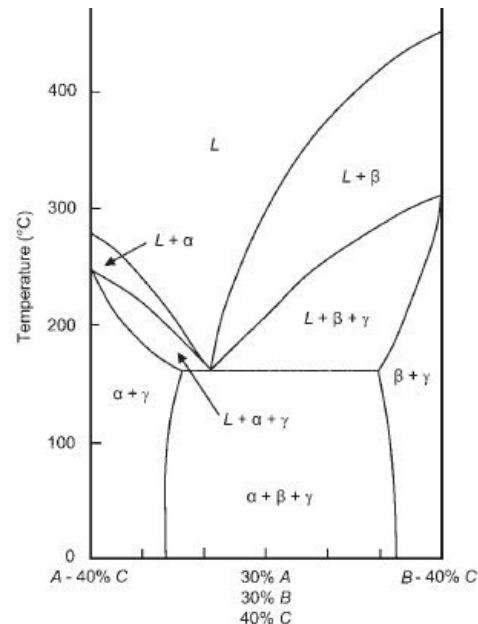
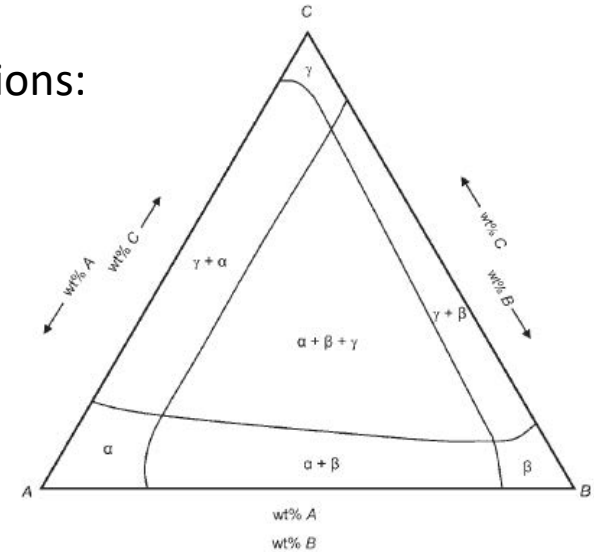
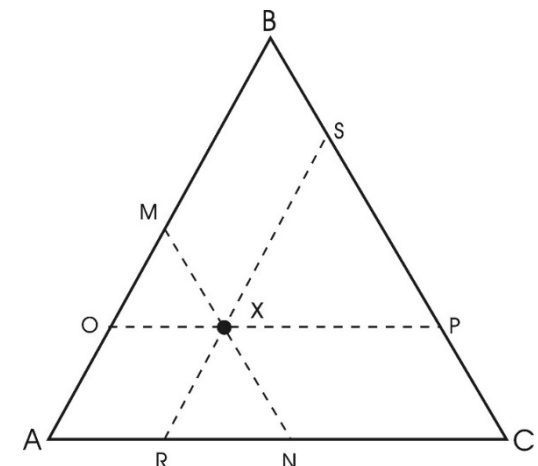


Fig. 10.5 Isopleth through hypothetical ternary phase diagram at a constant 40% C. Adapted from Ref 10.1

Isothermal sections:



phase fields



determining compositions

Free energies of a liquid and three solid phases of a ternary system

three binary eutectics

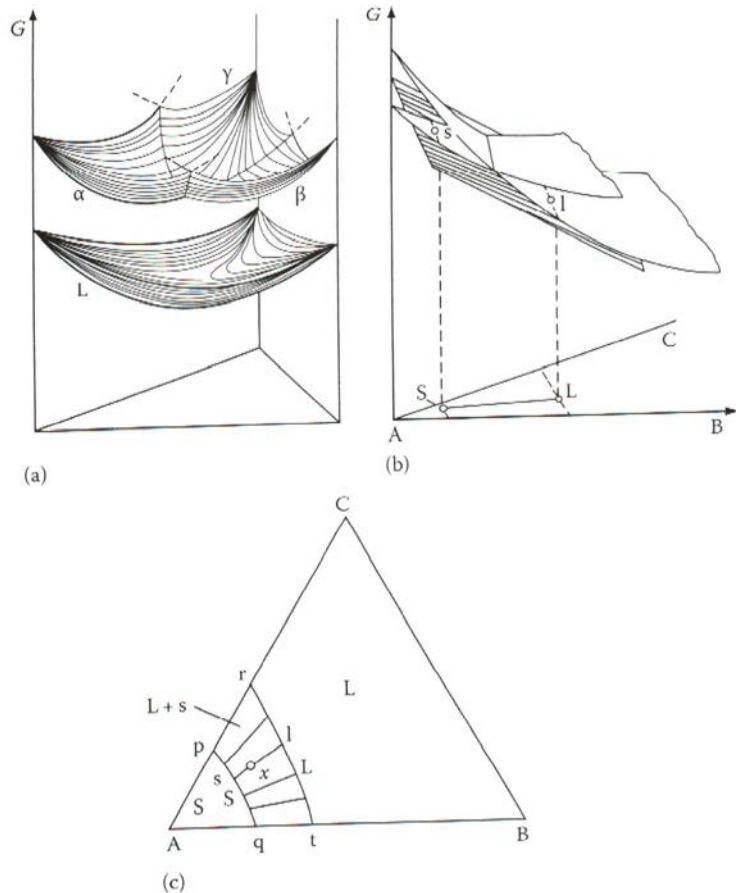
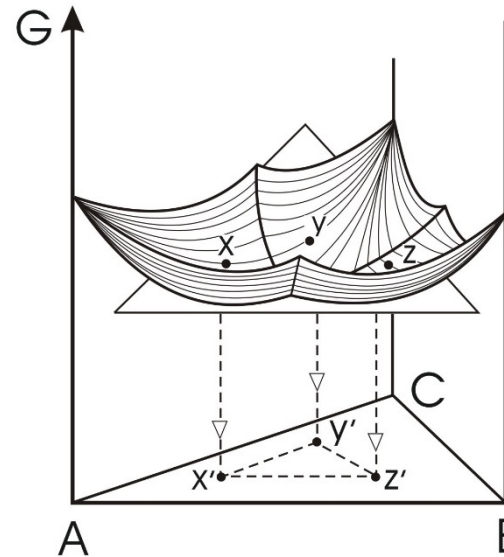


FIGURE 1.41

(a) Free energies of a liquid and three solid phases of a ternary system. (b) A tangential plane construction to the free energy surfaces defines equilibrium between s and l in the ternary system, (c) Isothermal section through a ternary phase diagram obtained in this way with a two-phase region (L1S) and various tie-lines. The amounts of l and s at point x are determined by the lever rule. (After P. Haasen, *Physical Metallurgy*, Cambridge University Press,



three phase equilibrium

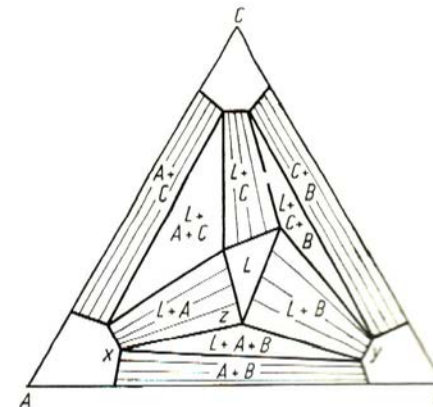
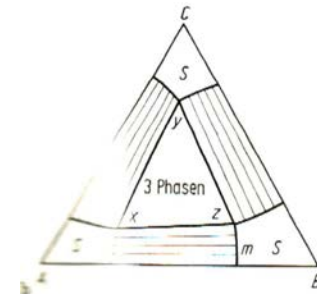


Abb. 5.23. Schnitt durch ternäres Zustandsdiagramm bei Temperatur oberhalb der ternären eutektischen, aber unterhalb aller binären T_E

Ternary phase diagrams

for enlargements

of small composition ranges

the rectangular diagram is useful

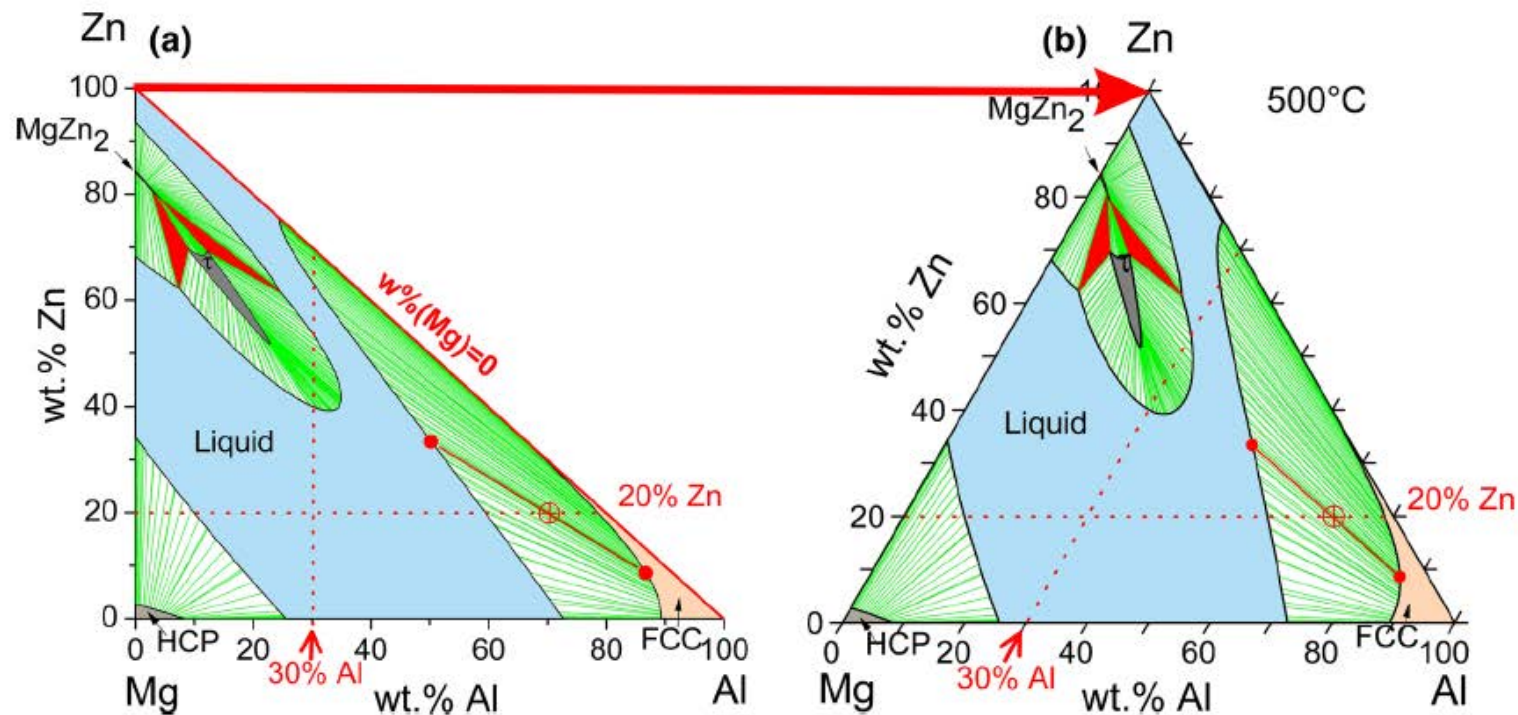
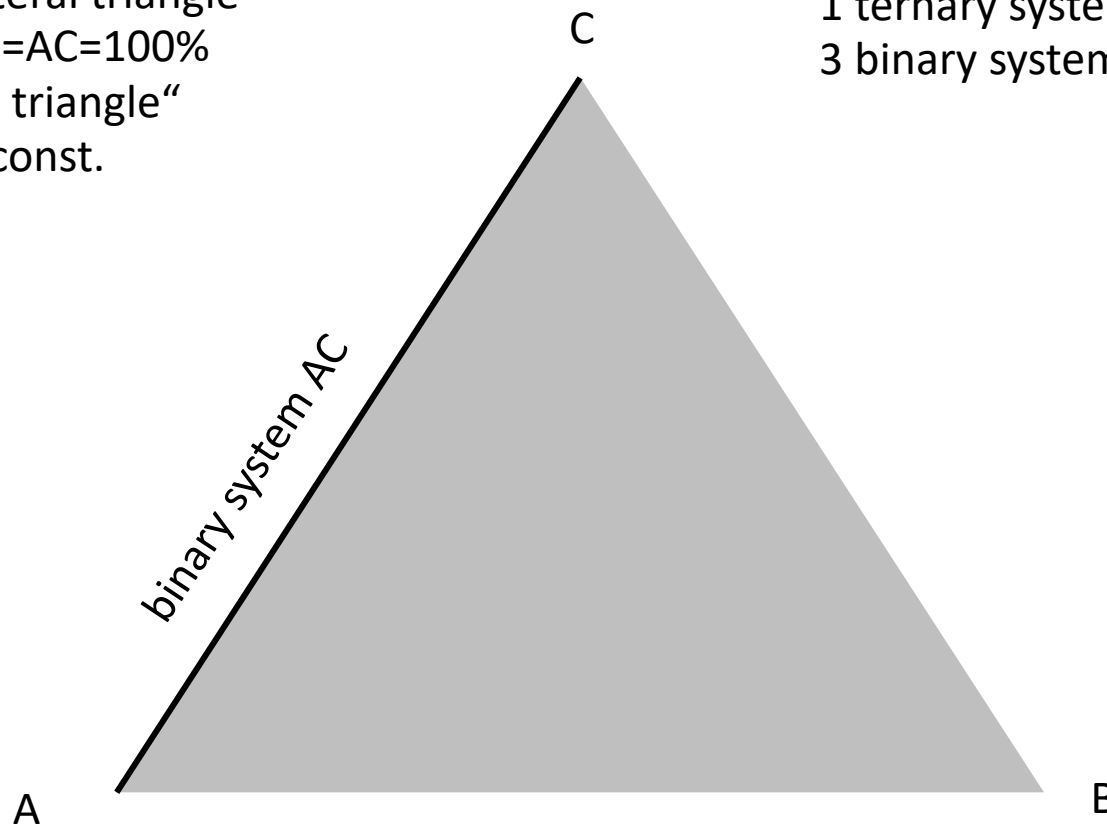


Fig. 7 Isothermal section of the Mg-Al-Zn phase diagram at 500 °C displayed in (a) rectangular coordinates, and (b) in the equilateral composition triangle. The L + FCC tie line passing through the state point of the alloy Mg₁₀Al₇₀Zn₂₀ (wt.%) is highlighted

Ternary phase diagram (isobaric, isothermal)

equilateral triangle
 $AC=AB=BC=100\%$
„Gibbs triangle“
 $p, T = \text{const.}$



1 ternary system, $C=3$ (A,B,C)
3 binary systems: A-B, A-C, B-C

Phase rule

$$F+P=C+2$$

$$p=\text{const}$$
$$F+P=C+1$$

Ternary phase diagrams: Determining compositions

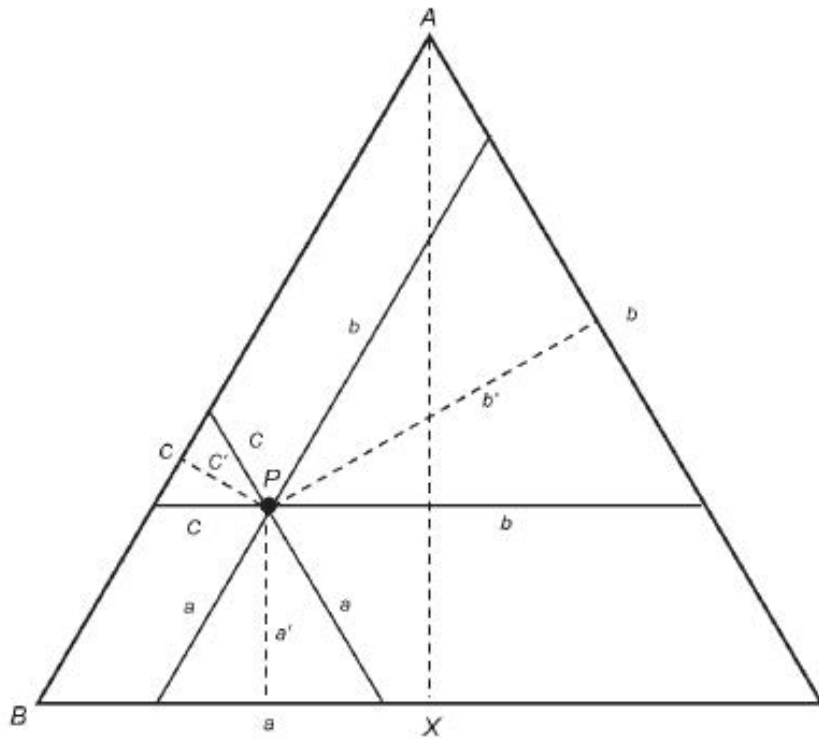


Fig. 10.7 The Gibbs triangle. Adapted from Ref 10.3

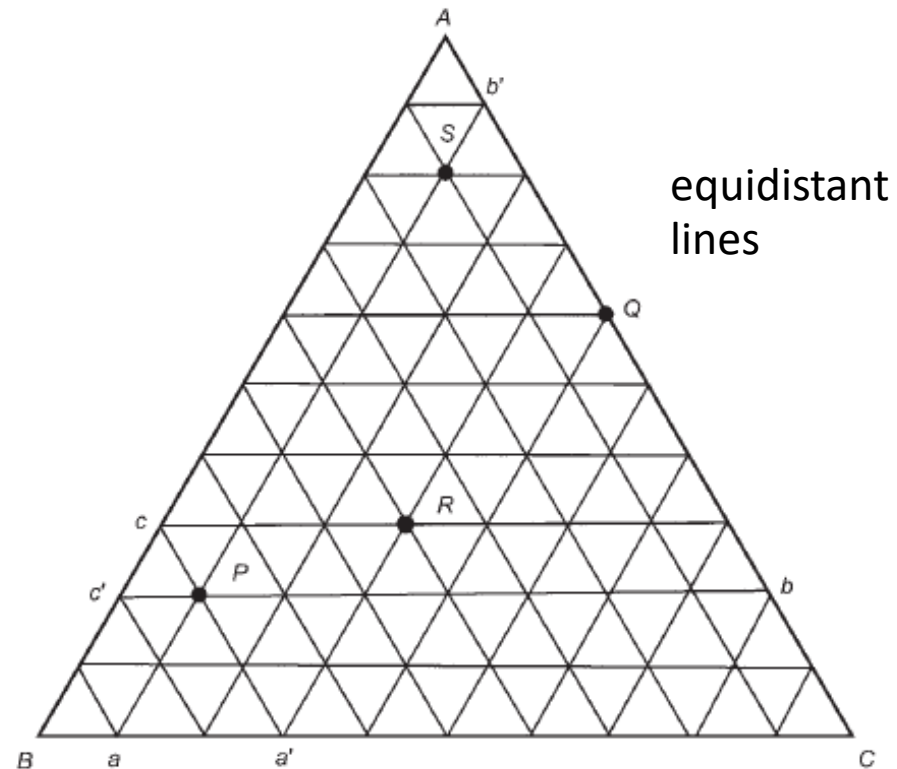
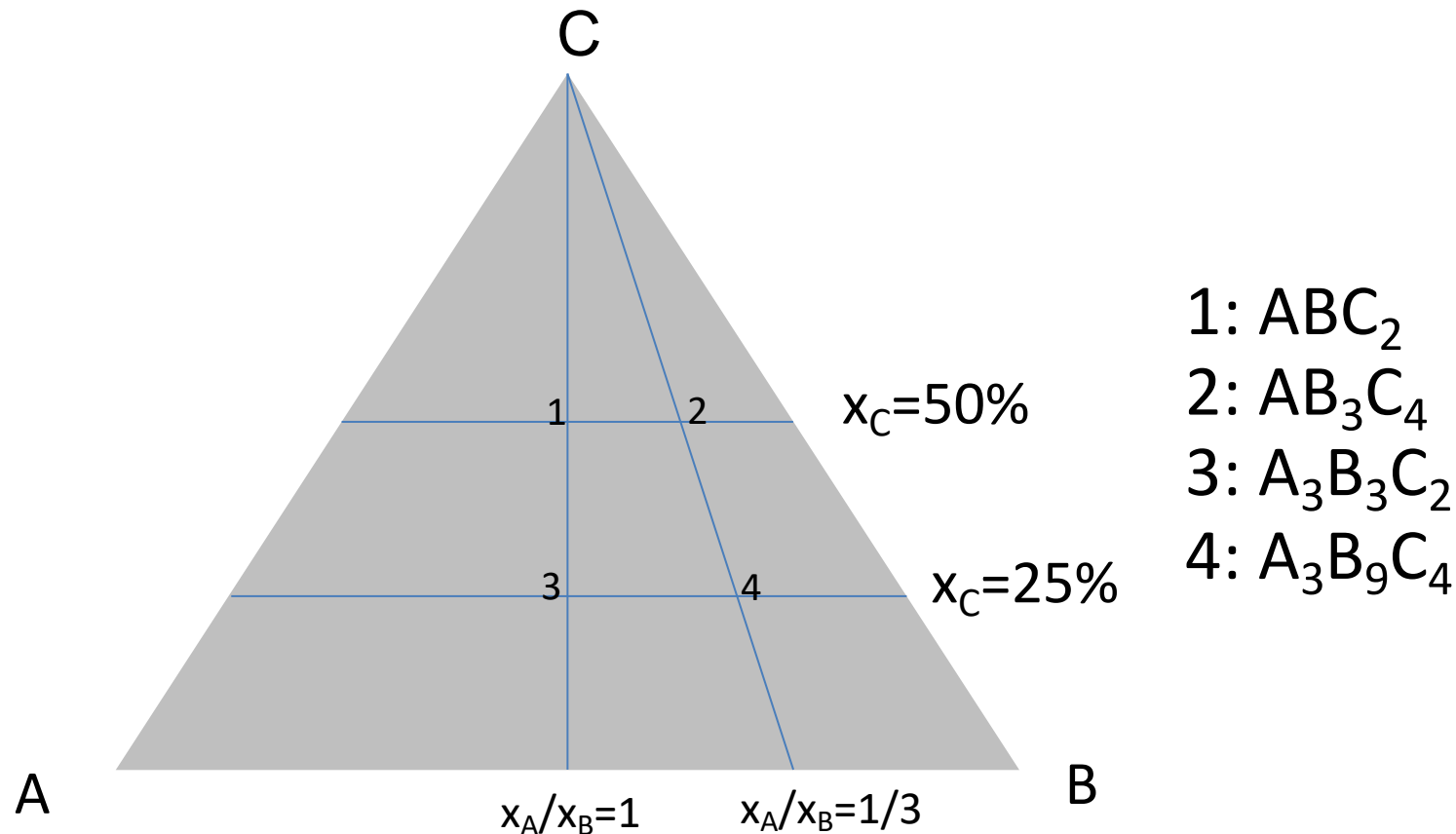


Fig. 10.8 The Gibbs triangle with composition lines. Adapted from Ref 10.3

the percentage of *C* by the line *Pc* (or *Pc'*), 10 units long. Other examples shown in Fig. 10.8 are: Alloy *R* = 30% *A* + 40% *B* + 30% *C*, Alloy *S* = 80% *A* + 10% *B* + 10% *C*, and Alloy *Q* = 60% *A* + 0% *B* + 40% *C*.

Ternary phase diagrams: ternary stoichiometric phases and lines of constant binary ratios



A ternary system can comprise stoichiometric ternary phases, e.g. ABC_2 and/or areas with variable composition

Ternary phase diagrams

components of ternary systems can be also compounds

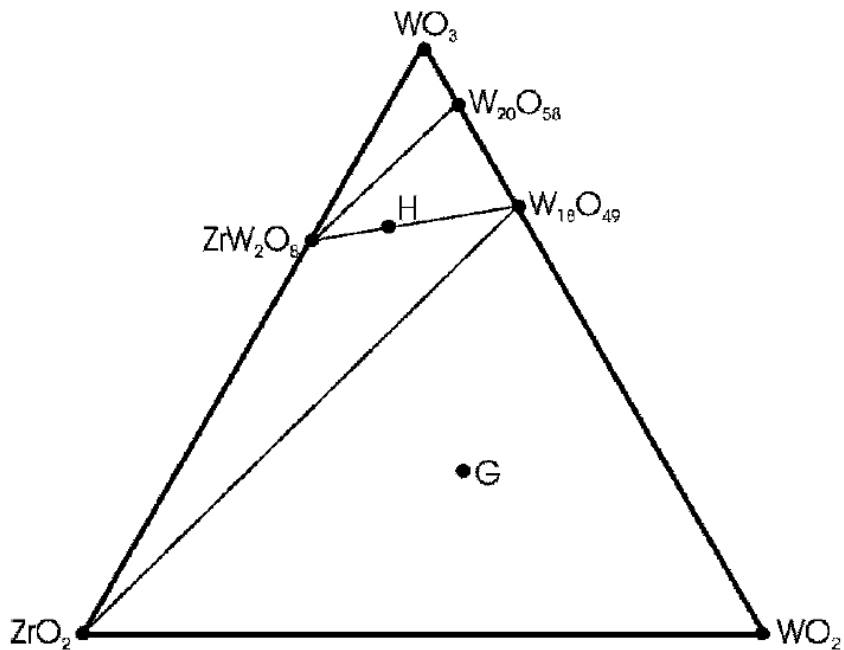


Figure 4.20 The simplified WO_3 – WO_2 – ZrO_2 phase diagram

tie lines and tie triangles

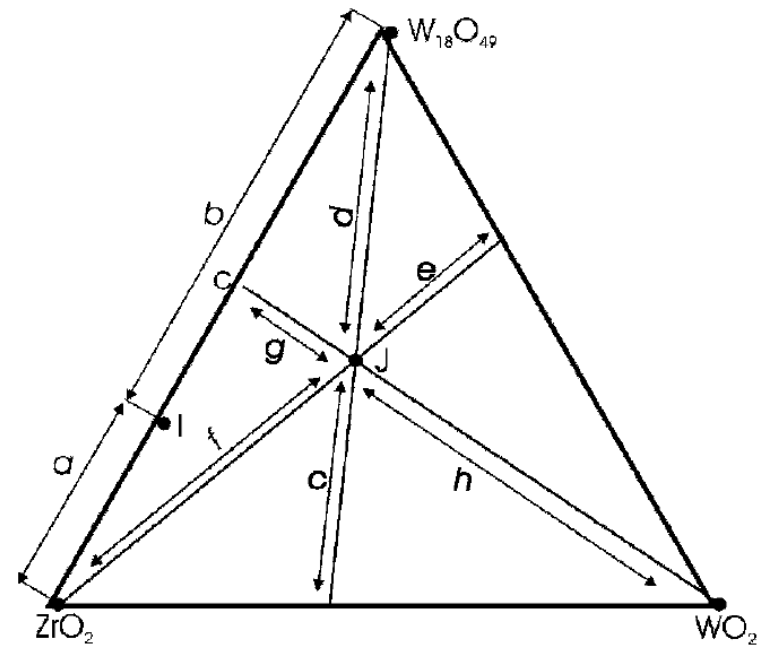
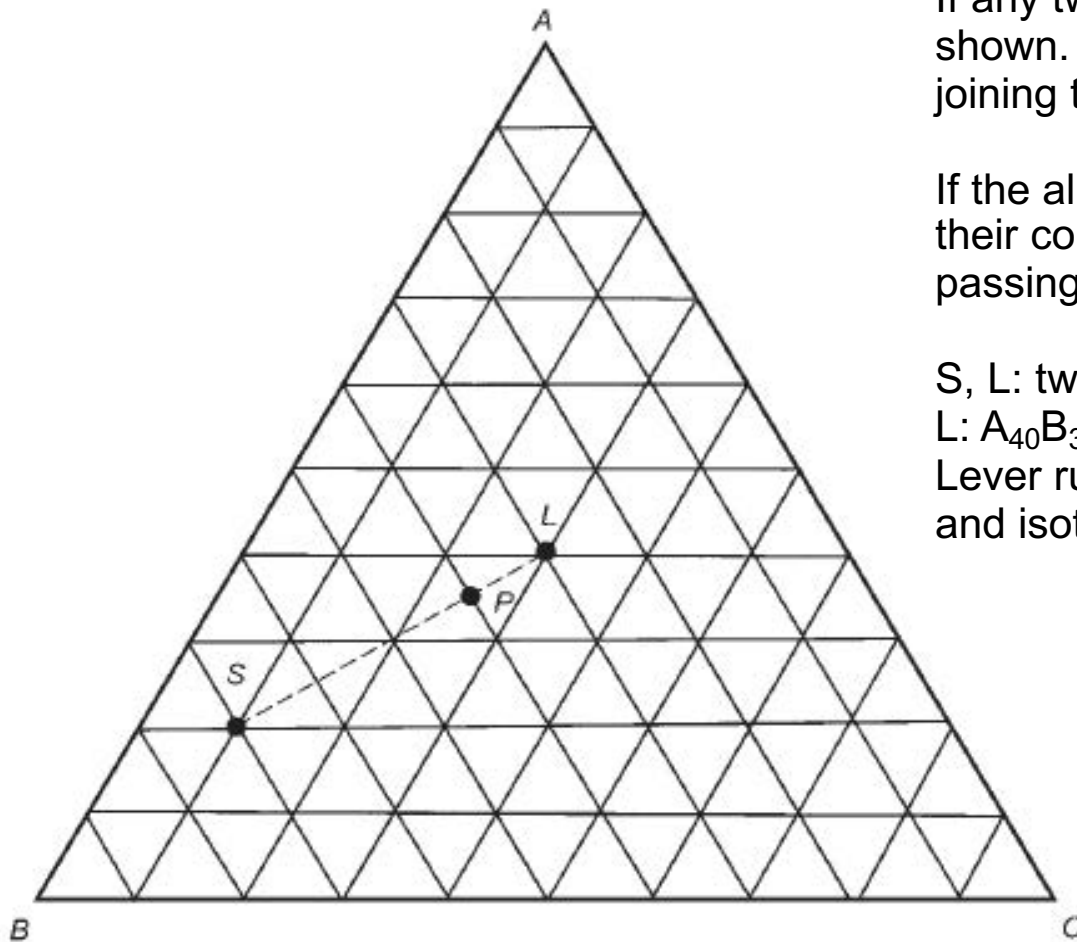


Figure 4.21 The method of determination of compositions on an isothermal phase diagram

Ternary phase diagrams: Tie lines, lever rules



If any two ternary alloys are mixed tie lines can be shown. Mixture compositions will lie on a straight line joining the original compositions.

If the alloy decomposes in two fractions, ' their composition will lie on the ends of a straight line passing through the original composition.

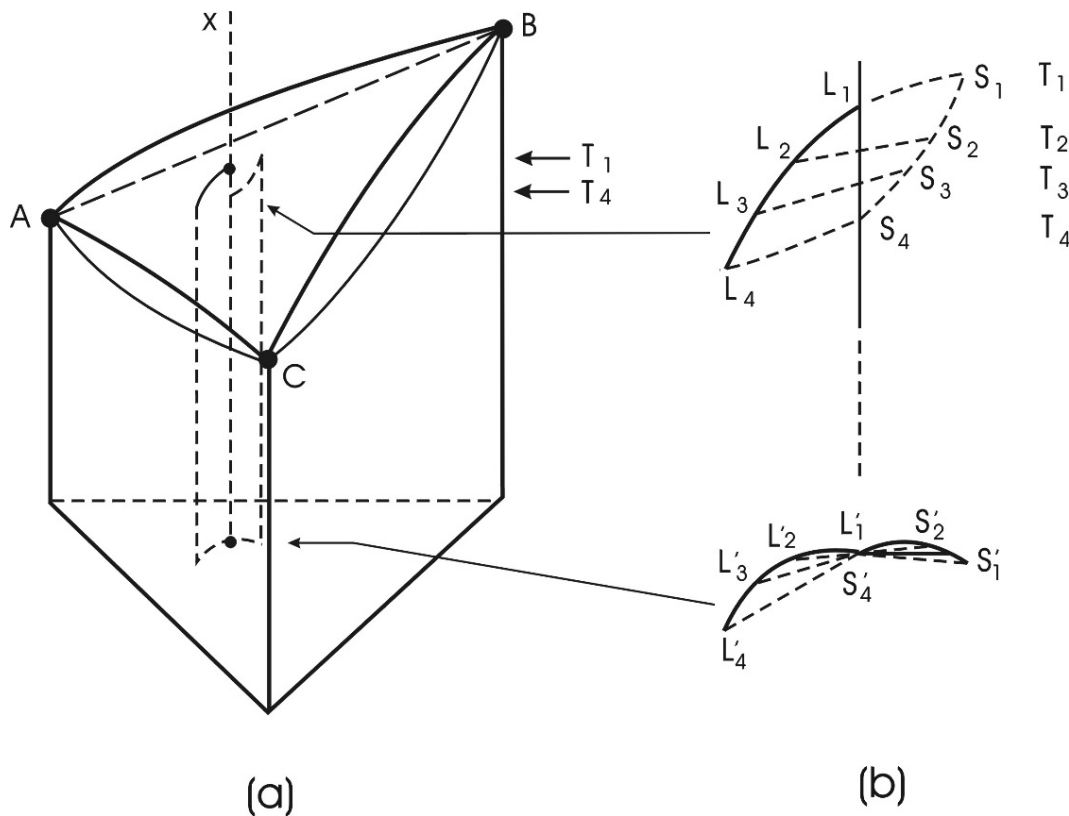
S, L: two ternary alloys of composition $S = A_{20}B_{70}C_{10}$,
 $L = A_{40}B_{30}C_{30}$. P: 25%S+75%L
 Lever rule can be applied as the diagram is isobaric and isothermal.

$$\% S = \frac{PL}{SL} \times 100$$

$$\% L = \frac{SP}{SL} \times 100$$

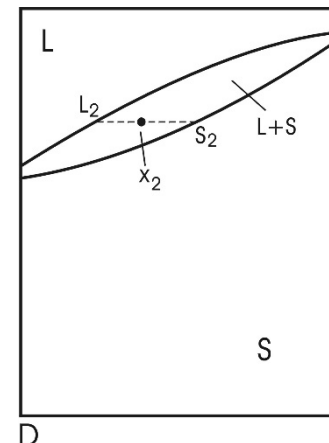
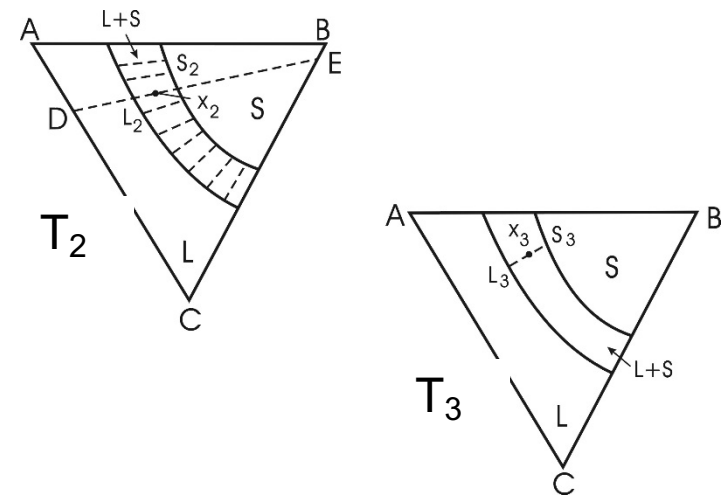
Fig. 10.9 The Gibbs triangle with tie line. Adapted from Ref 10.3

Ternary isomorphous phase diagram: example of solidification of an alloy x



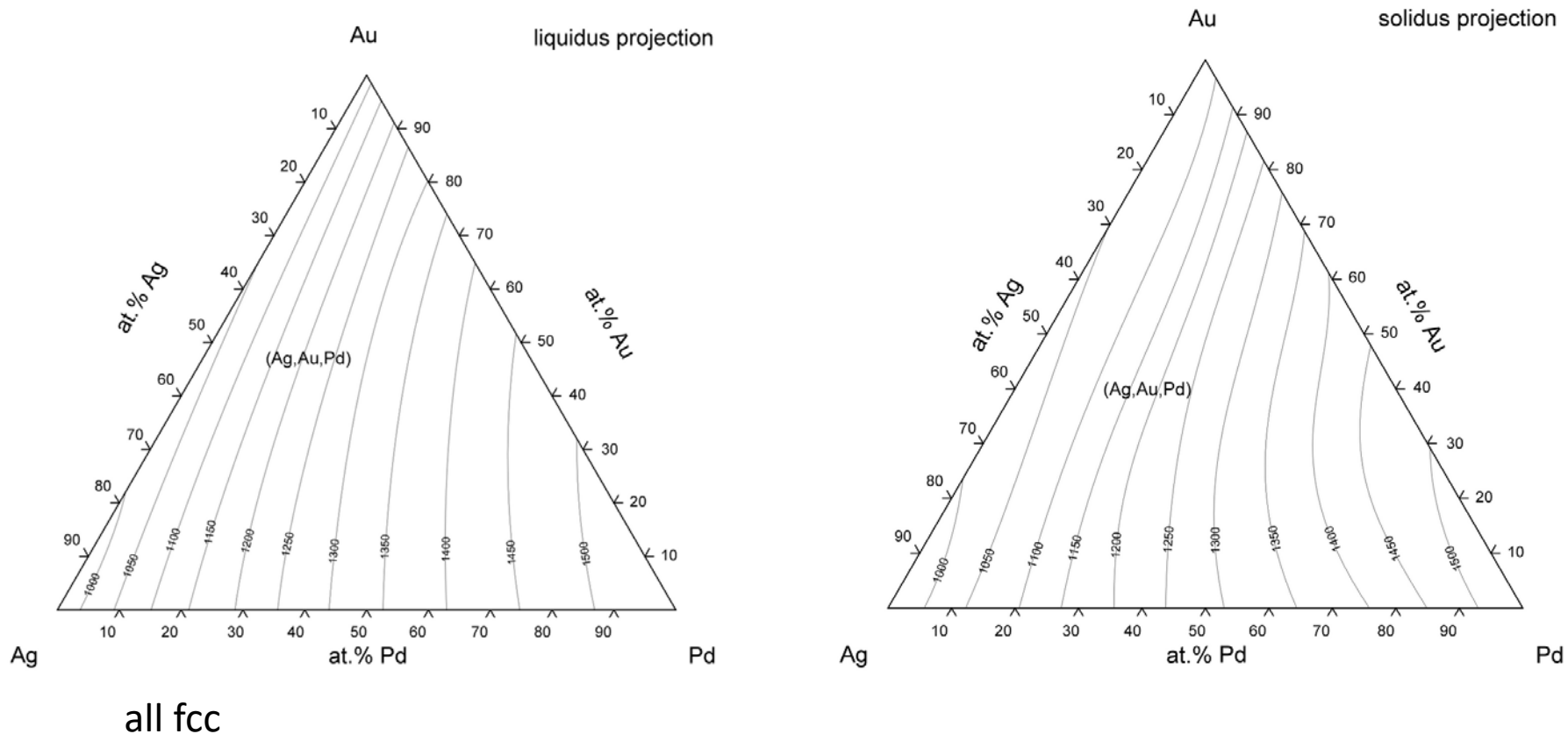
L1, L2, ... on liquidus surface
S1, S2, ... on solidus surface

isothermal cuts



vertical section
= quasibinary cut
= isopleth

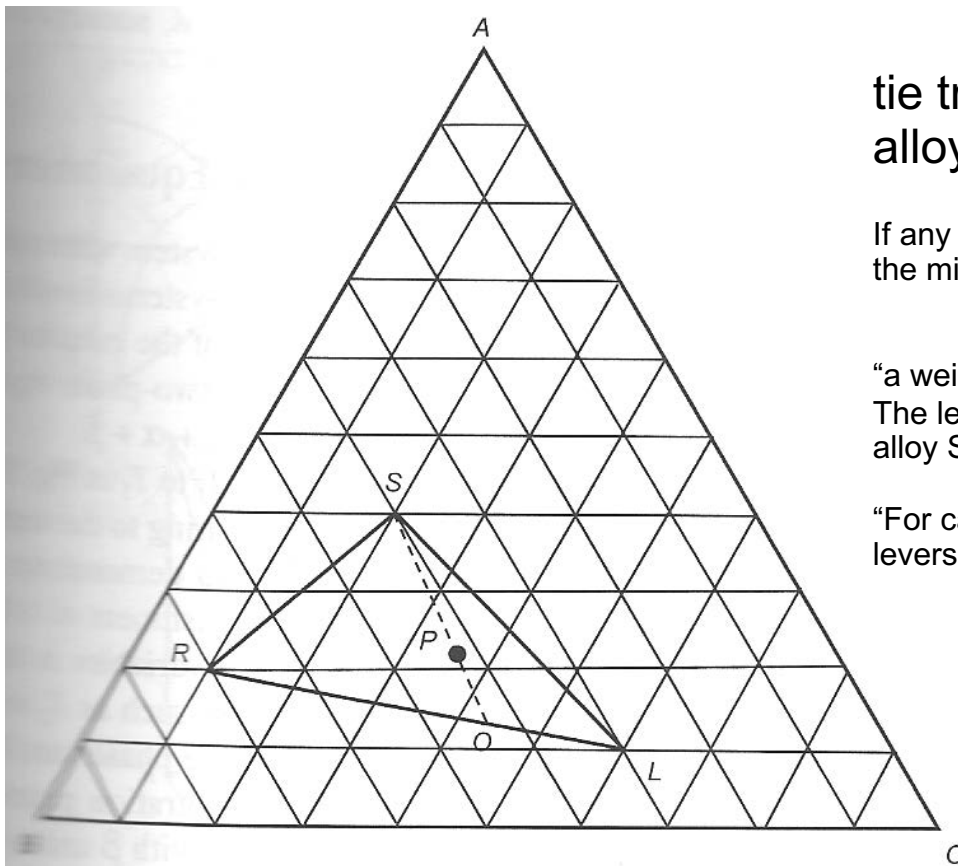
Ternary isomorphous phase diagram: projection of liquidus temperatures



Three phase equilibrium in ternary systems:

Tie triangle

Three phase equilibrium in ternary systems occurs over a temperature range (bivariant)



tie triangle:
alloys S, R, L in equilibrium

If any 3 alloys of a ternary system are mixed,
the mixture composition will lie within the triangle.

“a weightless plane triangle (here: RSL) supported on a point fulcrum at P”
The lever plane balances: “if 20% of alloy R is placed on point R, 30% of alloy S and 50% of alloy L on point L”

“For calculation, it is convenient to resolve the planar lever into two linear levers (here SPO, ROL)”

Fig. 10.19 Analysis of a tie triangle. Adapted from Ref 10.3

Three phase equilibrium in a ternary system (I)

one isomorphous binary and two eutectic binaries

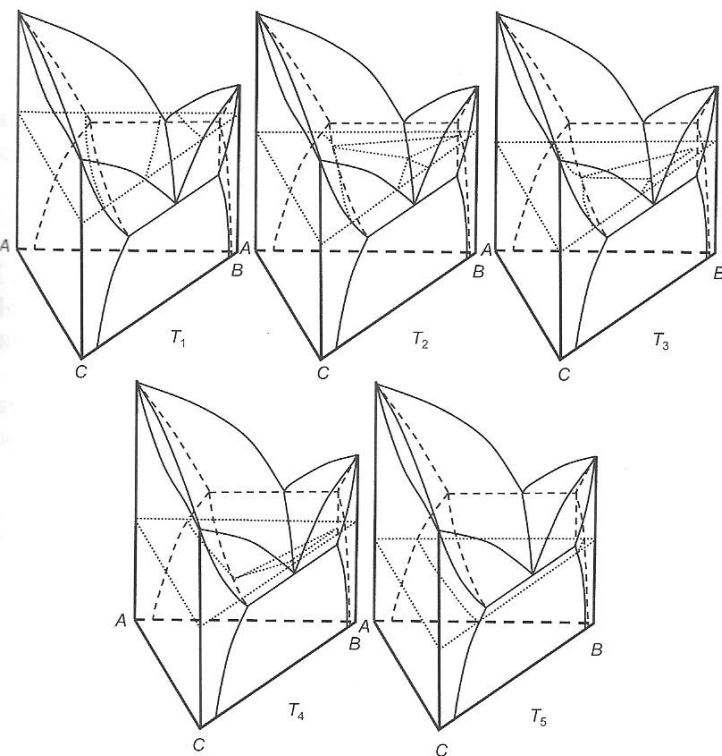


Fig. 10.21 Development of isotherms shown in Fig. 10.22. Adapted from Ref 10.3

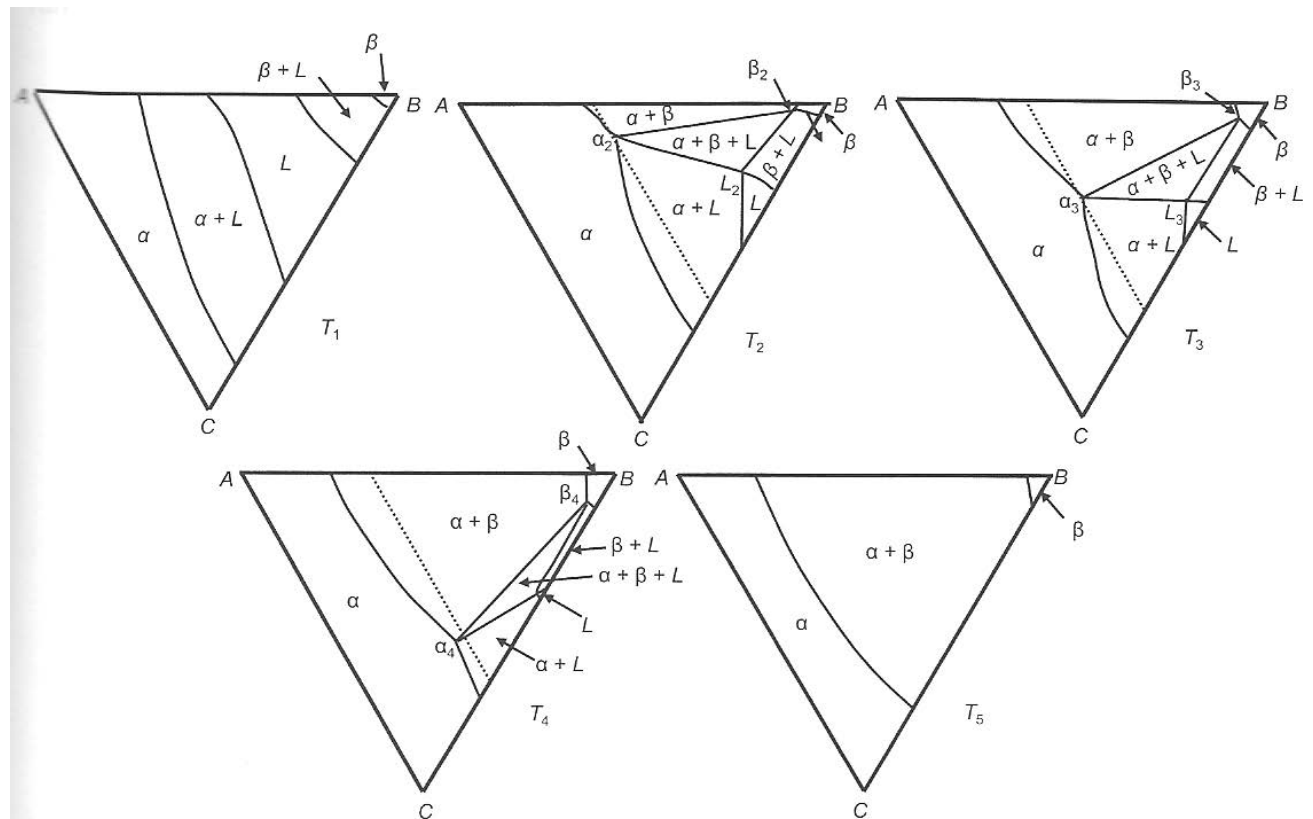


Fig. 10.22 Isotherms through the space diagrams of Fig. 10.21. Adapted from Ref 10.3

Three phase equilibrium in a ternary system (II)

2 eutectic binaries, 1 isomorphous binary

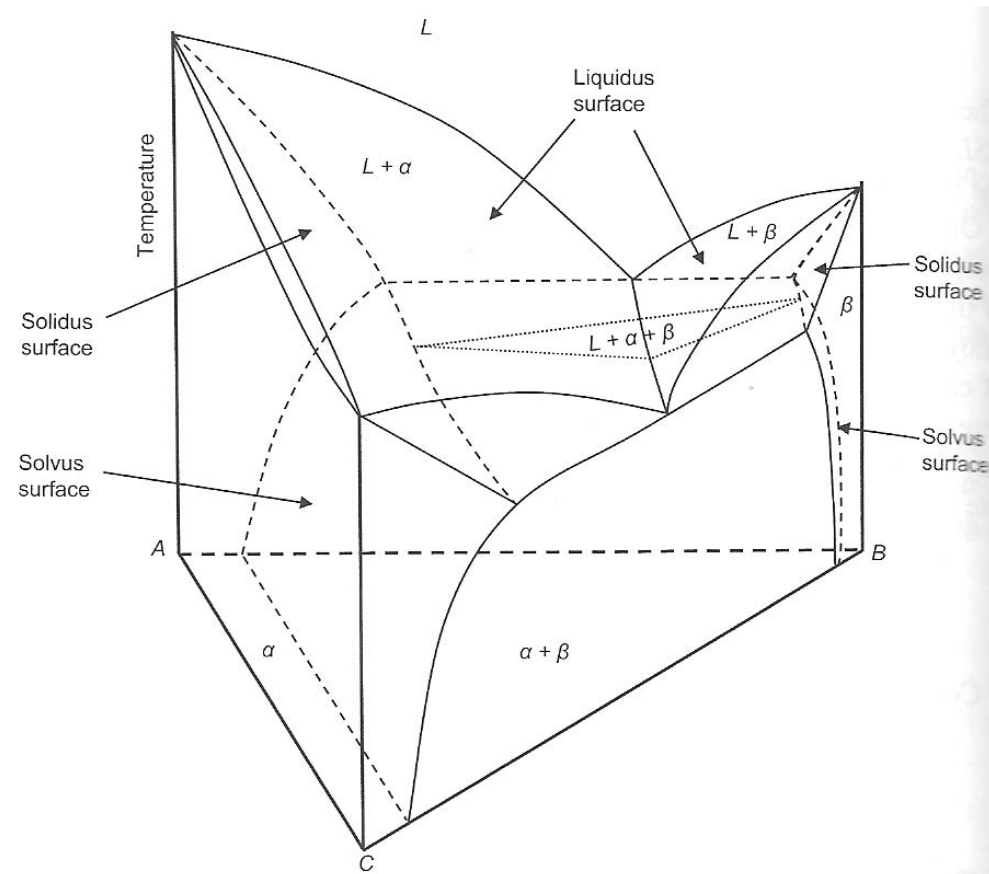
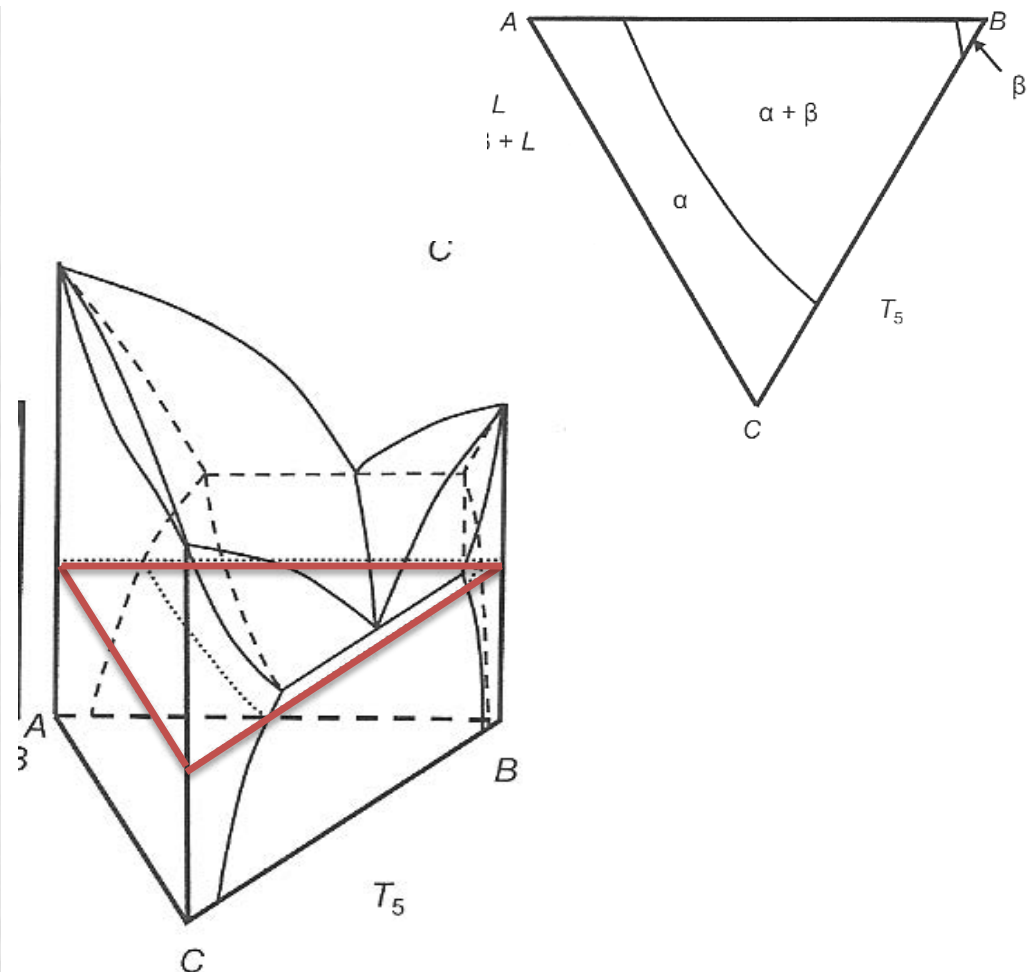


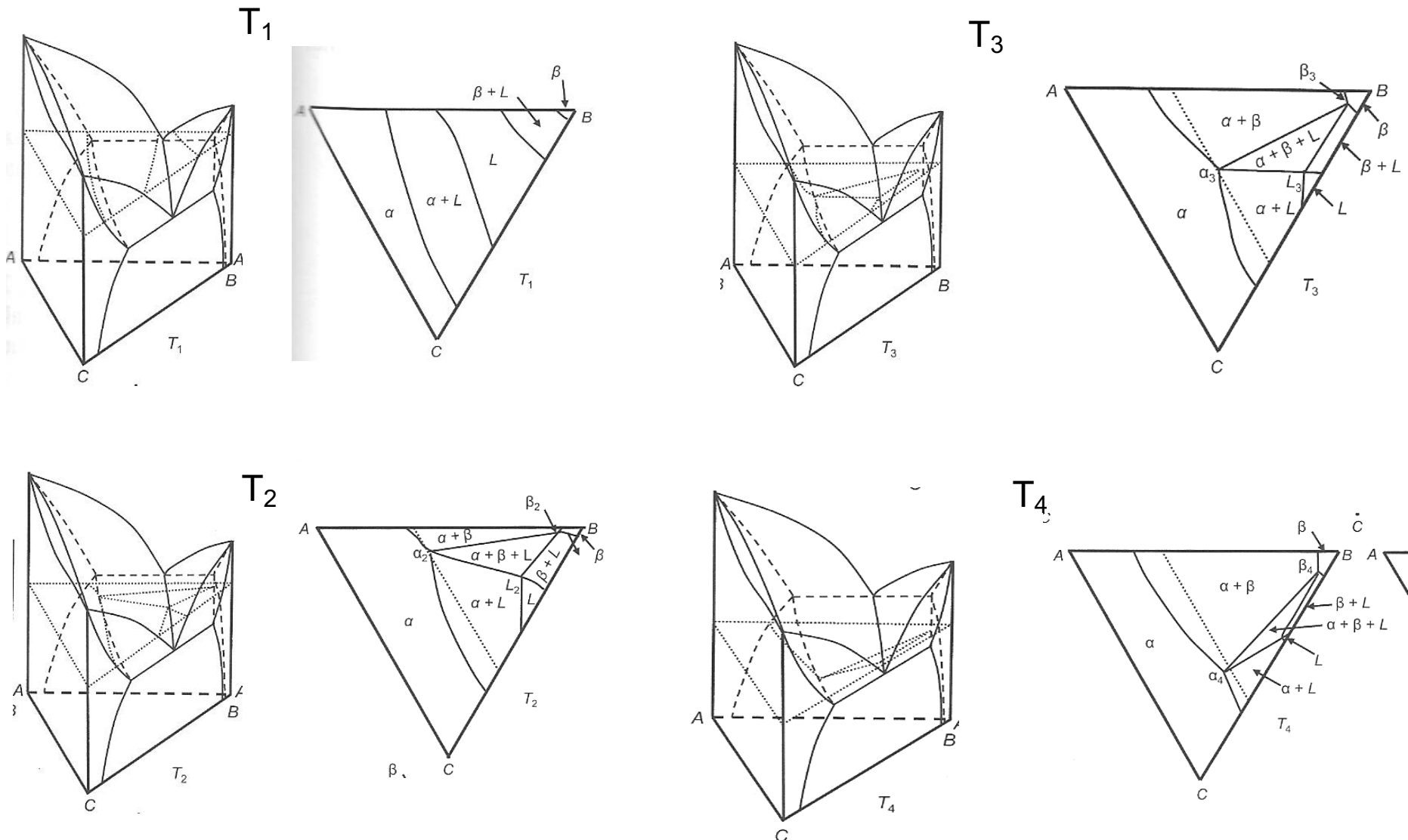
Fig. 10.20 Three-phase equilibria in a ternary system with a eutectic reaction. Adapted from Ref 10.3

isothermal section at $T < T_{\text{solidus}}$



Three phase equilibrium in a ternary system (III)

$$T_1 > T_2 > T_3 > T_4$$



Three phase equilibrium in a ternary system (IV)

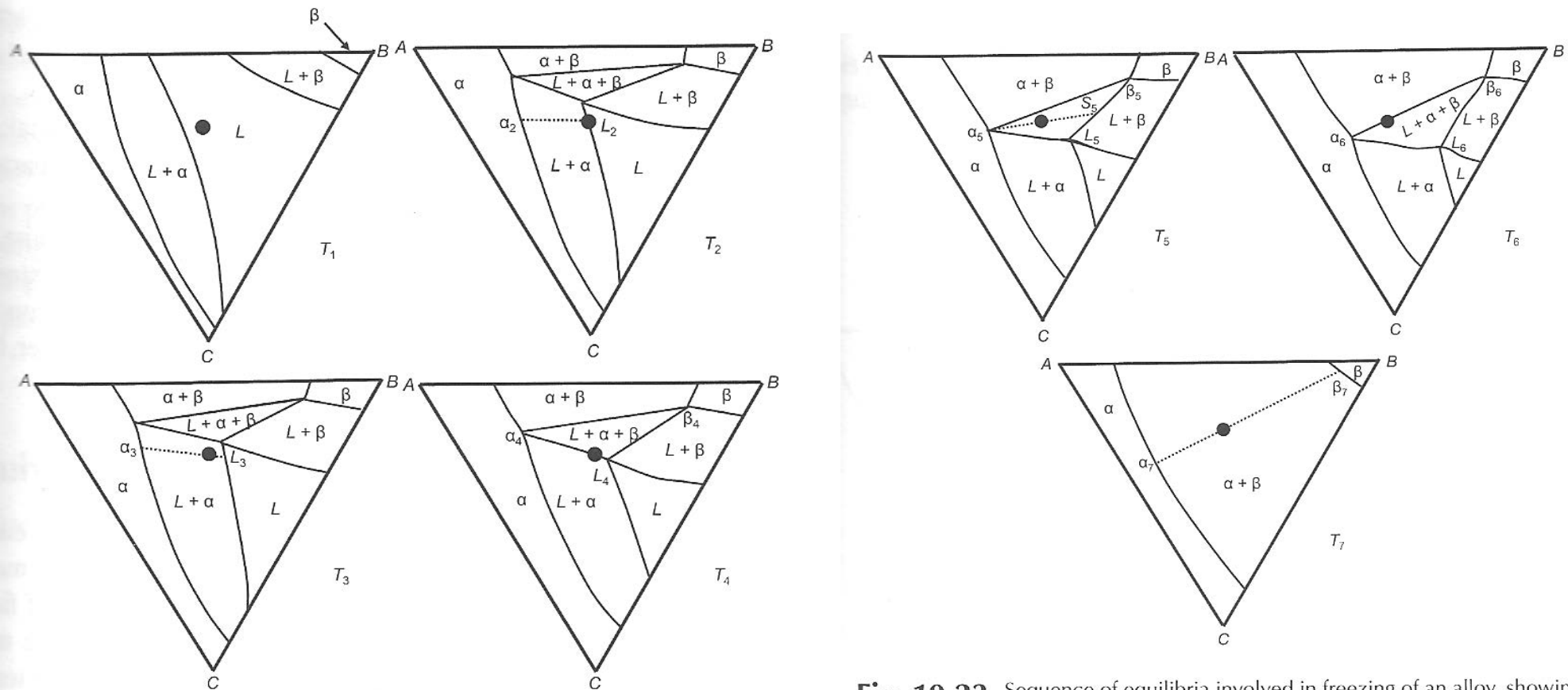


Fig. 10.23 Sequence of equilibria involved in freezing of an alloy, showing gross composition in each isotherm. Adapted from Ref 10.3

Peritectic system with three-phase equilibrium

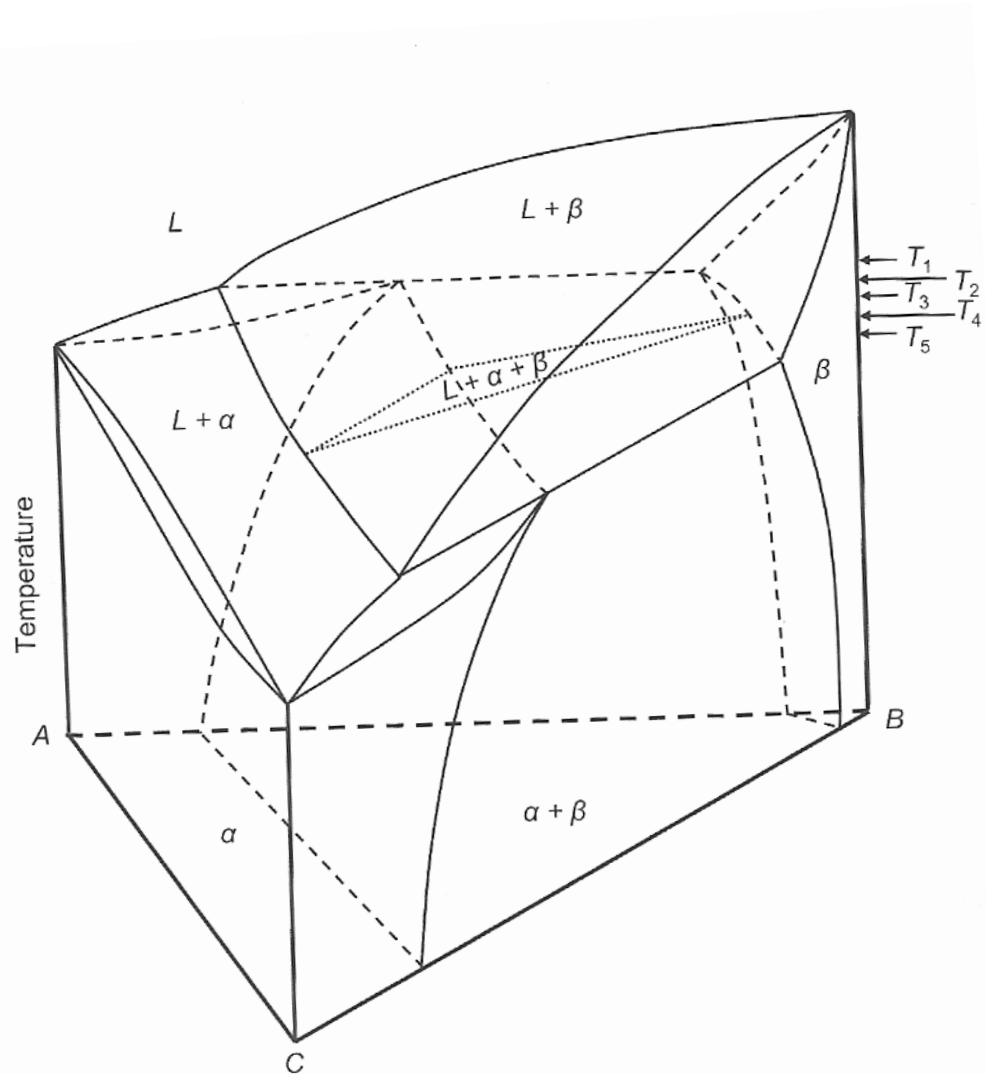


Fig. 10.24 Three-phase equilibria in a ternary system with a peritectic reaction. Adapted from Ref 10.3

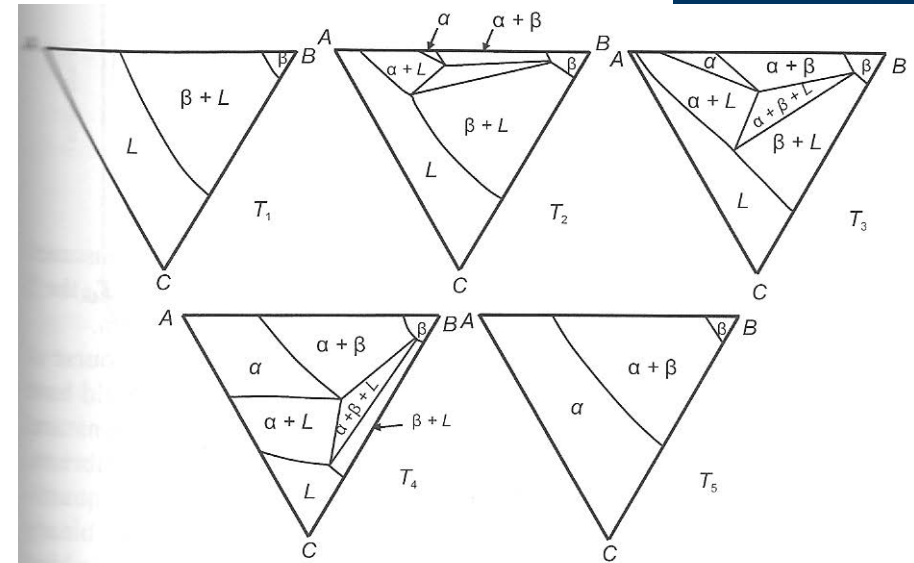


Fig. 10.25 Isotherms through the space diagram of Fig. 10.24. Adapted from Ref 10.3

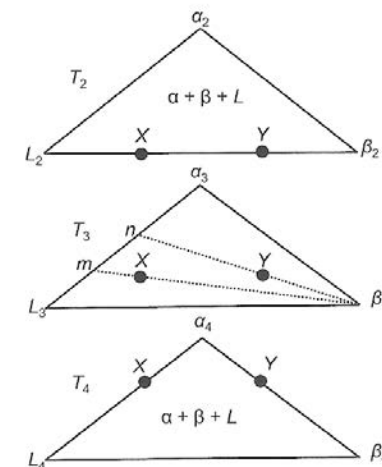


Fig. 10.26 Tie triangles at three different temperatures. Adapted from Ref 10.3

Ternary four-phase equilibrium

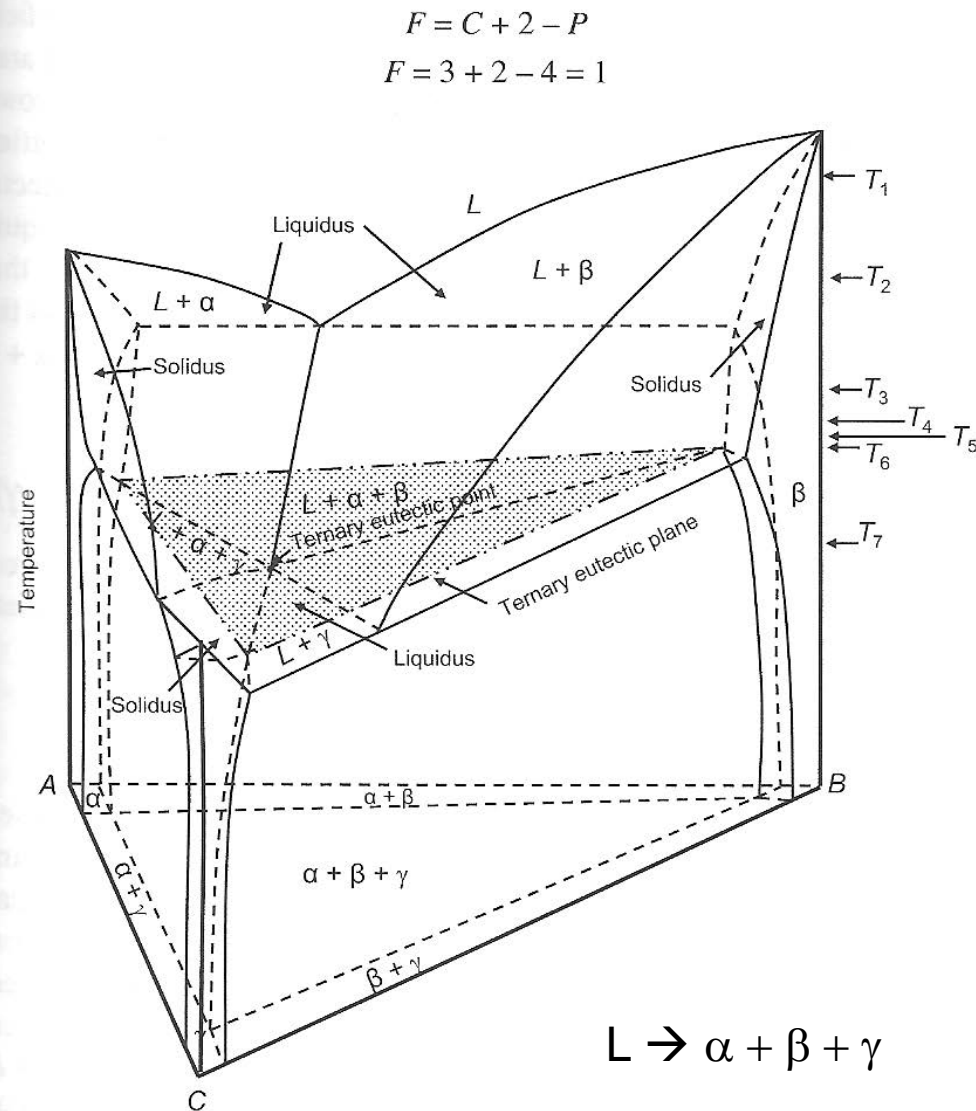


Fig. 10.27 Temperature-composition space model of a ternary eutectic system with the reaction $L \rightarrow \alpha + \beta + \gamma$. Adapted from Ref 10.3

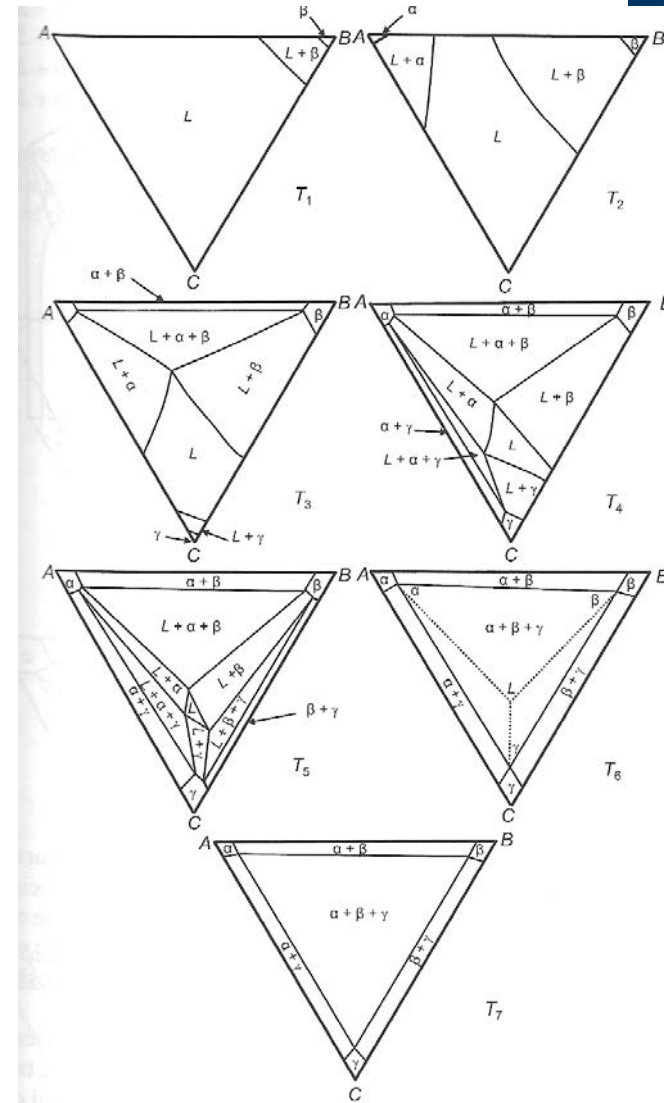
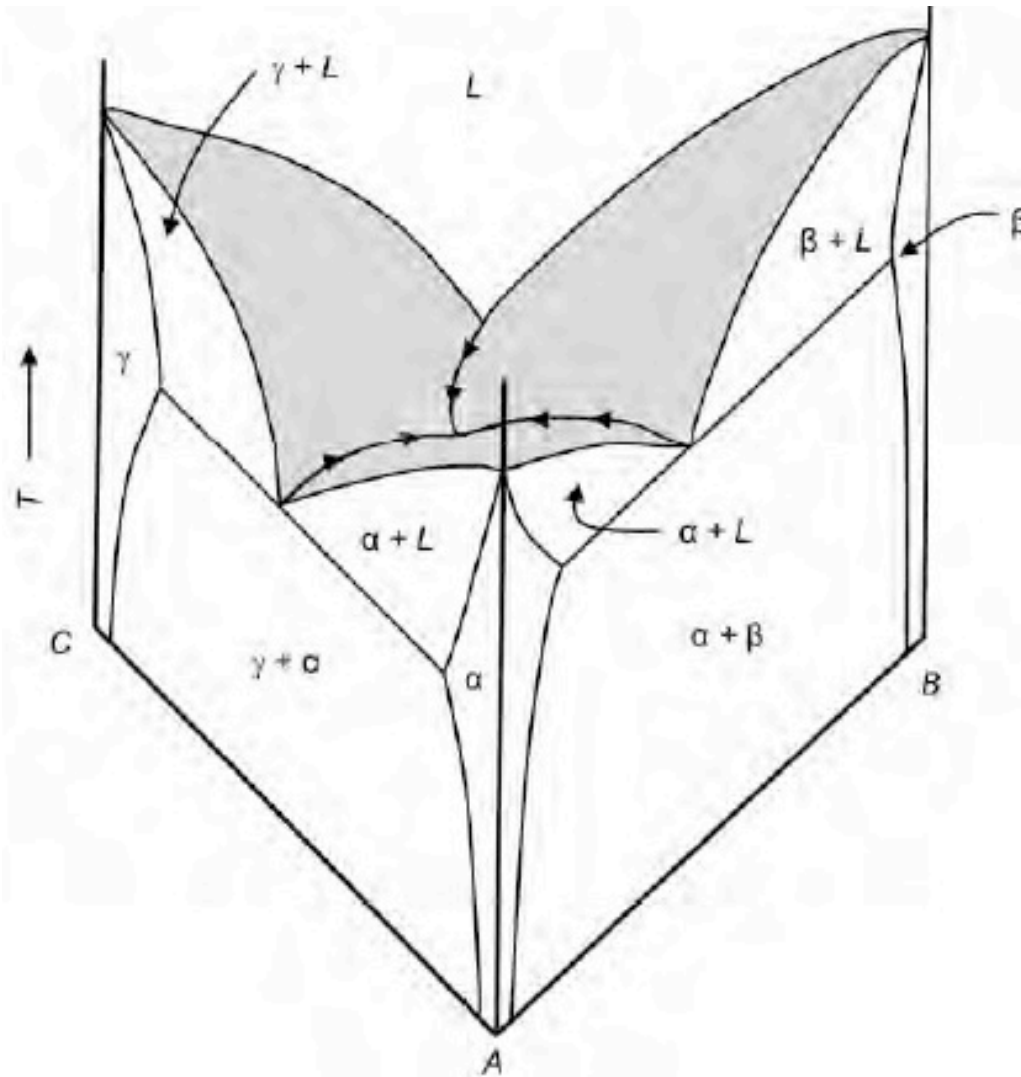


Fig. 10.28 Isotherms taken through space model in Fig. 10.27. Adapted from Ref 10.3

Ternary phase diagram (isobaric, eutectic)



three eutectic binaries

ternary eutectic:

$$L = \alpha + \beta + \gamma$$

Fig. 10.2 Hypothetical ternary phase diagram. Binary phase diagrams are present along the three faces. Adapted from Ref 10.1

Ternary phase diagram (isobaric, eutectic)

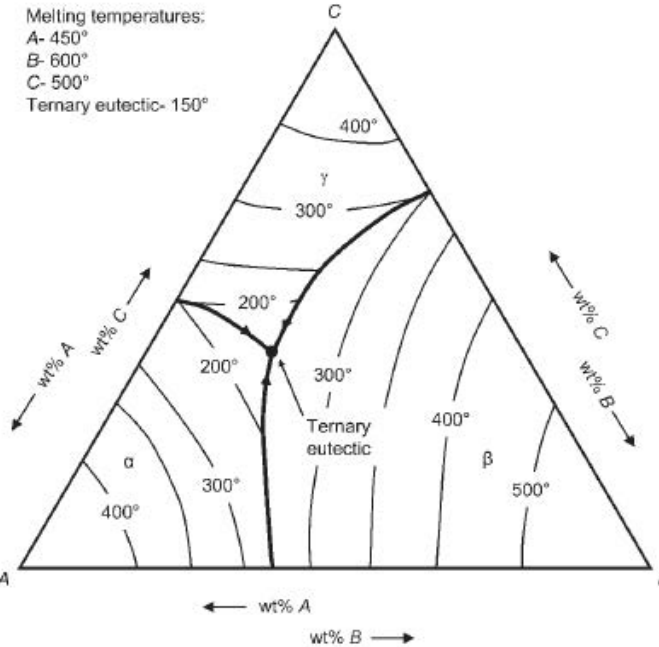
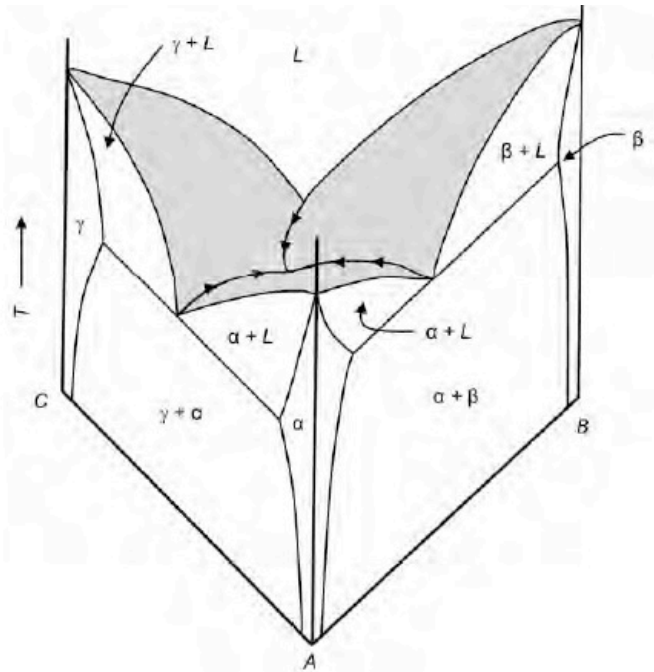
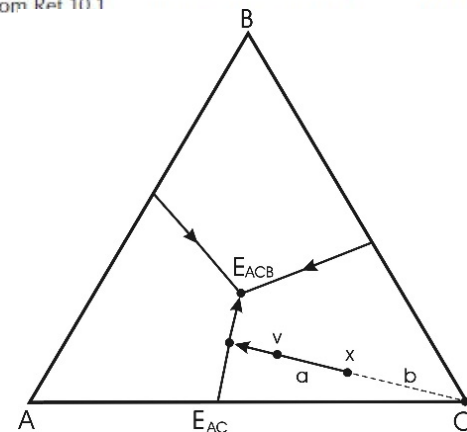
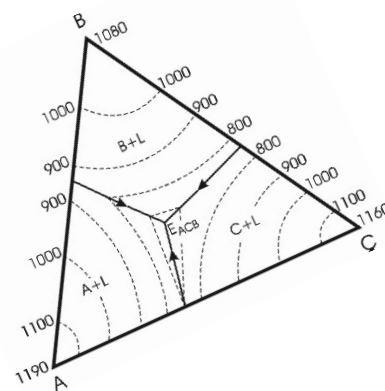
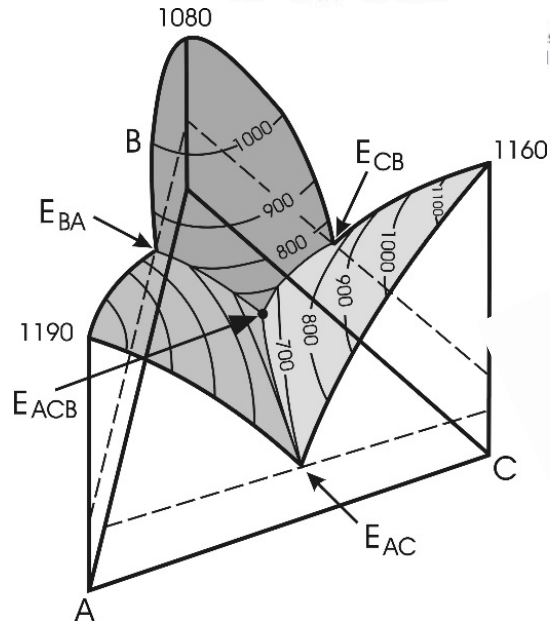


Fig. 10.3 Liquidus plot for hypothetical ternary phase diagram. Adapted from Ref 10.1



Example: Fe-Cr-Ni system

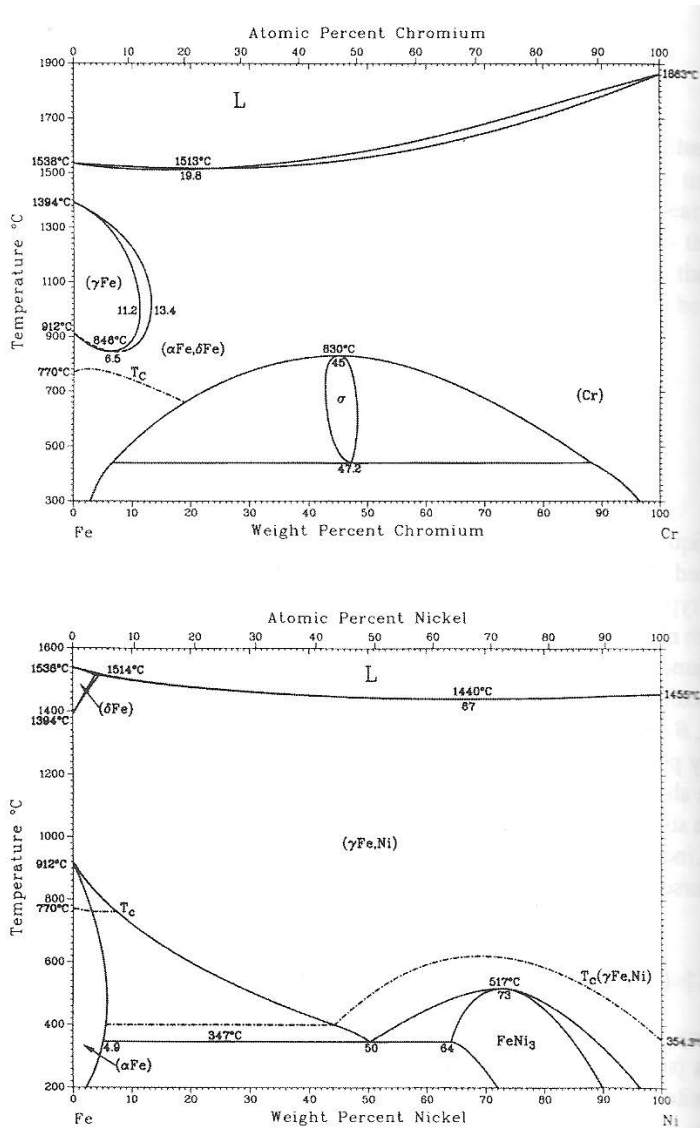


Fig. 10.37 Two representative binary iron phase diagrams, showing ferrite stabilization (iron-chromium) and austenite stabilization (iron-nickel). Source: Ref 10.4 as published in Ref 10.5

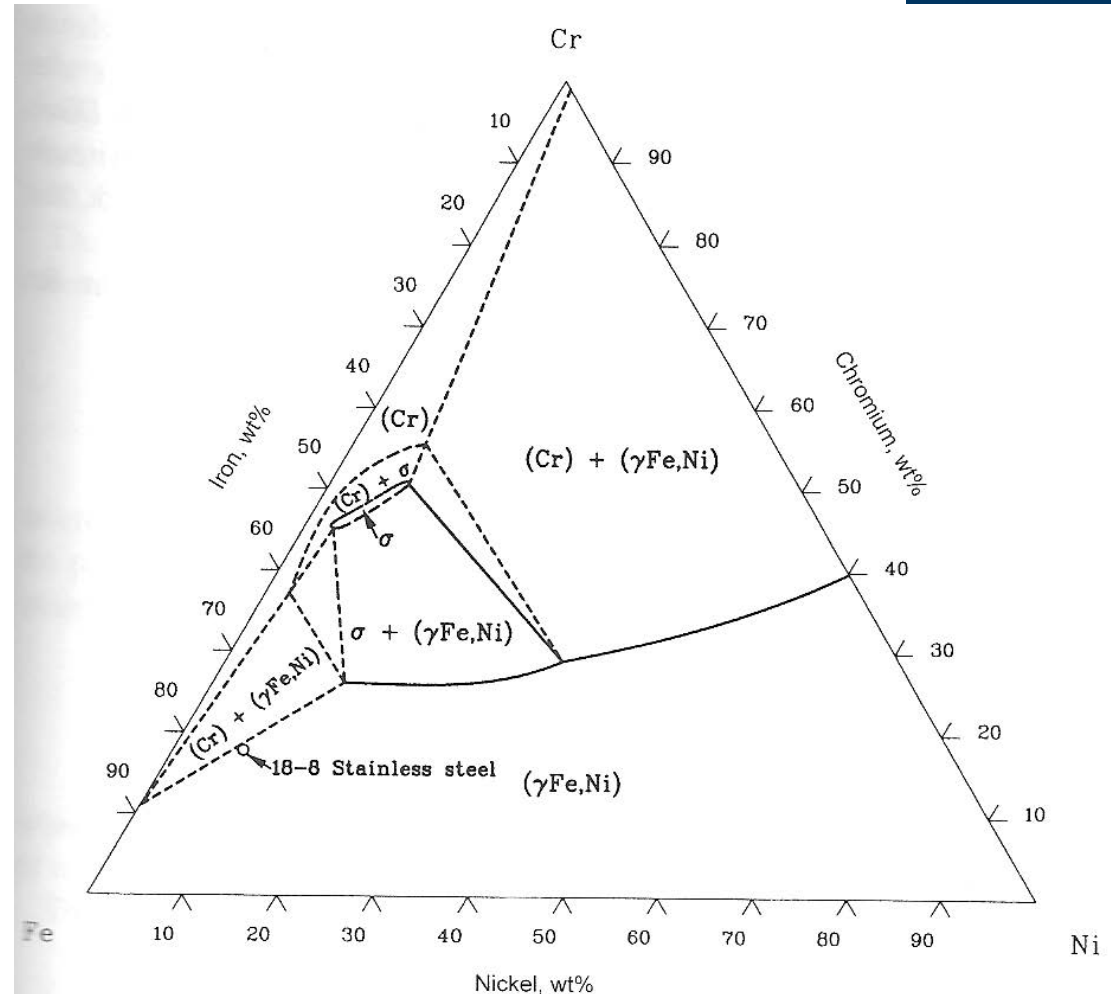
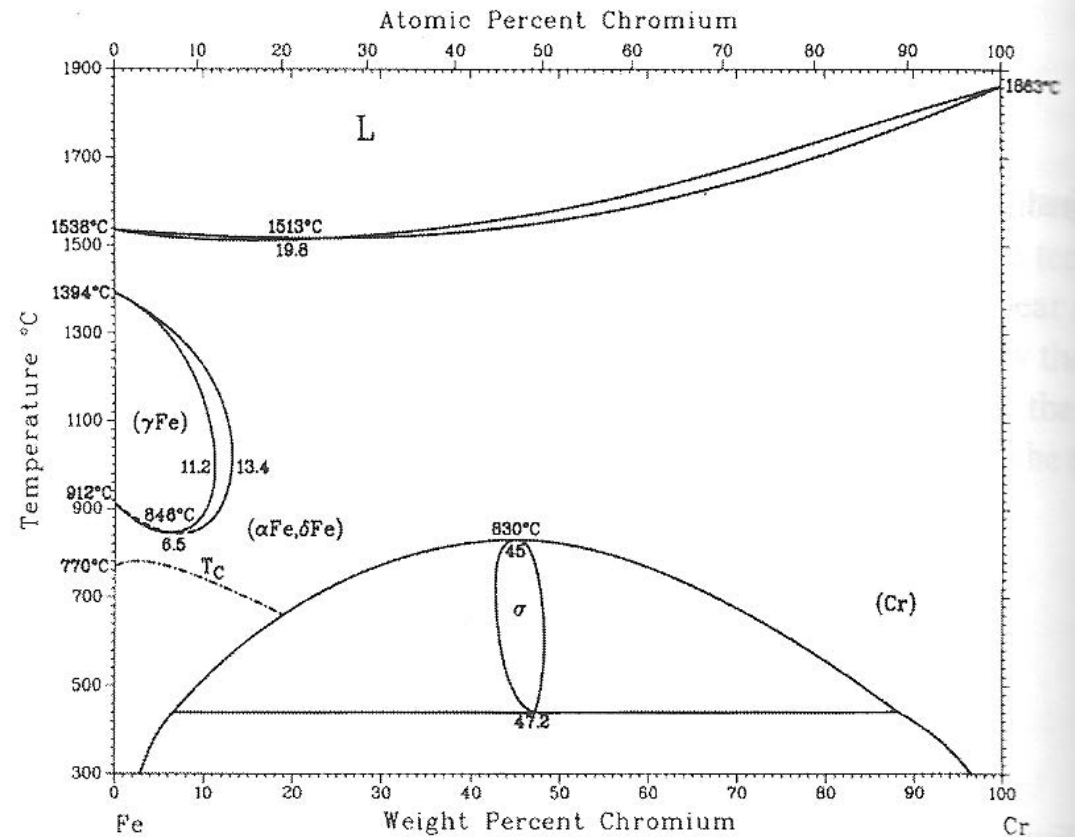
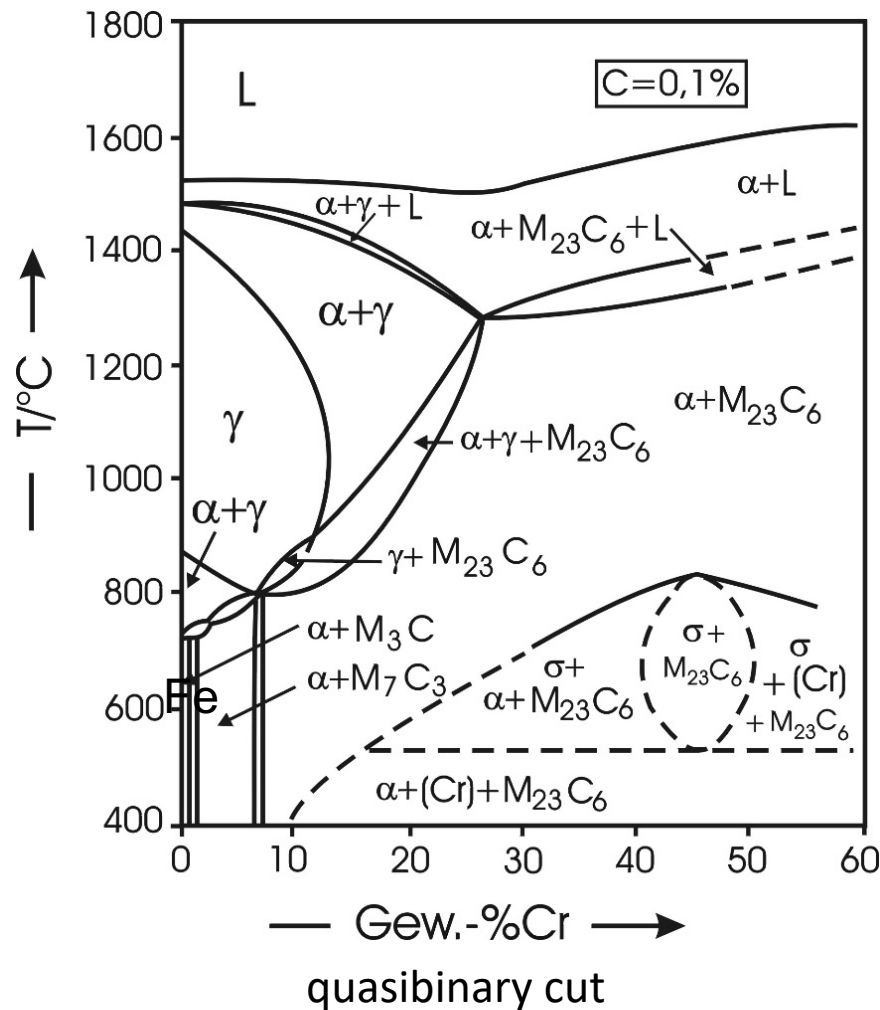


Fig. 10.38 The isothermal section at 900 °C (1652 °F) of the Fe-Cr-Ni ternary phase diagram, showing the nominal composition of 18-8 stainless steel. Source: Ref 10.6 as published in Ref 10.5

Ternary phase diagrams: quasibinary section

Fe-Cr-C



Quasi-ternary phase diagrams

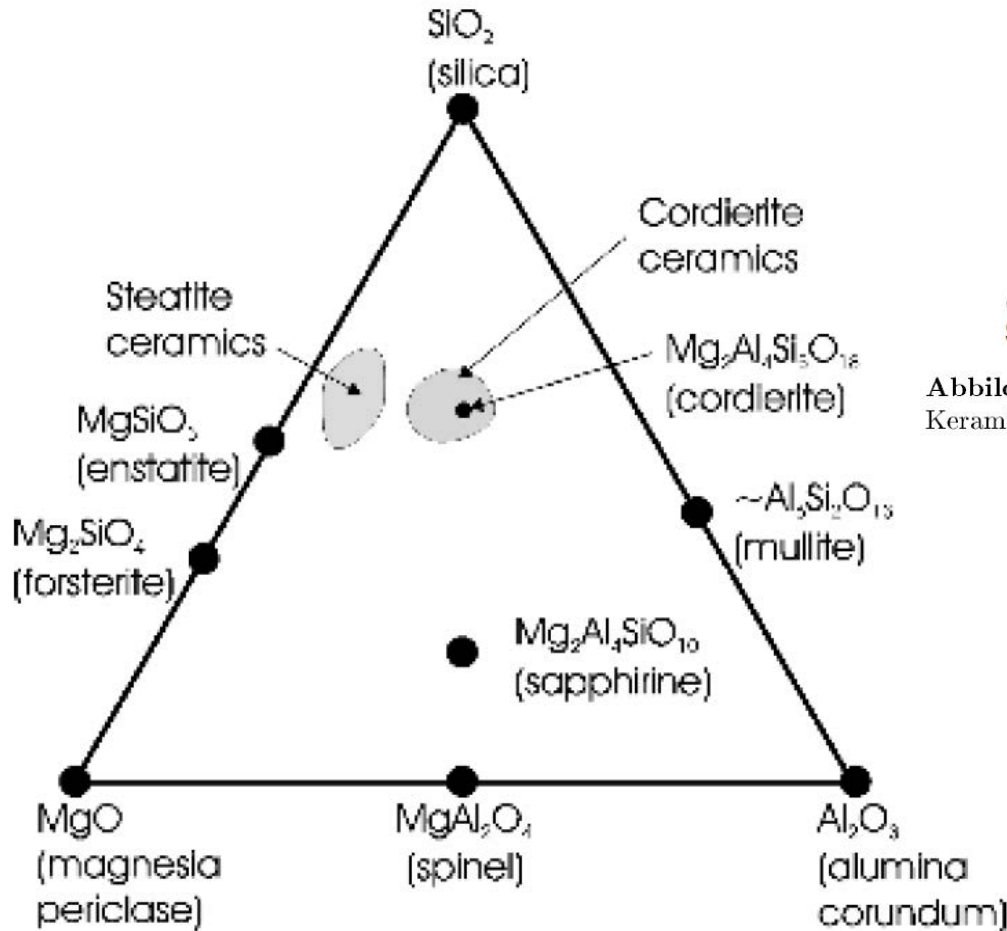


Figure 4.22 Schematic phase diagram for the $\text{MgO}-\text{Al}_2\text{O}_3-\text{SiO}_2$ system, which contains several important ceramic materials

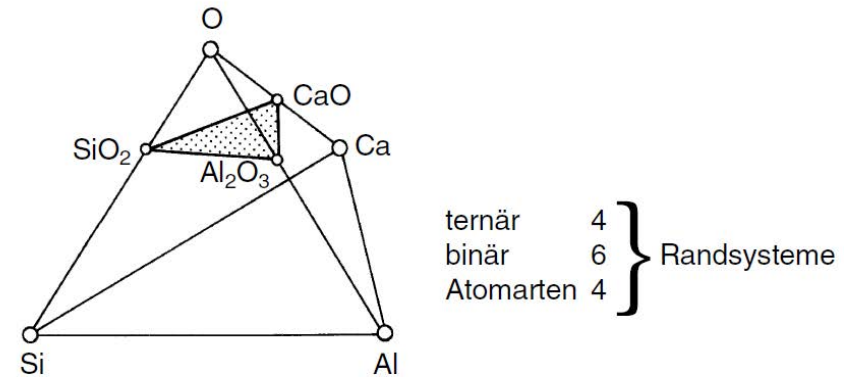
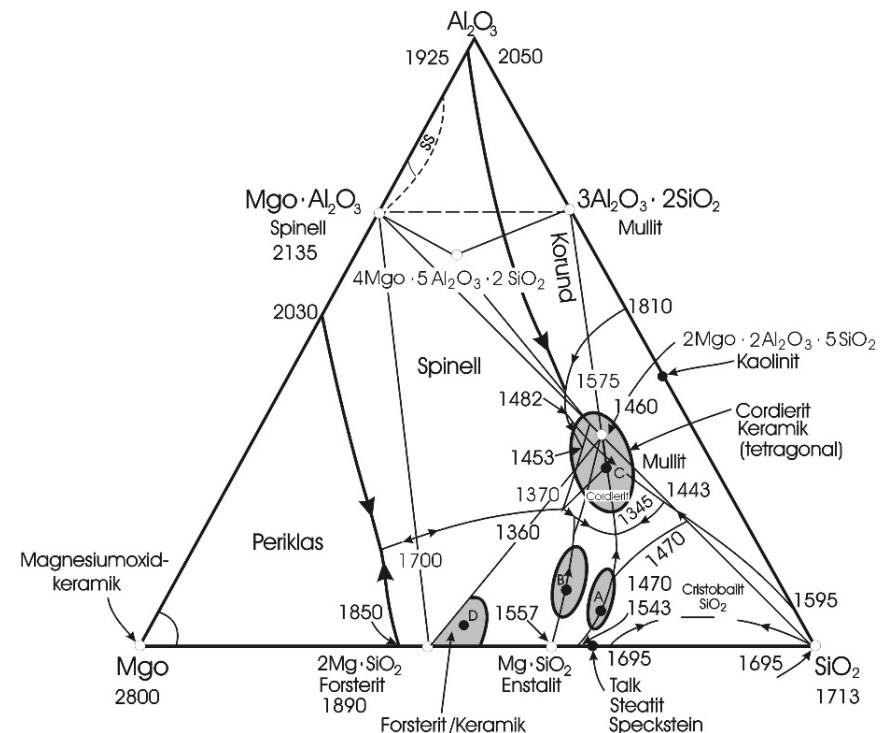
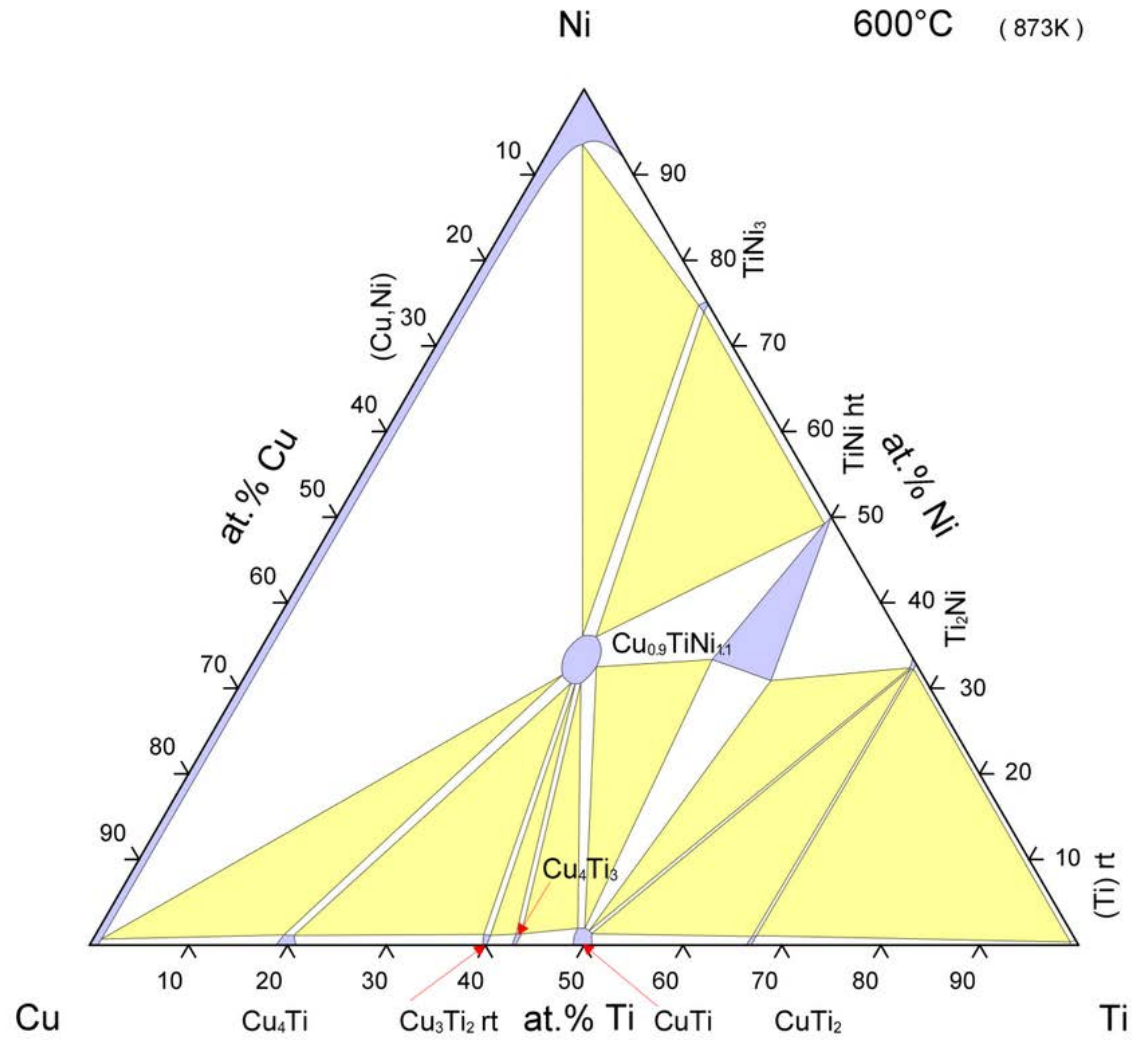


Abbildung 4.51. Schematische Darstellung eines quasiternären Systems wichtiger Keramiken.

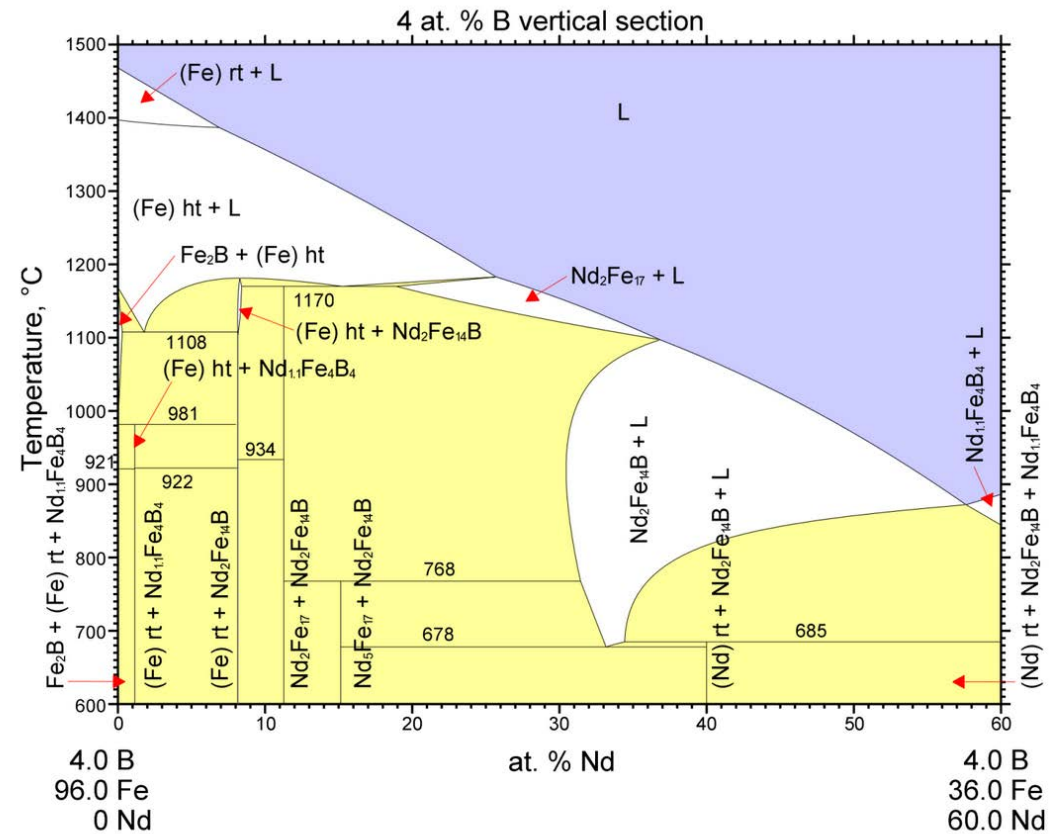
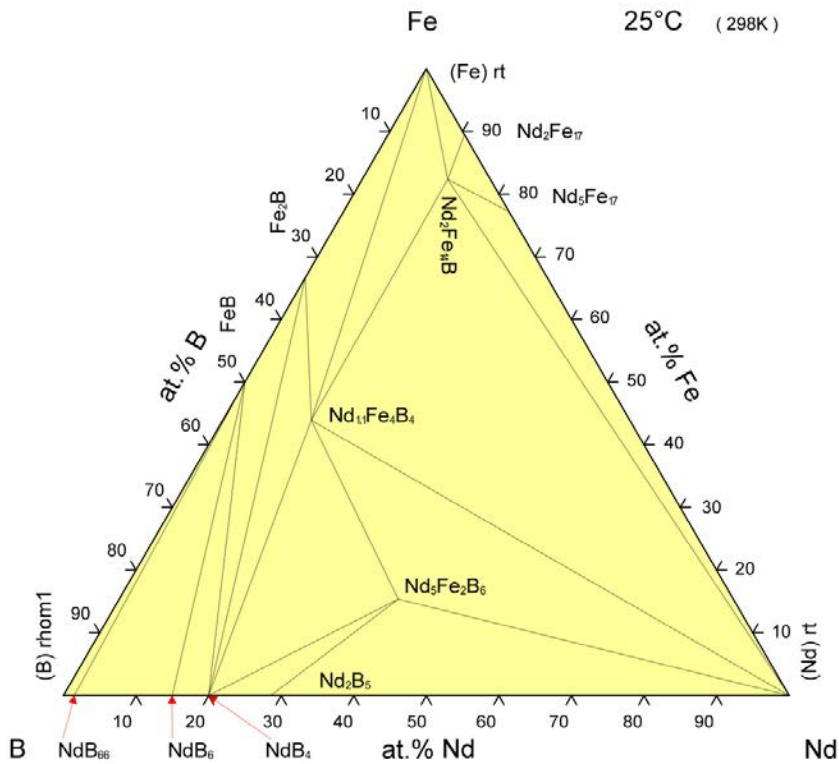


Ti-Ni-Cu



© ASM International 2007. Diagram No. 200884

Nd-Fe-B



© ASM International 2007. Diagram No. 200483

$\text{Nd}_2\text{Fe}_{14}\text{B}$

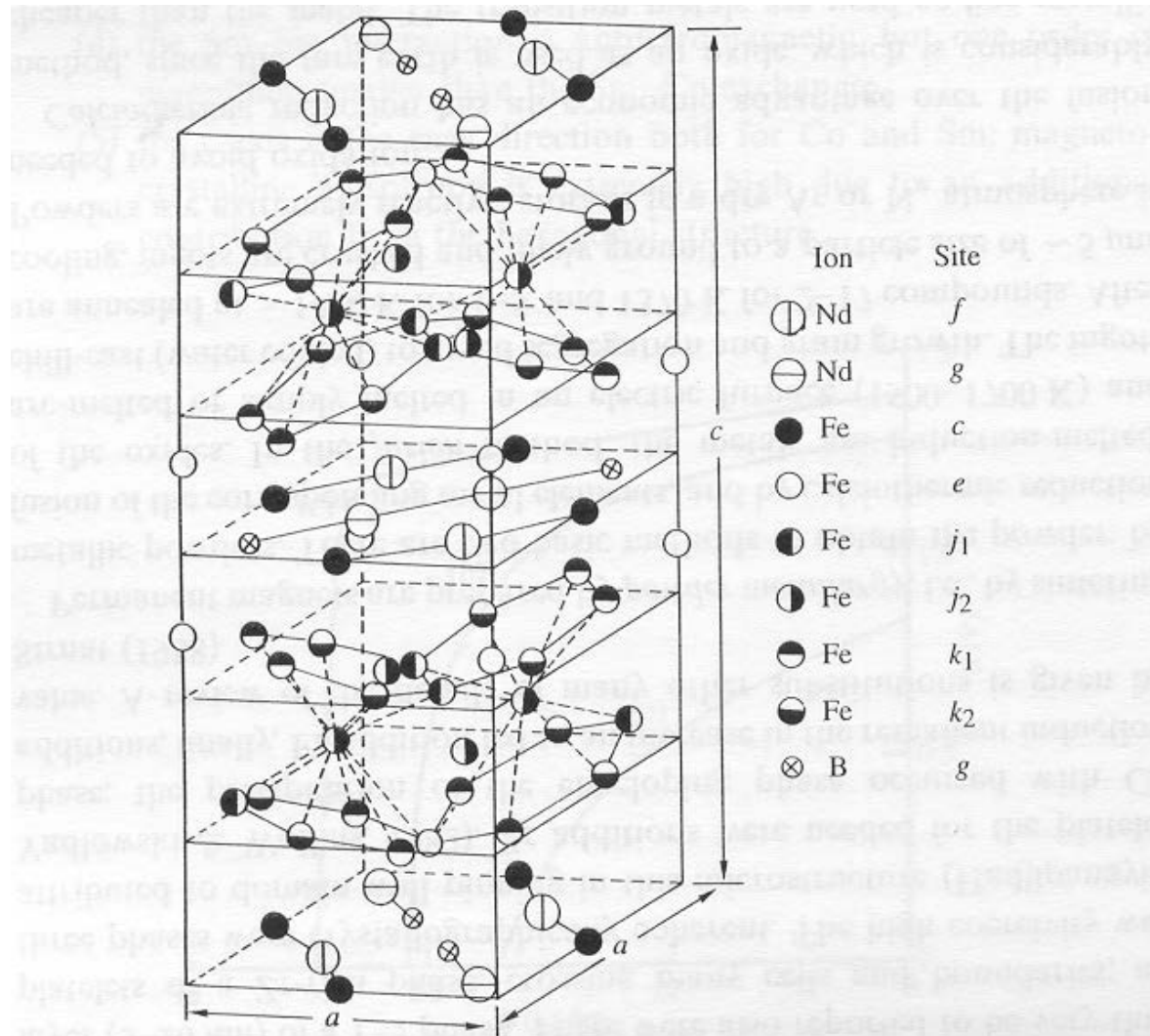
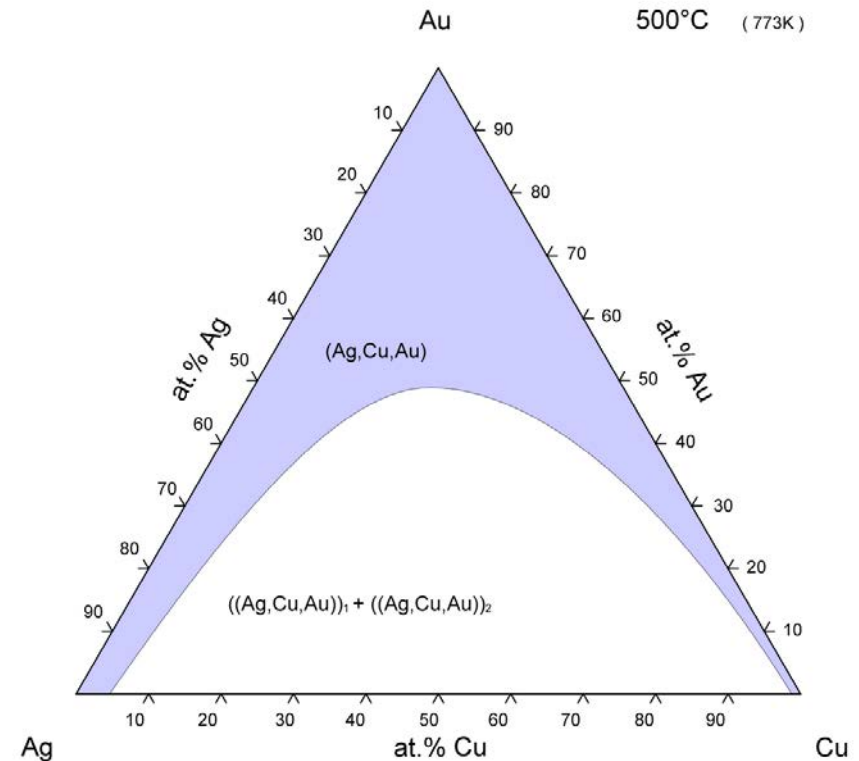
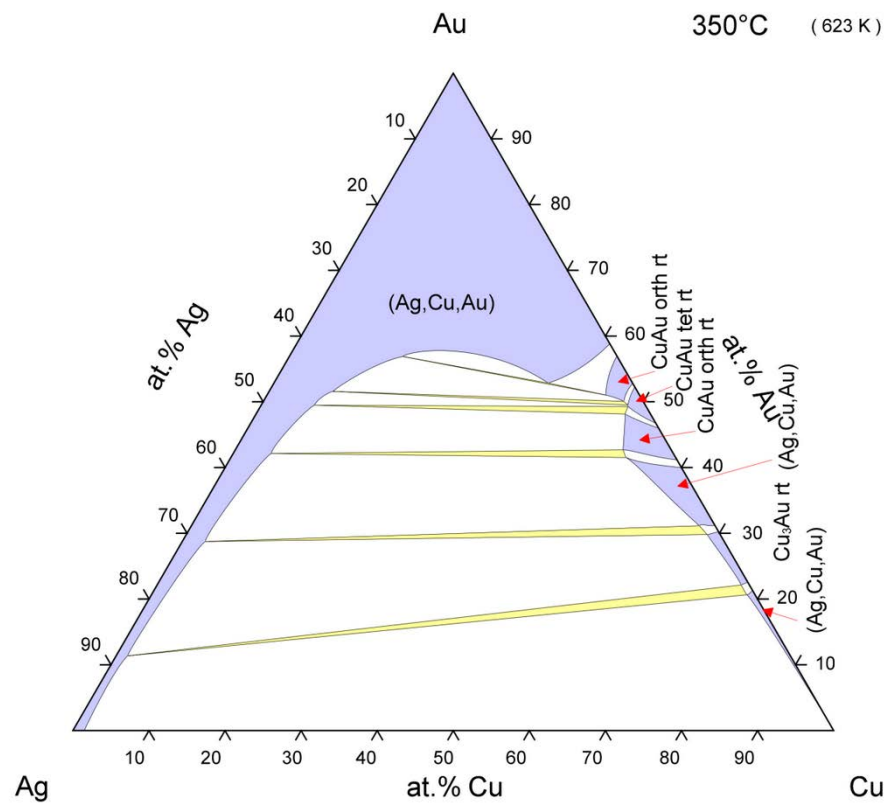
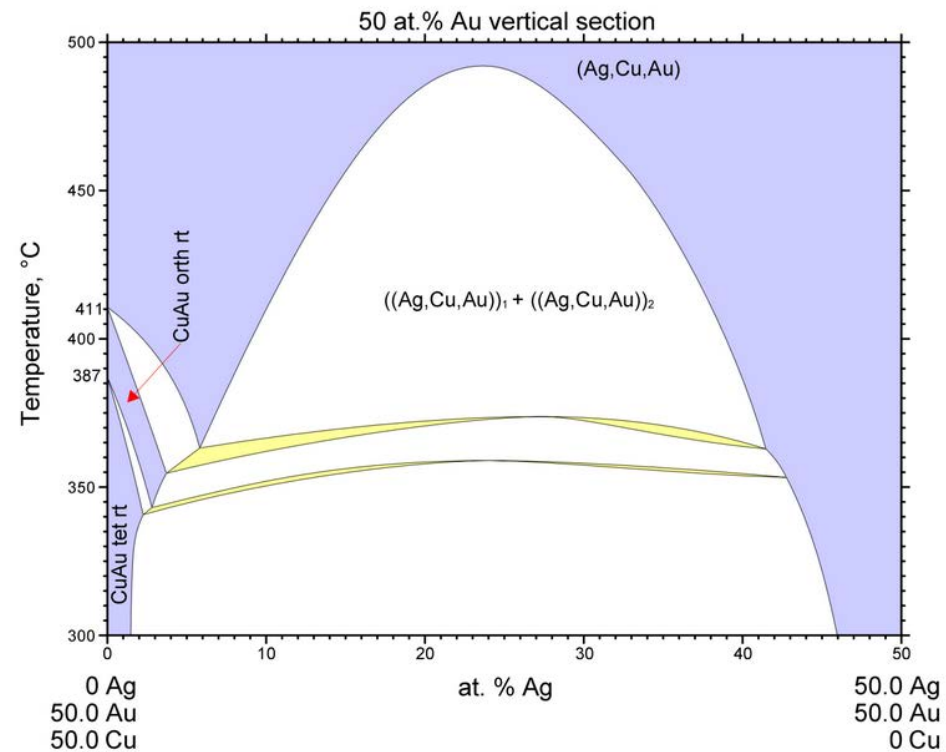
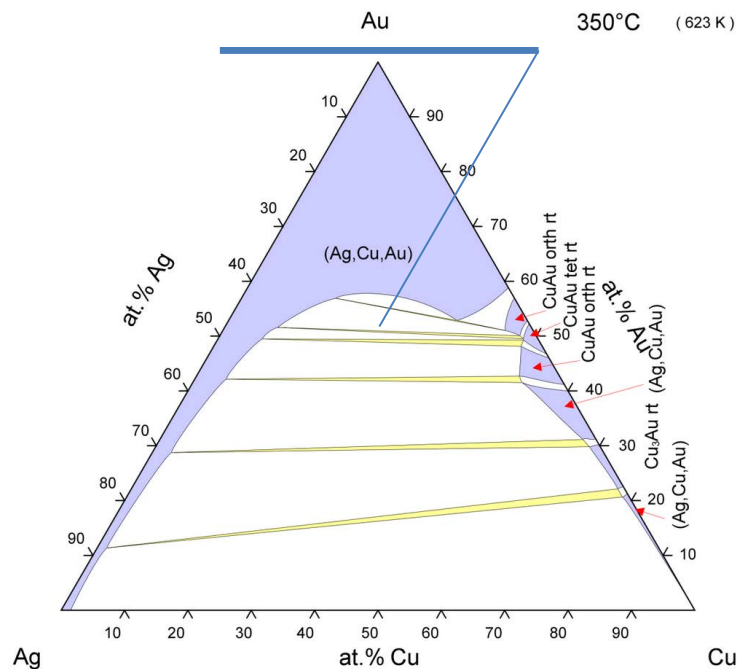


Fig. 6.30. The unit cell of $\text{Nd}_2\text{Fe}_{14}\text{B}$, showing the different Fe sites. (Adapted from Croat *et al.*, 1984)

Ag-Au-Cu: isothermal diagrams



Ag-Au-Cu: vertical section



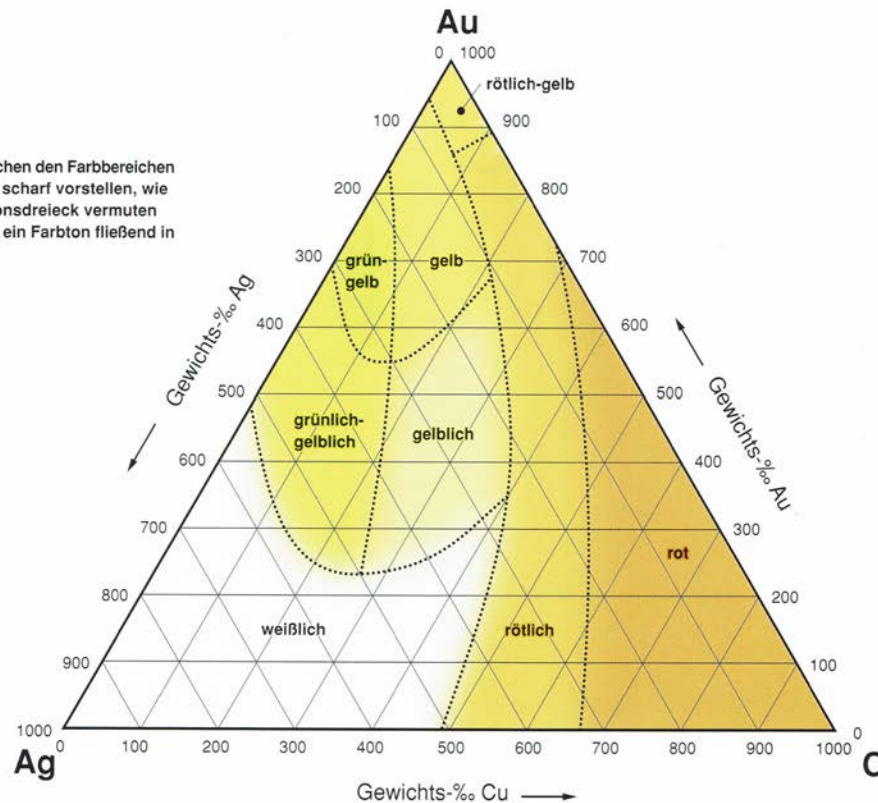
© ASM International 2006. Diagram No. 1300495

Ternary phase diagram of jewelry metals: color

FARBBEREICHE der Gold-Silber-Kupfer-Legierungen (nach Dries und Leuser)

Die Übergänge zwischen den Farbbereichen darf man sich nicht scharf vorstellen, wie es das Konzentrationsdreieck vermuten lässt. Vielmehr geht ein Farbton fließend in den anderen über.

Der intensivste Grünton einer Gold-Silber-Kupfer-Legierung (Cadmium-frei) tritt bei Au646 / Ag354 / Cu0 auf.



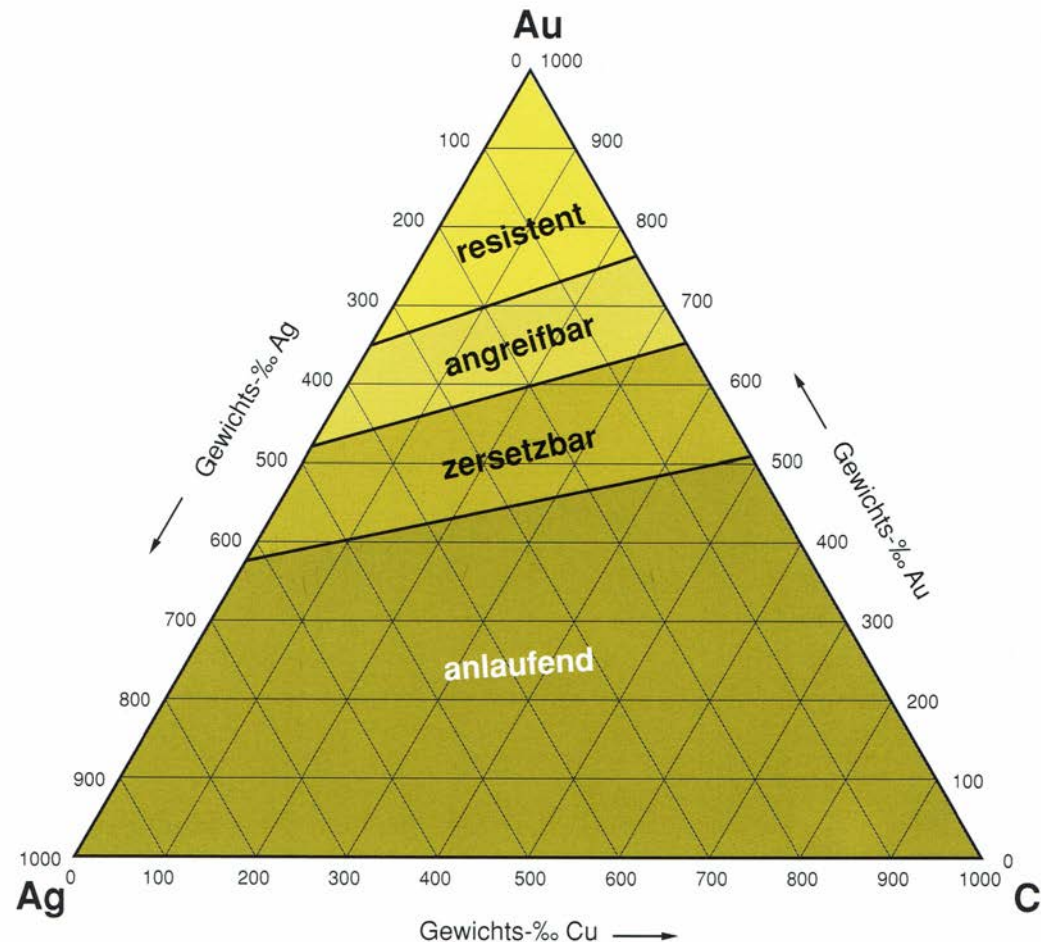
Ternary phase diagram of jewelry metals: chemical resistance

CHEMISCHE BESTÄNDIGKEIT der Gold-Silber-Kupfer-Legierungen (nach Tammann und Sterner-Rainer)

Folgende vier Bereiche chemischer Beständigkeit, die scharf gegeneinander abgegrenzt sind, lassen sich unterscheiden:

- Legierungen mit mehr als 50 Atom-% Gold (dies entspricht etwa 750 Gewichts-%) sind gegen chemische Einflüsse vollkommen resistent und lassen sich nur in Königswasser lösen.
- Legierungen mit 37,5 bis 50 Atom-% Gold (ca. 650 bis 750 Gew.-%) werden von starken Säuren (z.B. Salpetersäure) angegriffen. Dabei lösen sich die Kupfer- und Silberanteile soweit auf, bis sich ein Goldanteil von 50 Atom-% gebildet hat. Dies hat gleichzeitig eine Farbänderung ins Goldgelbe zur Folge.
- Legierungen mit Goldgehalten von 25 bis 37,5 Atom-% (ca. 530 bis 650 Gew.-%) werden in starken Säuren völlig zersetzt, indem die Zusatzmetalle in Lösung gehen, das Gold jedoch als Bodensatz zurückbleibt.
- Legierungen, deren Goldanteil unter 25 Atom-% liegt (unter ca. 530 Gew.-%), laufen unter dem Einfluss von Schwefel sowie an der Luft infolge des in der Atmosphäre vorhandenen Schwefelwasserstoffs leicht an.

Unabhängig vom Goldgehalt sind die blassen, also mehr silberhaltigen Legierungen beständiger als die rötlichen, mehr kupferhaltigen Legierungen mit gleichem Feingehalt, da das Edelmetall Silber neben dem Gold zur Beständigkeit mit beiträgt.



Control questions I

See also aims of the lecture questions

- (1) Give a definition for “phase”.
- (2) What is an intermediate phase?
- (3) What physical variable is kept constant in most of the phase diagrams used in materials science?
Please explain why.
- (4) Explain stability and metastability.
- (5) Why is the metastable phase diagram for Fe used more frequently than the stable one?
Which two compounds/elements define the composition axis?
- (6) What is the triple point and in what type of phase diagrams can it be found?
- (7) How many phases can occur in a unary system?
- (8) What are the prerequisites for a solid solution?
- (9) Sketch a peritectic, eutectic and eutectoid reaction.
- (10) What is the lever rule? Write down and explain the formula.
- (11) Explain Vegard’s law. For what type of phase is it applicable?
- (12) What is a tie line?
- (13) What are the liquidus and the solidus line?
- (14) What is a liquidus projection?
- (15) Draw a Gibbs triangle and indicate a line of constant composition ration $A/B=1$.

Control questions II

- (1) Name all polymorphs of pure Fe and the respective crystal structures.
- (2) How can temperature-dependent phase transformations be determined?
- (3) Give three examples for binaries with complete miscibility.
- (4) Name a practical application for the eutectic point in the Au-Si system.
- (5) Give three examples for prominent metal-gas system.
- (6) Why does solubility generally increase in solid solutions with temperature?
- (7) What is a congruent melting point?
- (8) Determine phase compositions in a ternary system
- (9) Apply the phase rule for a ternary system.
- (10) How many phases exist in the eutectic point of an isobaric ternary system?
- (11) What is an isopleth?
- (12) Explain isothermal and isopleth cuts in a ternary system
- (13) Indicate solidus and solvus surfaces in a three-dimensional ternary phase diagram
- (14) What is a tie triangle?
- (15) For what can you use an Ellingham diagram?