Net Present Value (NPV)

Net Present Value (NPV)

Using the NPV allows to assess the profitability of an investment by discounting all expected future cash flows (expenditures and revenues) to the present moment of time.

Because of monetary depreciation, it is necessary to discount the annual cash flows depending on the interest rate and the project lifetime.

Why is the NPV used?

- To make an informed decision based on a calculated monetary value, which issues if an investment is economically viable.

In general, the NPV is given as

$$\mathrm{NPV} = \sum_{t=0}^{n} \left(\frac{\mathrm{Revenue}_t - \mathrm{Expenditure}_t}{(1+i)^t} \right) = \sum_{t=0}^{n} \left(\frac{\mathrm{Net\ Cash\ Flow}_t}{(1+i)^t} \right)$$

i: interest rate
t: time period
n: project lifetime

Where $\frac{1}{(1+i)^t}$ corresponds to the discount factor for which also tabulated values are given for different interest rates depending on the project lifetime.

Net Present Value (NPV)

If there is no other cash flow than the investment in time period 0 and there are no more investments needed afterwards, then it simplifies to

$$\mathrm{NPV} = -\mathrm{Investment} + \sum_{t=1}^{n} \left(\frac{\mathrm{Net\ Cash\ Flow}}{(1+i)^t} \right)$$

i: interest rate
t: time period
n: project lifetime

And further for additionally constant cash flows to

$$\mathrm{NPV} = -\mathrm{Investment} + \mathrm{Net\ Cash\ Flow} \cdot \sum_{t=1}^{n} \left(\frac{1}{(1+i)^t} \right)$$

Net Present Value (NPV)

Economic significance

- NPV > 0: economically profitable
- NPV = 0: no economic significance
- NPV < 0: economically not profitable

➢ When comparing different potential project investments, one might choose the one **with the highest NPV**.

Net Present Value (NPV)

Sample calculation

You would like to decide whether to invest in a project or not. For that, you calculate the NPV. Note that there is just one investment to be made in time period 0.

$$\mathrm{NPV} = \sum_{t=0}^{n}\left(\frac{\mathrm{Revenue}_t - \mathrm{Expenditure}_t}{(1+i)^t}\right) = \sum_{t=0}^{n}\left(\frac{\mathrm{Net\ Cash\ Flow}_t}{(1+i)^t}\right)$$

Project data

	Var.	Value
interest rate	i	3%
project lifetime	n	20 years
investment in t_0	I	20,000 €
yearly expenditure	E	100 €
yearly revenue	R	1,000 €

NPV (constant cash flows):

$$\mathrm{NPV} = -\mathrm{Investment} + \mathrm{Net\ Cash\ Flow} \cdot \sum_{t=1}^{n=20}\left(\frac{1}{(1+i)^t}\right)$$

$$\sum_{t=1}^{n=20}\left(\frac{1}{(1+i)^t}\right) = \frac{1}{(1+0.03)^1} + \frac{1}{(1+0.03)^2} + \cdots + \frac{1}{(1+0.03)^{20}} \cong 14.877$$

$$\mathrm{NPV} = -\mathrm{Investment} + \mathrm{Net\ Cash\ Flow} \cdot 14.877 = -20{,}000\,€ + (1{,}000\,€ - 100\,€) \cdot 14.877 = -6{,}610€$$

Since the NPV is < 0, the project is not profitable.

Net Present Value (NPV)

Sample calculation

Suddenly, the circumstances for the previous investment decision change. You can now expect a revenue increase in year 3 of your project. You recalculate the NPV and rethink your investment decision.

NPV (variable cash flows):

$$\mathrm{NPV} = -\mathrm{Investment} + \sum_{t=1}^{n=20} \left(\frac{\mathrm{Net\ Cash\ Flow}}{(1+i)^t} \right)$$

$$\text{Net Cash Flow in } t_1 \text{ and } t_2 = 1{,}000\ € - 100\ € = 900\ €$$

$$\text{Net Cash Flow after } t_2 = 1{,}600\ € - 100\ € = 1{,}500\ €$$

$$\mathrm{NPV} = -20{,}000\ € + \frac{900\ €}{(1+0.03)^1} + \frac{900\ €}{(1+0.03)^2} + \frac{1.500\ €}{(1+0.03)^3} + \cdots + \frac{1.500\ €}{(1+0.03)^{20}} = 1{,}168\ €$$

Since the NPV is < 1, the project is profitable.

Project data

	Var.	Value
interest rate	i	3%
project lifetime	n	20 years
investment in t_0	I	20,000 €
yearly expenditure	E	100 €
revenue $t_1 – t_2$	R	1,000 €
revenue $t_3 – t_{20}$	R	1,500 €

Internal Rate of Return (IRR)

Internal Rate of Return (IRR)

Another investment appraisal method is the IRR, which is directly related to the NPV by showing which theoretical interest rate would be required to equal the NPV to zero for its specific temporal conditions.

Why is the IRR used?

- The IRR is more comparative than the NPV by weighing up which one of several prospective investments is the most worthwhile.

Mathematically the IRR is given as:

$$\sum_{t=0}^{n}\left(\frac{\text{Net Cash Flow}_t}{(1+i)^t}\right)=0 \qquad \text{wherein } i=\text{IRR}$$

Internal Rate of Return (IRR)

Calculating the IRR manually is unfavourable when n > 2, which causes the need for calculation programs providing graphical or simulated solutions.

→ E.g. **in Excel** with the use of the **IRR function**:
English: =*IRR(*[Net Cash Flows[1]]*)*
German: =*IKV(*[Net Cash Flows[1]])

[1]Net Cash Flows: Cash flows including the investment(s)

Economic significance

- The investment is worthwhile if the IRR is **greater than** the **Minimum Acceptable Rate of Return (MARR)**.
- If several potential investments are compared, choice of potential investment **with the highest IRR**.

Internal Rate of Return (IRR)

Sample calculation

Manually for n = 1:

$$\sum_{t=0}^{n}\left(\frac{\text{Net Cash Flowt}}{(1+i)^t}\right)=0 \qquad \text{wherein } i=\text{IRR}$$

$$\leftrightarrow -\text{Investment}+\sum_{t=1}^{n=1}\left(\frac{\text{Net Cash Flowt}}{(1+\text{IRR})^t}\right)=0$$

$$\text{NPV}=-\text{Investment}+\text{Net Cash Flow}\cdot\sum_{t=1}^{n=20}\left(\frac{1}{(1+i)^t}\right)$$

$$\leftrightarrow \text{IRR}=\frac{500\text{ Mio €}}{480\text{ Mio €}}-1=0.4167=4.167\% \qquad <12\%\text{ MARR } \rightarrow \text{investment is not worthwhile}$$

Project data

	Var.	Value
project lifetime	n	1 year
investment in t_0	I	480 Mio €
net cash flow in t_1		500 Mio €
MARR		12%

Internal Rate of Return (IRR)

Sample calculation

Computationally for n > 1:

$$\sum_{t=0}^{n}\left(\frac{\text{Net Cash Flow}t}{(1+i)^{t}}\right)=0 \qquad \text{wherein } i = \text{IRR}$$

$$\leftrightarrow \sum_{t=0}^{n=10}\left(\frac{\text{Net Cash Flow}t}{(1+i)^{t}}\right)=0$$

$$\leftrightarrow \text{e. g. in Excel: IRR} = \text{IRR}\left(\sum_{t=0}^{n=10}(\text{Net Cash Flow}t)\right)=14.472\%$$

$$> 12\% \text{ MARR } \rightarrow \text{investment is worthwhile}$$

Project data

	Var.	Value
project lifetime	n	**10** years
investment in t_0	I	480 Mio €
yearly net cash flow		**80** Mio €
MARR		12%

Annuity

Annuity

The annuity is based on the NPV method by apportioning it into regularly recurring cash flows over the project lifetime.

Why is the Annuity used?

- Pointing out an annual cash flow instead of one monetary value (NPV) as an additional consideration. Depending on the given framework conditions it may be even preferable to use in an economic assessment.

As result of the monetary depreciation it is again necessary to take the interest rate as well as the time period into account. So the annuity is given as

$$\begin{aligned}\text{Annuity} &= -\text{annualised investment cost} - \text{yearly expenditures} + \text{yearly revenues} \\ &= \sum_{t=0}^{n}\left(\frac{\text{Net Cash Flow}_t}{(1+i)^t}\right)\cdot a \qquad \text{with } a = \frac{(1+i)^n \cdot i}{(1+i)^n - 1} \\ &= \text{NPV}\cdot a\end{aligned}$$

a: annuity factor
i: interest rate
n: project lifetime

Its values are also tabulated in e.g. VDI 2067 sheet 1 depending on the interest rate and the time period.

Annuity

Economic significance (analogous to NPV)

- Annuity > 0: economically profitable
- Annuity = 0: no economic significance
- Annuity < 0: economically not profitable

➢ Choice of potential investment with the highest annuity.

Annuity

Sample calculation

You would like to decide whether to invest in a project or not. For that, you calculate the annuity.

$$\text{Annuity} = \sum_{t=0}^{n} \left(\frac{\text{Net Cash Flow}_t}{(1+i)^t}\right) \cdot a$$

$$\text{with } a = \frac{(1+i)^n \cdot i}{(1+i)^n - 1} = \frac{(1+0.04)^{20} \cdot 0.04}{(1+0.04)^{20} - 1} = 0.0736\,\frac{1}{a}$$

$$\text{with NPV} = -\text{Investment} + \text{Net Cash Flow} \cdot \sum_{t=1}^{n=20} \left(\frac{1}{(1+i)^t}\right)$$

$$= -20{,}000\text{ €} + (1{,}000\text{ €} - 100\text{ €}) \cdot 13.590 = -7{,}768\text{ €}$$

$$\text{Annuity} = \text{NPV} \cdot a = -7{,}768\text{ €} \cdot 0.0736\,\frac{1}{a} = -571\,\frac{\text{€}}{a}$$

Project data

	Var.	Value
interest rate	i	4%
project lifetime	n	20 years
investment in t_0	I	20,000 €
yearly expenditures	C_e	100 €
yearly revenues	C_r	1,000 €

If the project is invested in, the annuity would be -571 €/a. Therefore, the project is economically not profitable.

Annuity

Sample calculation

The annuity factor can also be looked-up in VDI 2067 sheet 1:

with a

$$= \frac{(1+i)^n \cdot i}{(1+i)^n - 1}$$

$$= \frac{(1+0.04)^{20} \cdot 0.04}{(1+0.04)^{20} - 1}$$

$$= 0.0736 \frac{1}{a}$$

tabulated

	Interest rate							
Lifetime	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
6	0.1846	0.1908	0.1970	0.2034	0.2098	0.2163	0.2229	0.2296
7	0.1605	0.1666	0.1728	0.1791	0.1856	0.1921	0.1987	0.2054
8	0.1425	0.1485	0.1547	0.1610	0.1675	0.1740	0.1807	0.1874
9	0.1284	0.1345	0.1407	0.1470	0.1535	0.1601	0.1668	0.1736
10	0.1172	0.1233	0.1295	0.1359	0.1424	0.1490	0.1558	0.1627
11	0.1081	0.1141	0.1204	0.1268	0.1334	0.1401	0.1469	0.1540
12	0.1005	0.1066	0.1128	0.1193	0.1259	0.1327	0.1397	0.1468
13	0.0940	0.1001	0.1065	0.1130	0.1197	0.1265	0.1336	0.1408
14	0.0885	0.0947	0.1010	0.1076	0.1143	0.1213	0.1284	0.1357
15	0.0838	0.0899	0.0963	0.1030	0.1098	0.1168	0.1241	0.1315
16	0.0796	0.0858	0.0923	0.0990	0.1059	0.1130	0.1203	0.1278
17	0.0760	0.0822	0.0887	0.0954	0.1024	0.1096	0.1170	0.1247
18	0.0727	0.0790	0.0855	0.0924	0.0994	0.1067	0.1142	0.1219
19	0.0698	0.0761	0.0827	0.0896	0.0968	0.1041	0.1117	0.1195
20	0.0672	0.0736	0.0802	0.0872	0.0944	0.1019	0.1095	0.1175

Project data

	Var.	Value
interest rate	i	**4%**
project lifetime	n	**20 years**
investment in t_0	I	20,000 €
yearly expenditures	C_e	100 €
yearly revenues	C_r	1,000 €

Discounted Payback Period (DPP)

Discounted Payback Period (DPP)

The dynamic DPP is applied to calculate the time it takes for the yearly revenues to pay off the investment and the yearly expenditures. From this point on, the total revenue becomes positive and the investment is profitable.

Why is the DPP used?

- To compare several potential investments based on the time required to make profits, if all investments are of similar economic viability.

As its name says, the DPP takes monetary depreciation into account. It depends on the interest rate and the project lifetime and is given as:

$$\mathrm{DPP} = \frac{\mathrm{Investment}}{\sum_{t=1}^{n} \mathrm{Discounted\ Cash\ Flows}_t}$$

Discounted Payback Period (DPP)

Economic significance

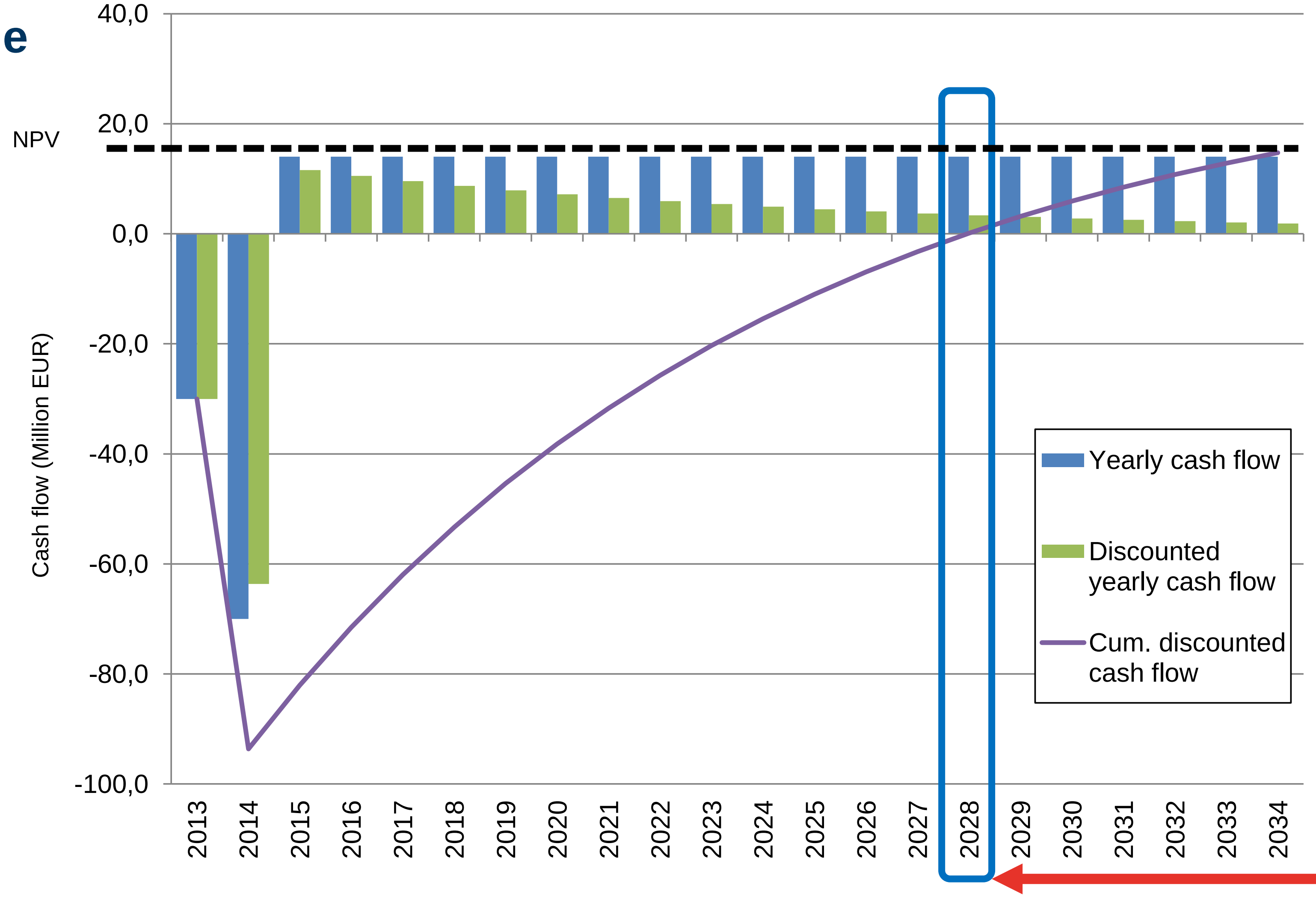
- DPP < project lifetime: economically profitable
- DPP = project lifetime: no economic significance
- DPP > project lifetime: economically not profitable

➢ Choice of shortest DPP to earn profits as fast as possible.

However: DPP should be used just as an additional profitability factor.

Discounted Payback Period (DPP)

Example



It takes approximately 14 years for the initial investment to be covered by the discounted cash flows.

Discounted Payback Period (DPP)

Sample calculation

You would like to decide in which potential project to invest. For that, you calculate the DPPs.

$$DPP = \frac{\text{Investment}}{\sum_{t=1}^{n} \text{Discounted Cash Flows}_t}$$

Project data

	Var.	Value
interest rate	i	3%
project lifetimes	n	10 years
project 1:		
investment in t_0	I	25,000 €
yearly expenditures	C_e	100 €
yearly revenues	C_r	5,000 €
project 2:		
investment in t_0	I	16,000 €
yearly expenditures	C_e	1,000 €
yearly revenues	C_r	3,000 €

Discounted Payback Period (DPP)

Sample calculation – project 1

1. Calculate the **discounted cash flows per year**.
2. **Accumulate** the discounted cash flows per year.
3. **Determine the year** in which the cumulative cash flows cover the investment. In year 6, the cumulative cash flows reach 26,544 €, which covers the initial investment of 25,000 €.
4. Calculate the **exact point in time**. In year 5, 25,000 € - 22,441 € = 2,559 € is still missing. The cash flow in year 6 is 4,104 €. Proportion of year 5 that is required: 2,559 € / 4,104 € ≈ 0.62 years.
5. Calculate the **discounted payback period**. DPP = 5 years + 0.62 years ≈ 5.62 years. This means that it takes approximately 5.62 years for the initial investment to be covered by the discounted cash flows.

Project data

	Var.	Value
interest rate	i	3%
project lifetimes	n	10 years
project 1:		
investment in t_0	I	25,000 €
yearly expenditures	C_e	100 €
yearly revenues	C_r	5,000 €
project 2:		
investment in t_0	I	16,000 €
yearly expenditures	C_e	1,000 €
yearly revenues	C_r	3,000 €

Discounted Payback Period (DPP)

Sample calculation – project 2

1. Calculate the **discounted cash flows per year**.
2. **Accumulate** the discounted cash flows per year.
3. **Determine the year** in which the cumulative cash flows cover the investment.
   In year 9, the cumulative cash flows reach 17,476 €, which covers the initial investment of 16,000 €.
4. Calculate the **exact point in time**.
   In year 8, 16,000 € - 15,534 € = 466 € is still missing.
   The cash flow in year 9 is 1,533 €.
   Proportion of year 9 that is required: 466 € / 1,533 € ≈ 0.30 years.
5. Calculate the **discounted payback period**.
   DPP = 8 years + 0.30 years ≈ 8.30 years.
   This means that it takes approximately 8.30 years for the initial investment to be covered by the discounted cash flows.

Project data

	Var.	Value
interest rate	i	3%
project lifetimes	n	10 years
project 1:		
investment in t_0	I	25,000 €
yearly expenditures	C_e	100 €
yearly revenues	C_r	5,000 €
project 2:		
investment in t_0	I	16,000 €
yearly expenditures	C_e	1,000 €
yearly revenues	C_r	3,000 €

Discounted Payback Period (DPP)

Sample calculation

You would like to decide in which potential project to invest. For that, you calculate the DPPs.

$$DPP_{project1} = 5.62\ a$$

$$DPP_{project2} = 8.30\ a$$

Since both DPPs are smaller than the project lifetimes, both potential investments are profitable. However, economically one would decide for the DPP with the shorter payback time, i.e. for project 1.

Project data

	Var.	Value
interest rate	i	3%
project lifetimes	n	10 years
project 1:		
investment in t_0	I	25,000 €
yearly expenditures	C_e	100 €
yearly revenues	C_r	5,000 €
project 2:		
investment in t_0	I	16,000 €
yearly expenditures	C_e	1,000 €
yearly revenues	C_r	3,000 €

Recaps

Net Present Value (NPV)

In general, the NPV is given as

$$\mathrm{NPV} = \sum_{t=0}^{n} \left(\frac{\mathrm{Revenue}_t - \mathrm{Expenditure}_t}{(1+i)^t} \right) = \sum_{t=0}^{n} \left(\frac{\mathrm{Net\ Cash\ Flow}_t}{(1+i)^t} \right)$$

i: interest rate
t: time period
n: project lifetime

Where $\frac{1}{(1+i)^t}$ corresponds to the discount factor for which also tabulated values are given for different interest rates depending on the project lifetime.

If there is no other cash flow than the investment in time period 0 and there are no more investments needed afterwards, then it simplifies to

$$\mathrm{NPV} = -\mathrm{Investment} + \sum_{t=1}^{n} \left(\frac{\mathrm{Net\ Cash\ Flow}}{(1+i)^t} \right)$$

i: interest rate
t: time period
n: project lifetime

And further for additionally constant cash flows to

$$\mathrm{NPV} = -\mathrm{Investment} + \mathrm{Net\ Cash\ Flow} \cdot \sum_{t=1}^{n} \left(\frac{1}{(1+i)^t} \right)$$

Internal Rate of Return (IRR)

Mathematically the IRR is given as:

$$\sum_{t=0}^{n}\left(\frac{\text{Net Cash Flowt}}{(1+i)^{t}}\right)=0 \qquad \text{wherein } i = \text{IRR}$$

Calculating the IRR manually is unfavourable when n > 2, which causes the need for calculation programs providing graphical or simulated solutions.

→ E.g. **in Excel** with the use of the **IRR function**:
English: =*IRR(*[Net Cash Flows[1]]*)*
German: =*IKV(*[Net Cash Flows[1]])

[1]Net Cash Flows: Cash flows including the investment(s)

Annuity

The annuity is given as

$$\begin{aligned}\text{Annuity} &= \text{annualised investment cost} + \text{yearly expenditures} - \text{yearly revenues} \\ &= \sum_{t=0}^{n} \left(\frac{\text{Net Cash Flow}_t}{(1+i)^t} \right) \cdot a \qquad \text{with } a = \frac{(1+i)^n \cdot i}{(1+i)^n - 1} \\ &= \text{NPV} \cdot a\end{aligned}$$

a: annuity factor
i: interest rate
n: project lifetime

Its values are also tabulated in e.g. VDI 2067 sheet 1 depending on the interest rate and the time period.

Discounted Payback Period (DPP)

As its name says the DPP takes monetary depreciation into account. It depends on the interest rate and the project lifetime and is given as:

$$\text{DPP} = \frac{\text{Investment}}{\sum_{t=1}^{n} \text{Discounted Cash Flows}_t}$$

1. Calculate the **discounted cash flows per year**.
2. **Accumulate** the discounted cash flows per year.
3. **Determine the year** in which the cumulative cash flows cover the investment.
4. Calculate the **exact point in time**.
5. Calculate the **discounted payback period**.