Hydraulic groundwater modeling

- Week 11

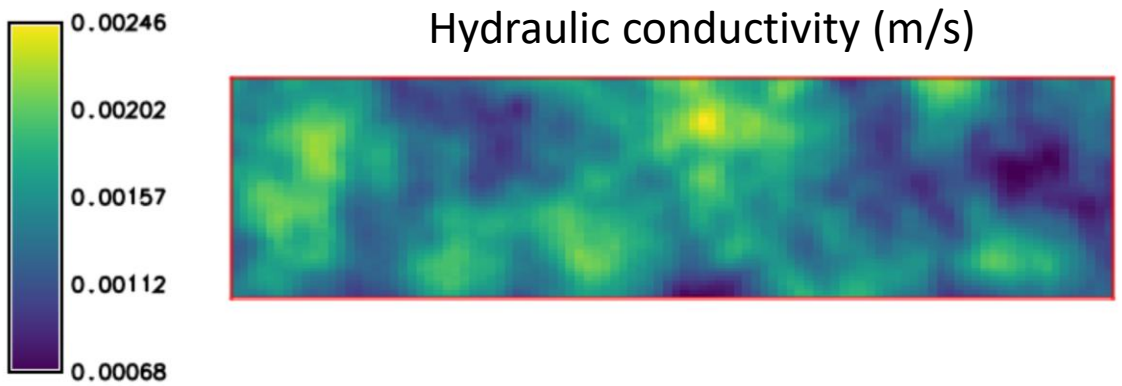
- Assessing the quality of a groundwater model

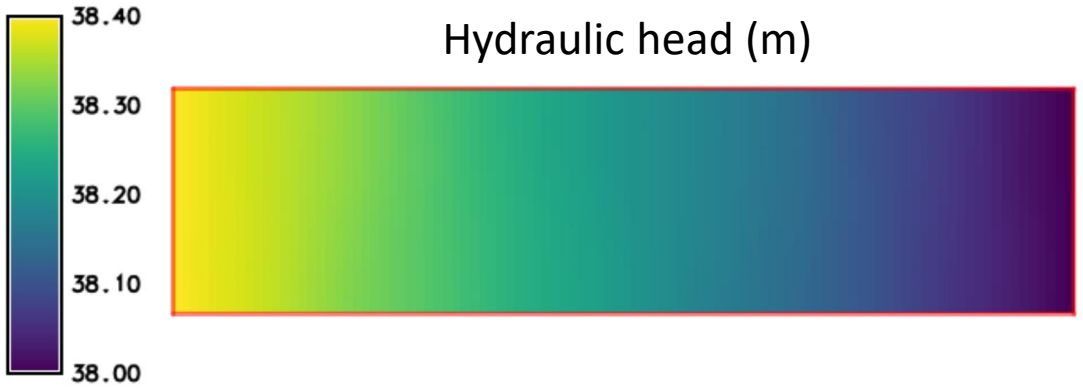
„Quality“ of a model

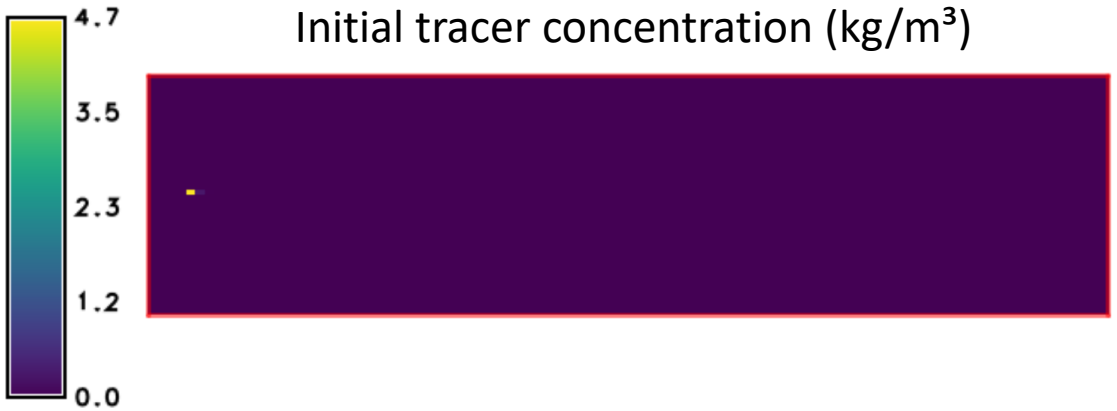
- Stability
- Accuraccy
- Applicability
- Limitations
- …

Example: Solute transport

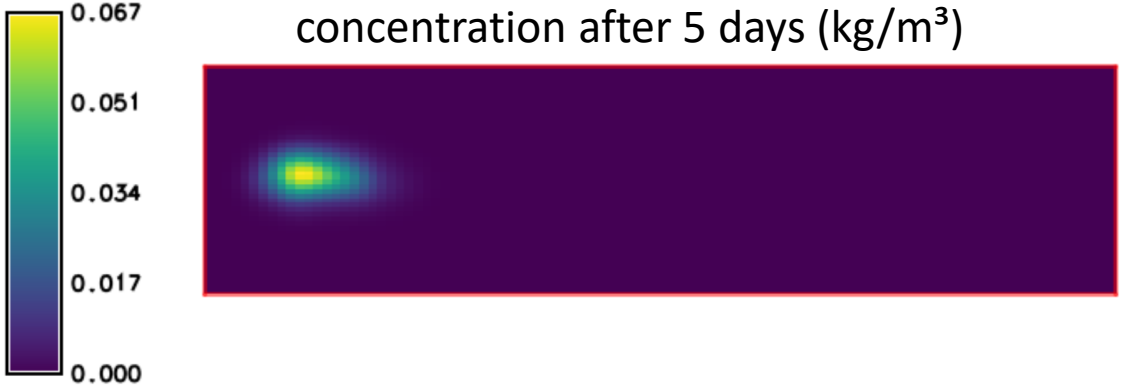
- Tracer injection into a steady but heterogeneous aquifer



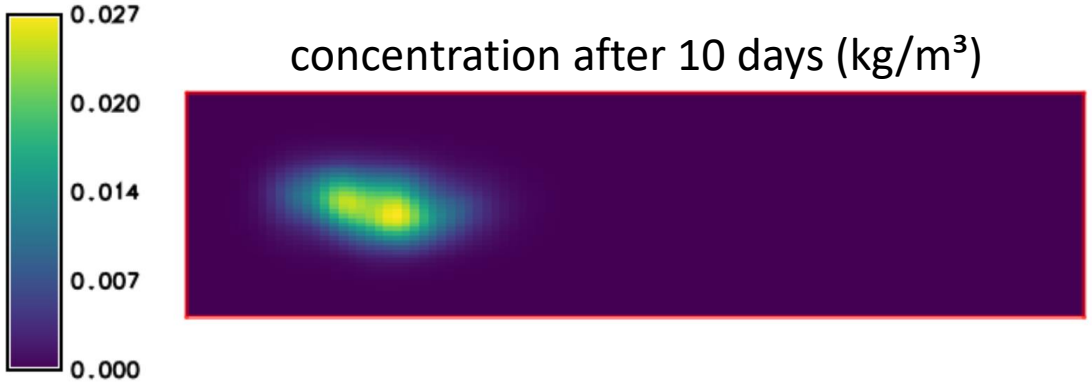




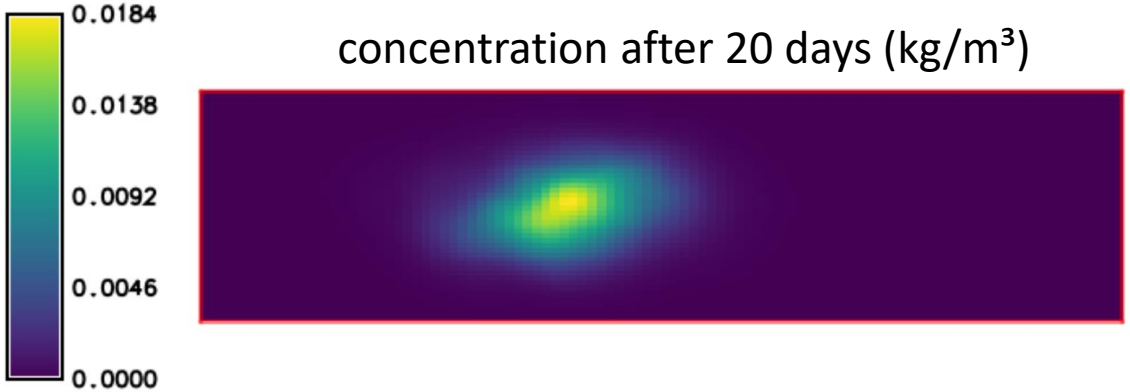
Simulation results



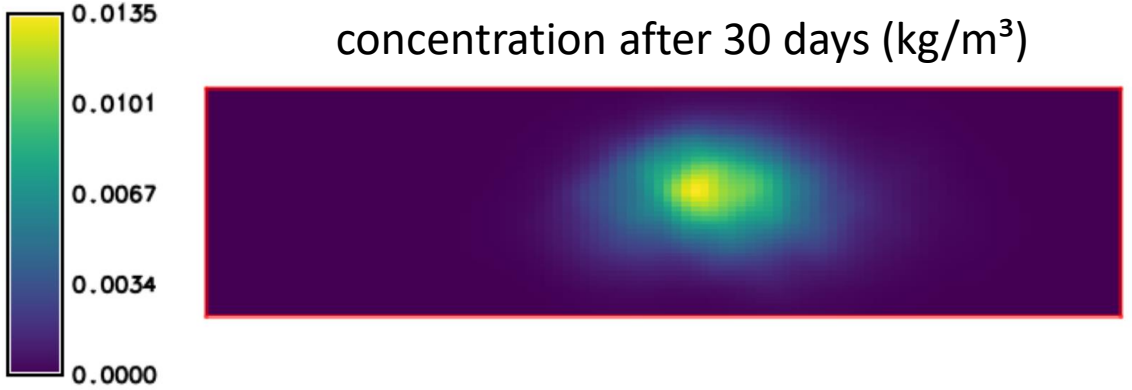
concentration after 5 days (kg/m³)



concentration after 10 days (kg/m³)



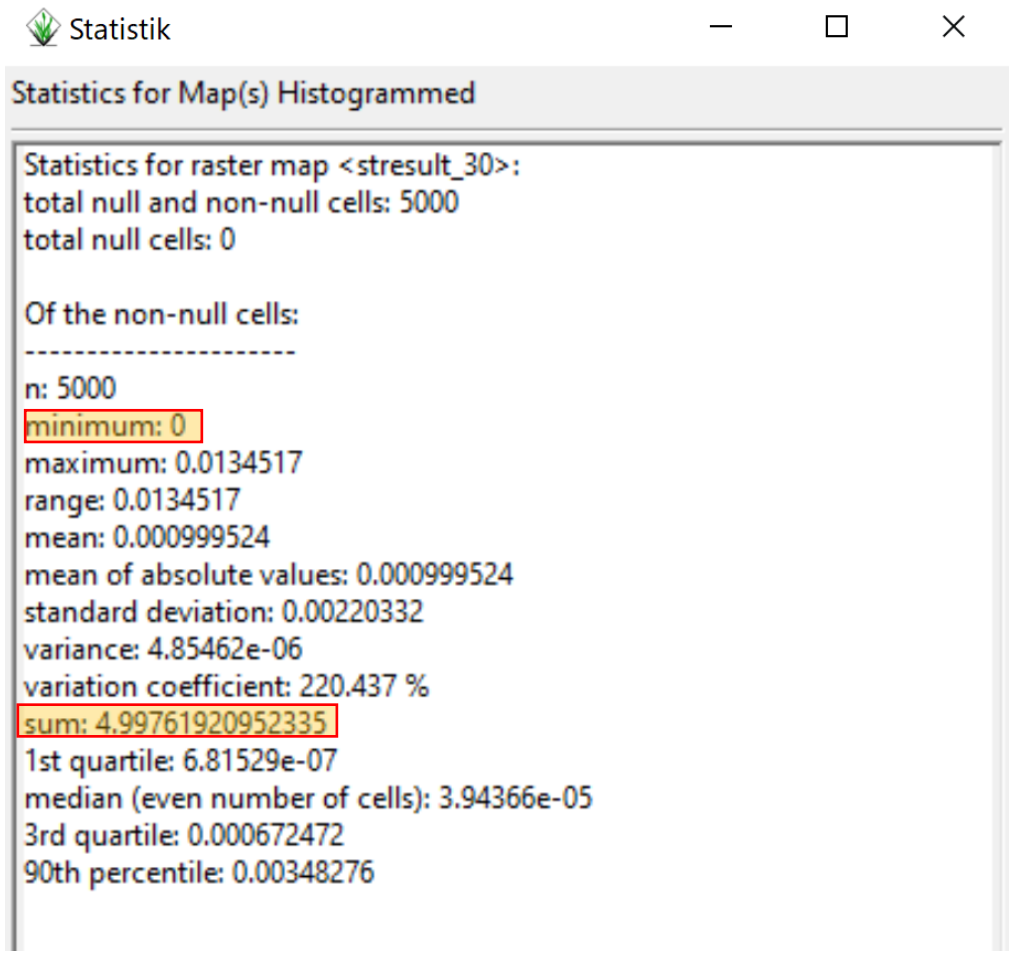
concentration after 20 days (kg/m³)



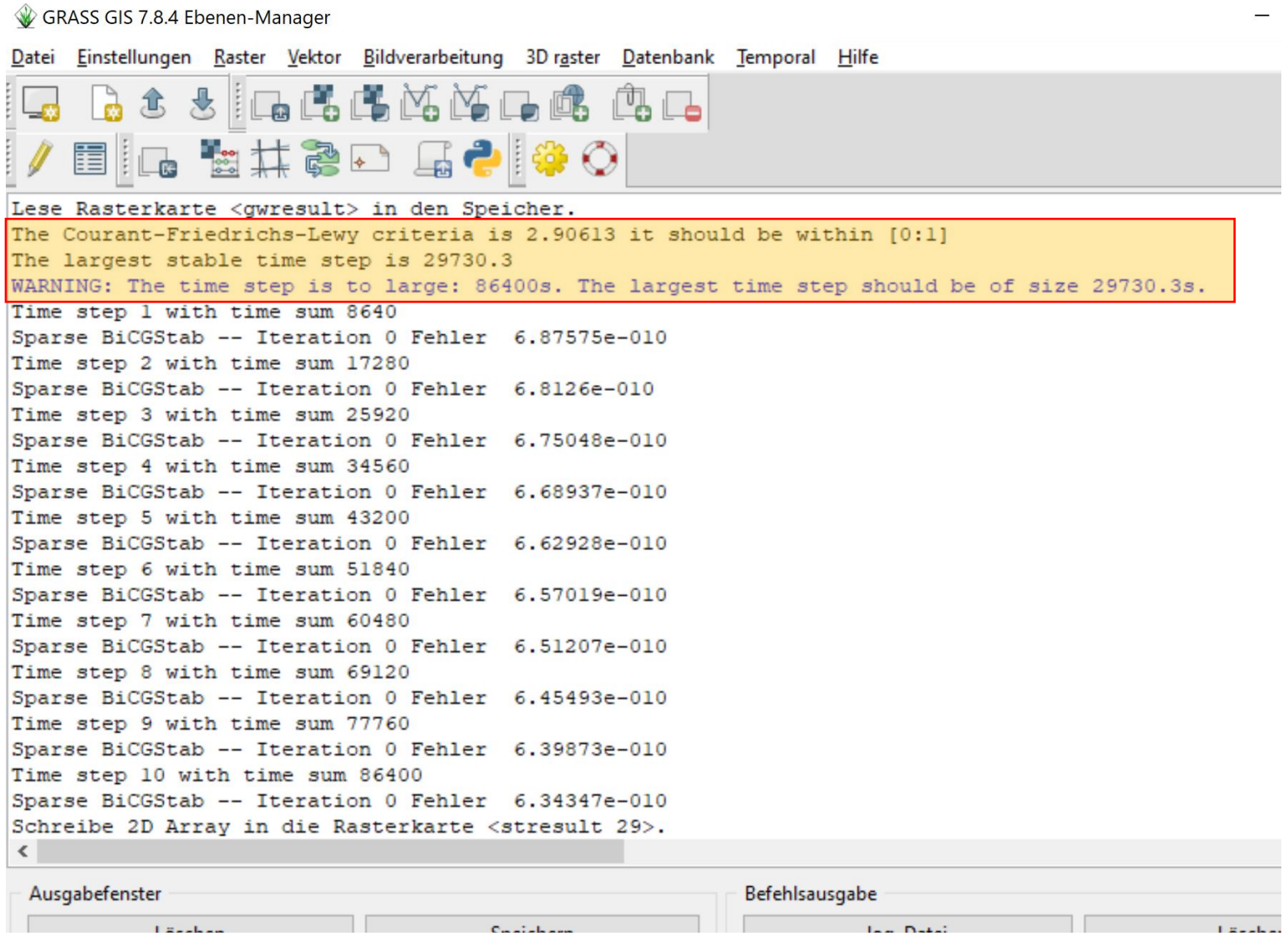
concentration after 30 days (kg/m³)

Simulation results - statistic

- Using the statistics tool:
  - The solution is bounded
  - The model is mass conserving



Look at the output!



The CFL stability criteria

- CFL criteria advection

$$\frac{u\ dt}{dx} < 1$$

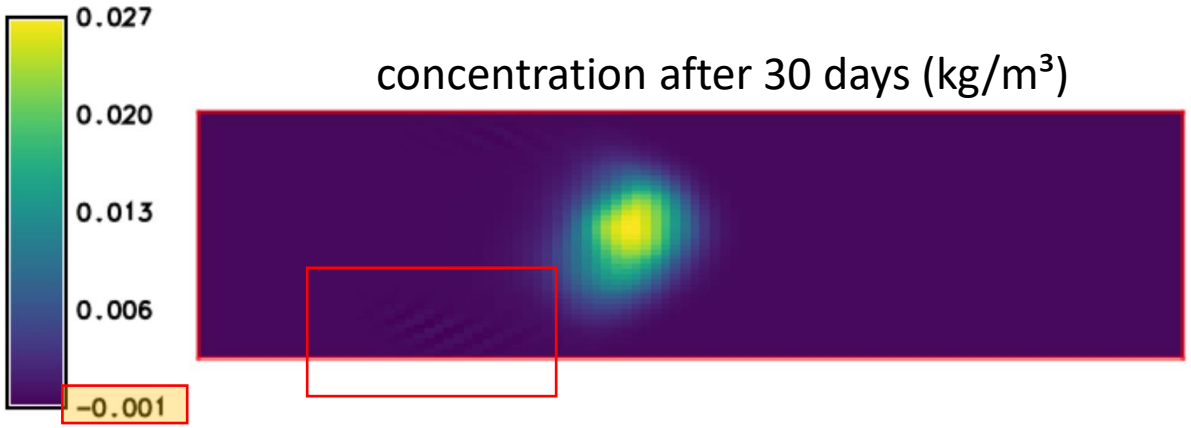
- CFL criteria diffusion (CSFT)
  - Note that D/dx is also velocity (m/s)
  - Usually diffusion much slower than advection

$$\frac{2\ D\ dt}{dx^2} < 1$$

CFL in r.solute.transport

- CFL for advection (source code)
- In GUI: use CFL flag (does not work in Python)
- With Python: Use loops
  - Divides given time step in number of separate steps
  - In shown example: 10 loops

Violating CFL criteria



concentration after 30 days (kg/m³)

```
Lese Rasterkarte <gwresult> in den Speicher.
The Courant-Friedrichs-Lewy criteria is 2.90613 it should be within [0:1]
The largest stable time step is 29730.3
WARNING: The time step is to large: 86400s. The largest time step should be of size 29730.3s.
Time step 1 with time sum 86400
Sparse BiCGStab -- Iteration 0 Fehler  8.90477e-009
Schreibe 2D Array in die Rasterkarte <stresult_30>.
```

Statistik

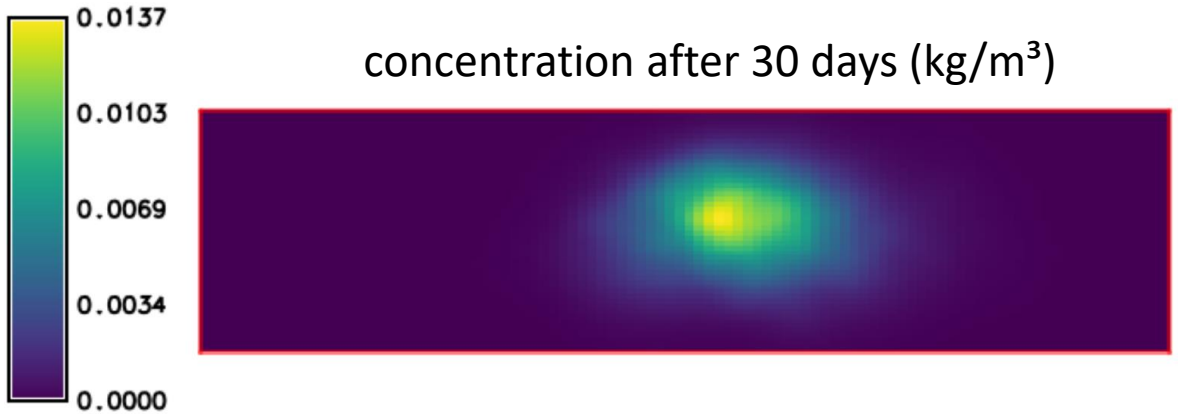
Statistics for Map(s) Histogrammed

Statistics for raster map <stresult_30>:
total null and non-null cells: 5000
total null cells: 0

Of the non-null cells:
---
n: 5000
minimum: -0.000622375
maximum: 0.0271283
range: 0.0277507
mean: 0.000999535
mean of absolute values: 0.00100736
standard deviation: 0.00364895
variance: 1.33148e-05
variation coefficient: 365.065 %
sum: 4.99767704014725
1st quartile: 0
median (even number of cells): 7.72278e-14
3rd quartile: 3.75629e-05
90th percentile: 0.0015221

Decreasing timestep dt

- Half of previous time step (now: 4320s)



Statistik

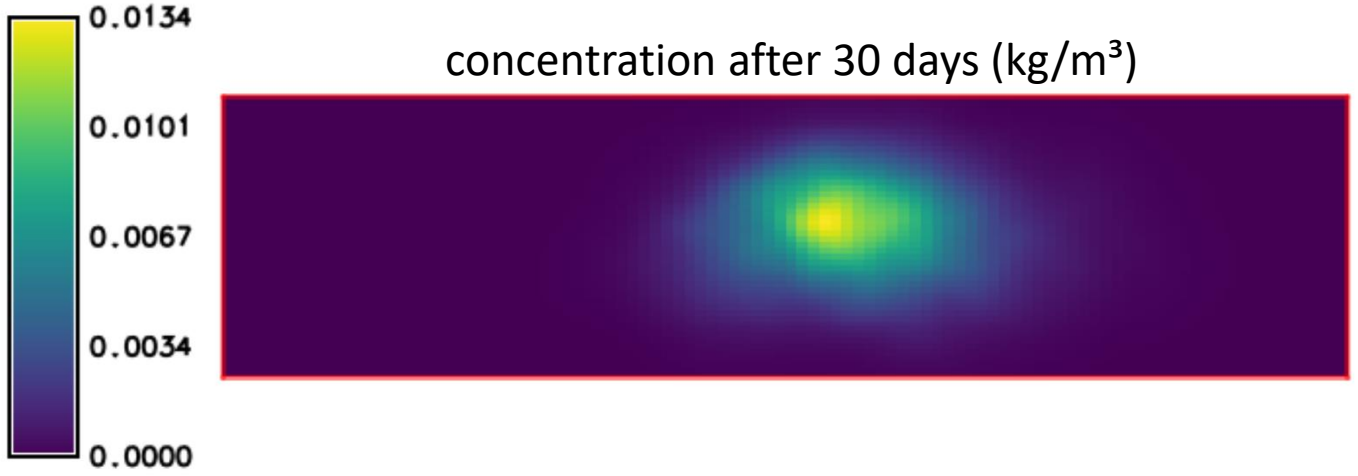
Statistics for Map(s) Histogrammed

Statistics for raster map <stresult_30>:
total null and non-null cells: 5000
total null cells: 0

Of the non-null cells:
---
n: 5000
minimum: 0
maximum: 0.0137242
range: 0.0137242
mean: 0.000999529
mean of absolute values: 0.000999529
standard deviation: 0.00223994
variance: 5.01733e-06
variation coefficient: 224.1 %
sum: 4.99764566533057

- Result: No fundamental change small changes to be expected

Increasing accuraccy



concentration after 30 days (kg/m³)

```
gs.run_command("r.solute.transport", solver="bicgstab", \
    top="top_unconf", bottom="null", phead="gwresult", \
    status="tstatus", q="null",hc_x="hydcond", hc_y="hydcond", rd="R", \
    cs="null", nf="poros", output="stresult_" + str(t + 1), \
    dt=86400, diff_x="diff", diff_y="diff", c="stresult_" + str(t), \
    al=0.01, at=0.01, overwrite=True, loops=10,
    error="0.000000000000001",
    maxit="100000000000")
```

Statistik

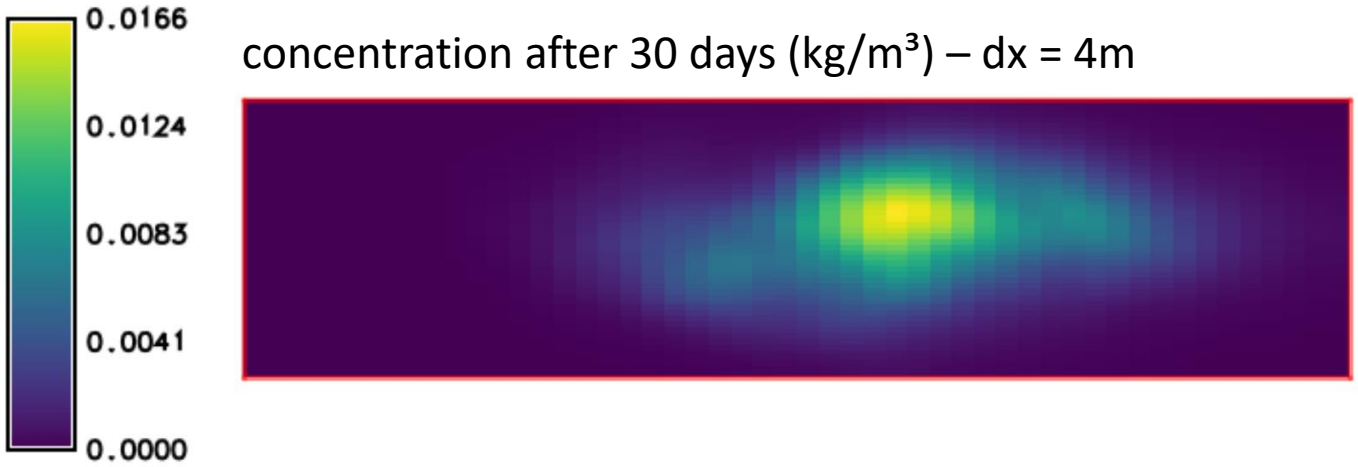
Statistics for Map(s) Histogrammed

Statistics for raster map <stresult_30>:
total null and non-null cells: 5000
total null cells: 0

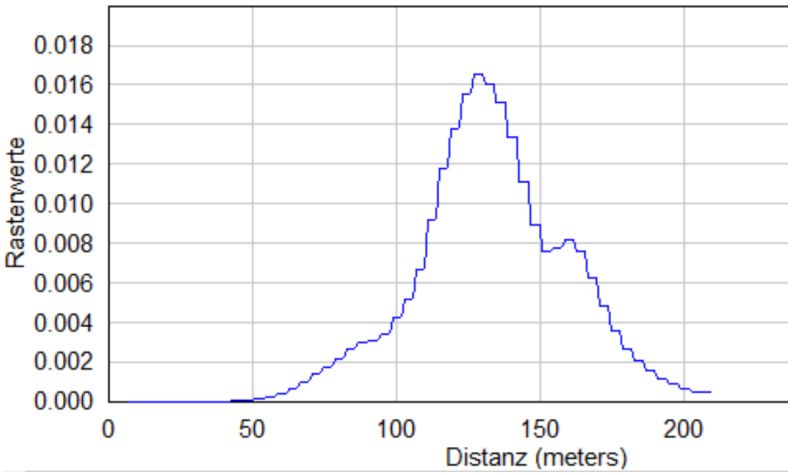
Of the non-null cells:
---
n: 5000
minimum: 0
maximum: 0.013419
range: 0.013419
mean: 0.000999518
mean of absolute values: 0.000999518
standard deviation: 0.00219781
variance: 4.83035e-06
variation coefficient: 219.887 %
sum: 4.99759223891779
1st quartile: 7.47963e-07

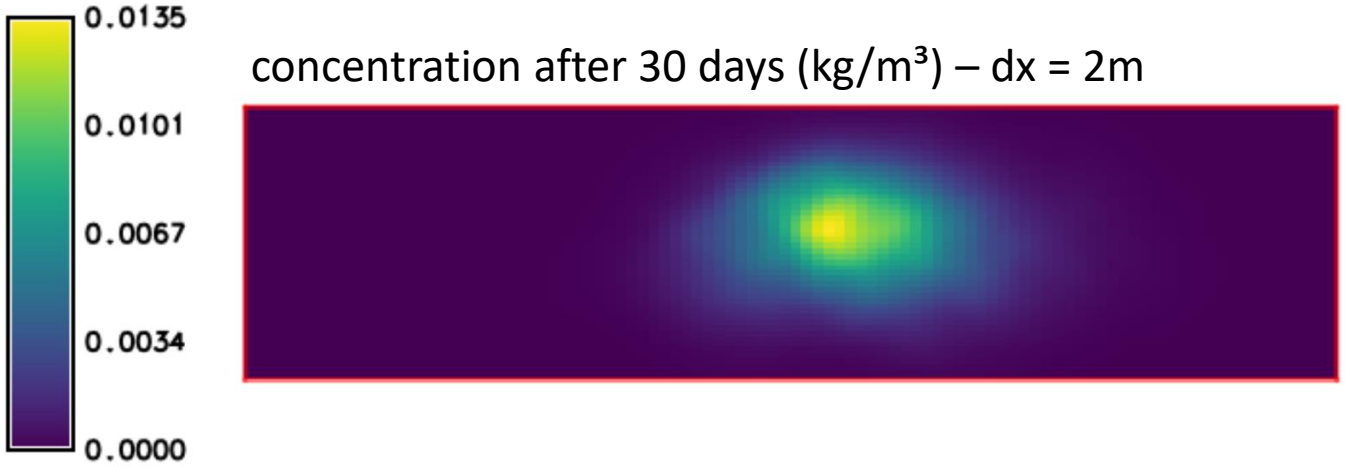
```
Sparse BiCGStab -- Iteration 2 Fehler  4.15716e-021
Time step 7 with time sum 60480
Sparse BiCGStab -- Iteration 0 Fehler  1.27293e-009
Sparse BiCGStab -- Iteration 1 Fehler  1.5883e-015
Sparse BiCGStab -- Iteration 2 Fehler  4.03333e-021
Time step 8 with time sum 69120
Sparse BiCGStab -- Iteration 0 Fehler  1.26355e-009
```

Reduce spatial resolution

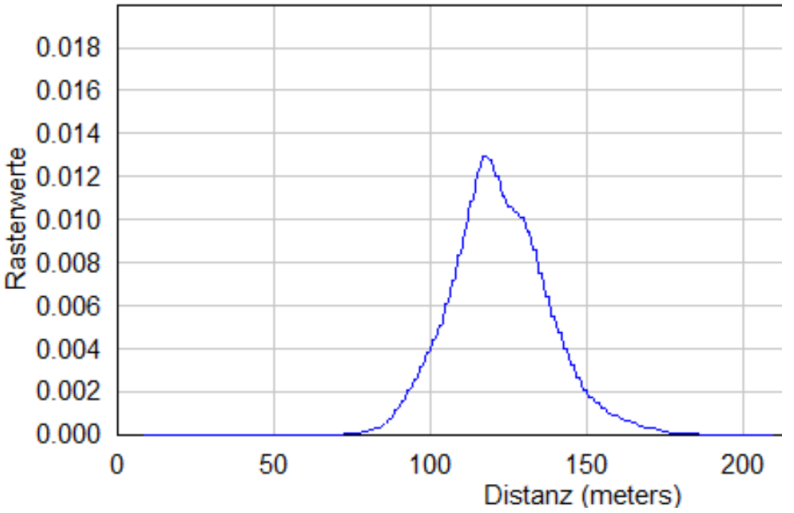


concentration after 30 days (kg/m³) – dx = 4m



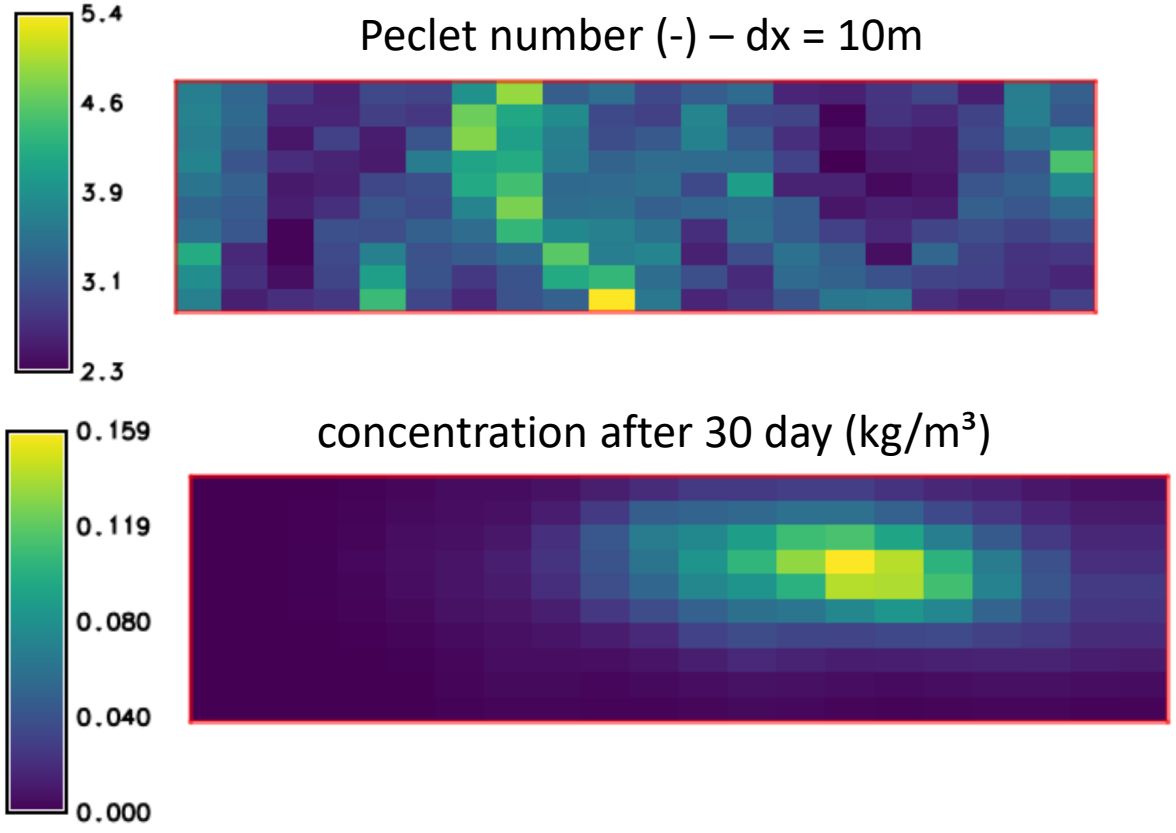


concentration after 30 days (kg/m³) – dx = 2m



Peclet number

- $Pe = \frac{dx\, v}{D} < 2$



Statistik

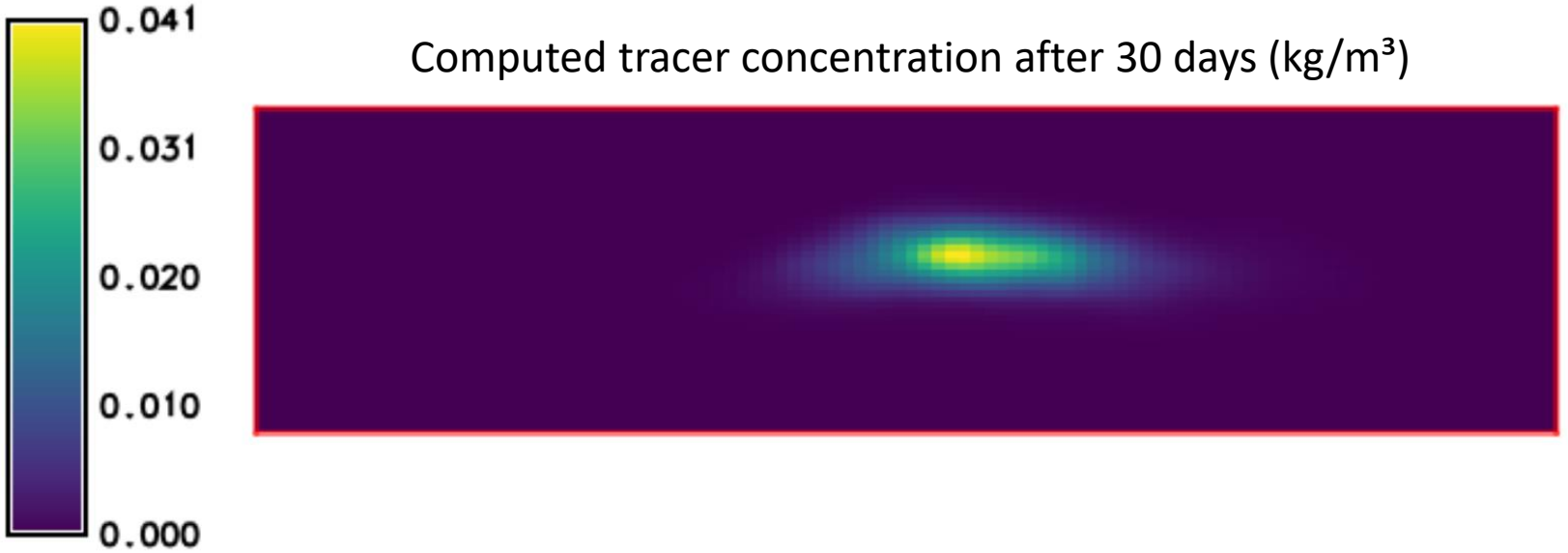
Statistics for Map(s) Histogrammed

Statistics for raster map <stresult_30>:
total null and non-null cells: 200
total null cells: 0

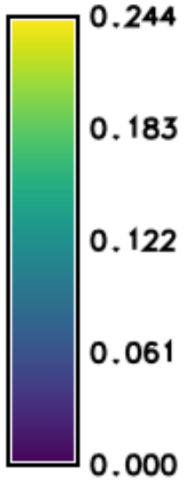
Of the non-null cells:
---
n: 200
minimum: 0
maximum: 0.159029
range: 0.159029
mean: 0.0238411
mean of absolute values: 0.0238411
standard deviation: 0.0325971
variance: 0.00106257
variation coefficient: 136.726 %
sum: 4.7682297694393
1st quartile: 0.00221652

Assessing numerical diffusion

- Assume pure advection
- In Grass GIS r.solute.transport diffusivity has to be larger than 0 but can be set to a very small number (here: 0.00000000000000001)



Numerical diffusion - homogen



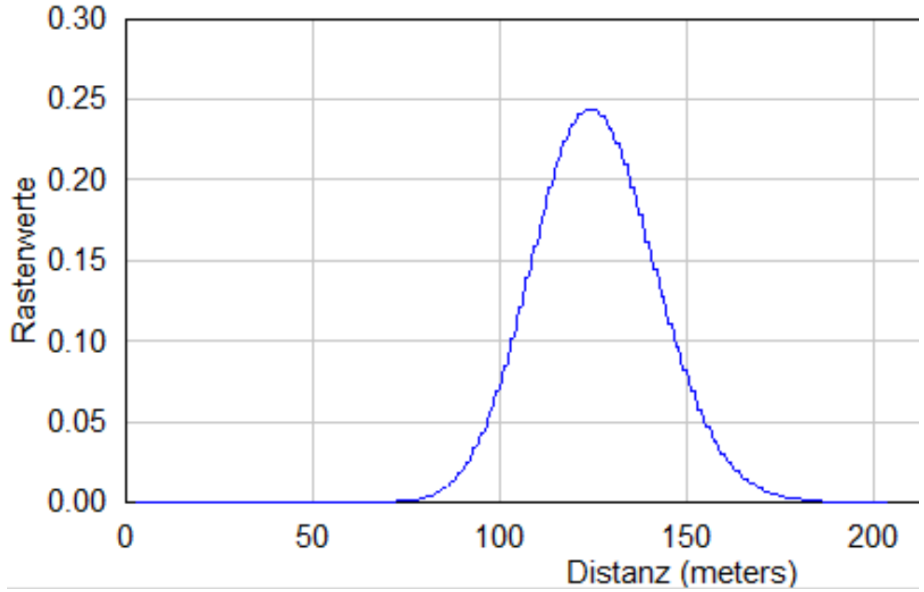
Computed tracer concentration after 30 days (kg/m³)

Statistik

Statistics for Map(s) Histogrammed

Statistics for raster map <stresult_30>:
total null and non-null cells: 5000
total null cells: 0

Of the non-null cells:
---
n: 5000
minimum: 0
maximum: 0.244033
range: 0.244033
mean: 0.000999994
mean of absolute values: 0.000999994
standard deviation: 0.0131062
variance: 0.000171774
variation coefficient: 1310.63 %
sum: 4.99997019828779
1st quartile: 8.51055e-116



Lessons learned

- Assessing the quality of a simulation is maningfold
- Conceptual decissions need to be assessed separately!
- Boundedness and conservativity can be assessed comparably easy
- Know (and check!) your stability criterias
- Accuraccy difficult to assess without validation
- Check for convergence in spatial and temporal resolution