Hydraulic groundwater modeling

- Week 7

- Geostatistics for groundwater modeling

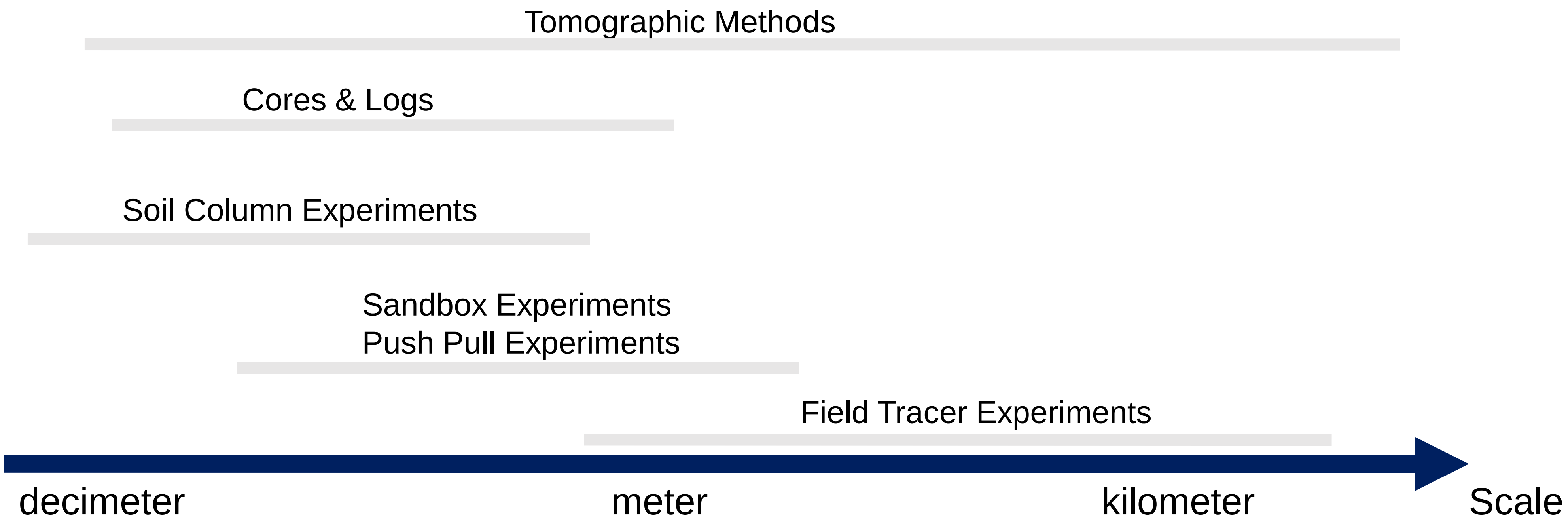
Geostatistics in groundwater modeling

- Task:
  - Design models based on measurements covering a fraction of the entire area
  - Exact description of a system is neither feasible nor economically possible
  - The results are necessarily uncertain
  - The uncertainty is not an intrinsic property of the systems; it is the result of incomplete knowledge by the observer.

- Aim:
  - Characterization of spatial systems that are incompletely known

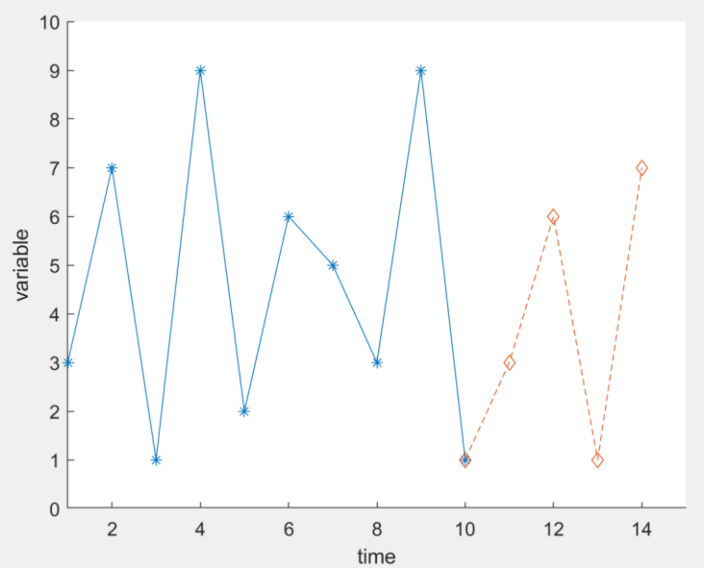
Assessing spatial heterogeneity

- Depending on scale, different experiments can help to assess spatial heterogeneity

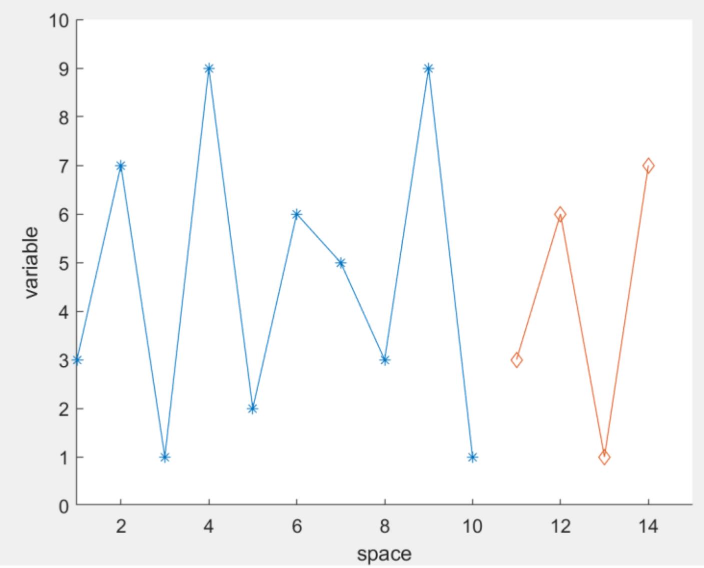


Inter-/Extrapolation

- Time series:
  - Users want to extrapolate

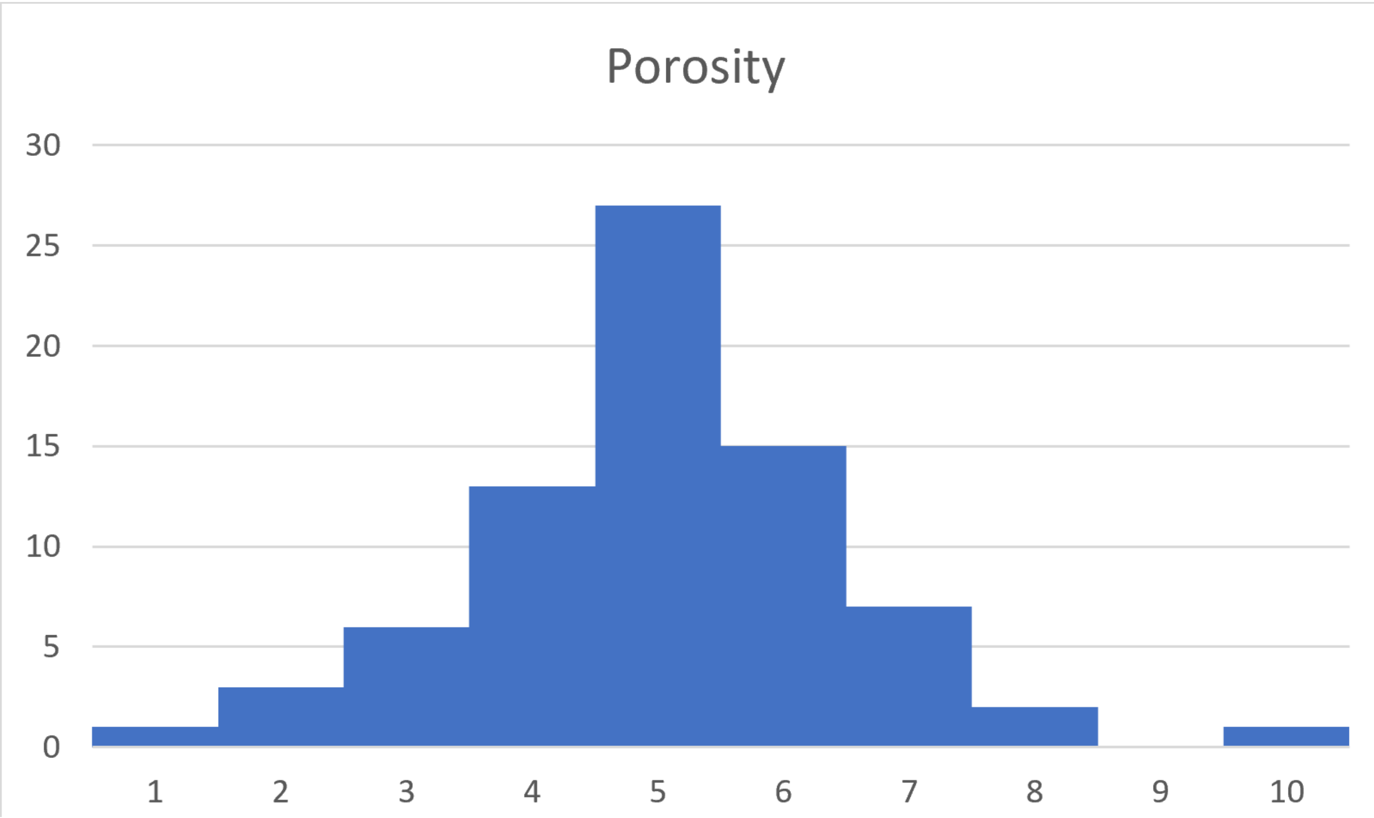


- Geostatistics:
  - Users want to interpolate

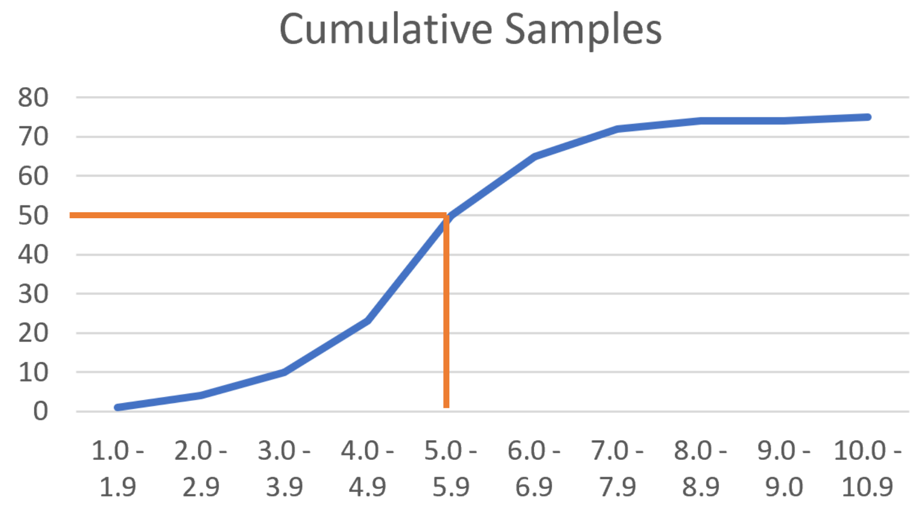


Univariate statistics

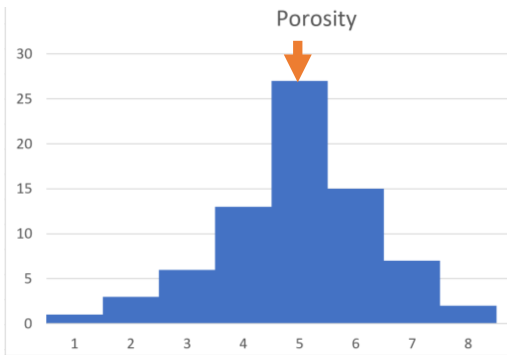
range (%)	# of samplex	% of samples
1.0 - 1.9	1	1,3%
2.0 - 2.9	3	4,0%
3.0 - 3.9	6	8,0%
4.0 - 4.9	13	17,3%
5.0 - 5.9	27	36,0%
6.0 - 6.9	15	20,0%
7.0 - 7.9	7	9,3%
8.0 - 8.9	2	2,7%
9.0 - 9.0	0	0,0%
10.0 - 10.9	1	1,3%
SUM	75	100



Mean, Median and Mode



- Arithmetic Mean
  - The additive average over all observations.
- Median
  - Value that evenly splits the number of observations into a lower half of smaller observations and an upper half of larger measurements.
- Mode
  - The most probable or frequent value, or, equivalently, the center point of the class containing the most observations



All mean

- Arithmetic mean
  - The additive average over all observations $$\hat{x}_{arit} = \frac{1}{n}\sum_{i=1}^{n} x_i$$
- Geometric mean
  - Useful to compare items with different value range $$\hat{x}_{geom} = \sqrt[n]{\prod_{i=1}^{n} x_i}$$
- Harmonic mean
  - Appropriate for average of rates $$\hat{x}_{harm} = \frac{n}{\frac{1}{x_1} + \cdots + \frac{1}{x_n}}$$

Examples for geometric & harmonic mean

- Geometric mean:
  - Useful in characterizing distributions where you expect significant variation across many orders of magnitude (such as hydraulic conductivity)

- Harmonic mean:
  - Estimate density of an alloy based on constituents' densities & mass fractions

Example: hydraulic conductivity

- List of measured hydraulic conductivities
  - Weighted arithmetic mean: $3.0*10^{-4}$
  - Weighted geometric mean: $3.3*10^{-7}$

Conductivity [m/s]	Volume fraction
10^{-9}	0.4
10^{-6}	0.3
10^{-3}	0.3

- Change the volume fractions to
  - Weighted arithmetic mean: $2.0*10^{-4}$
  - Weighted geometric mean: $2.8*10^{-7}$

Conductivity [m/s]	Volume fraction
10^{-9}	0.6
10^{-6}	0.2
10^{-3}	0.2

Measures of spread

- Variance (arithmetic)
  - Average squared dispersion around the mean

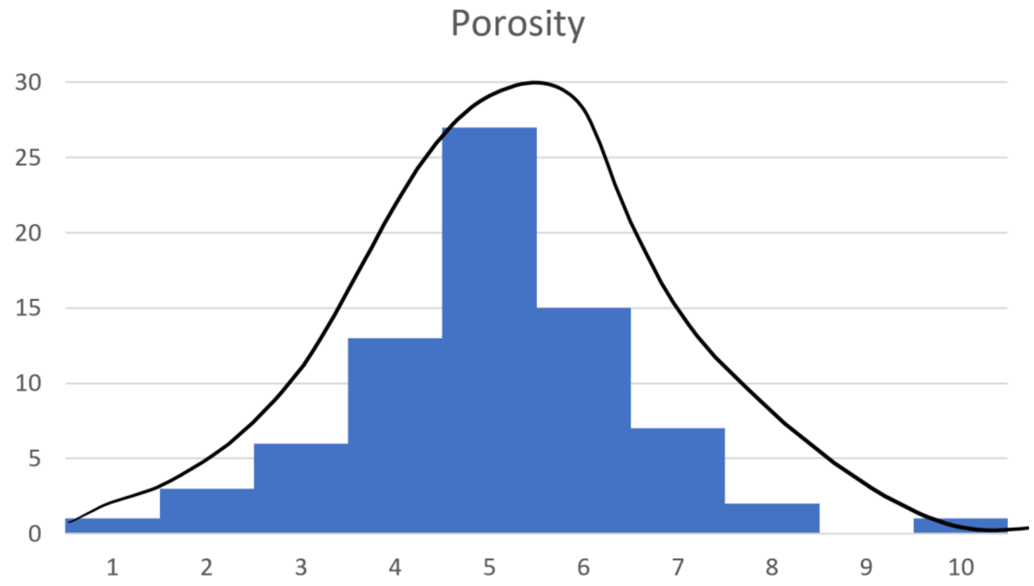
$$V = \frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{x})^2$$

- Standard deviation
  - Square root of the variance

$$\sigma = \sqrt{V}$$

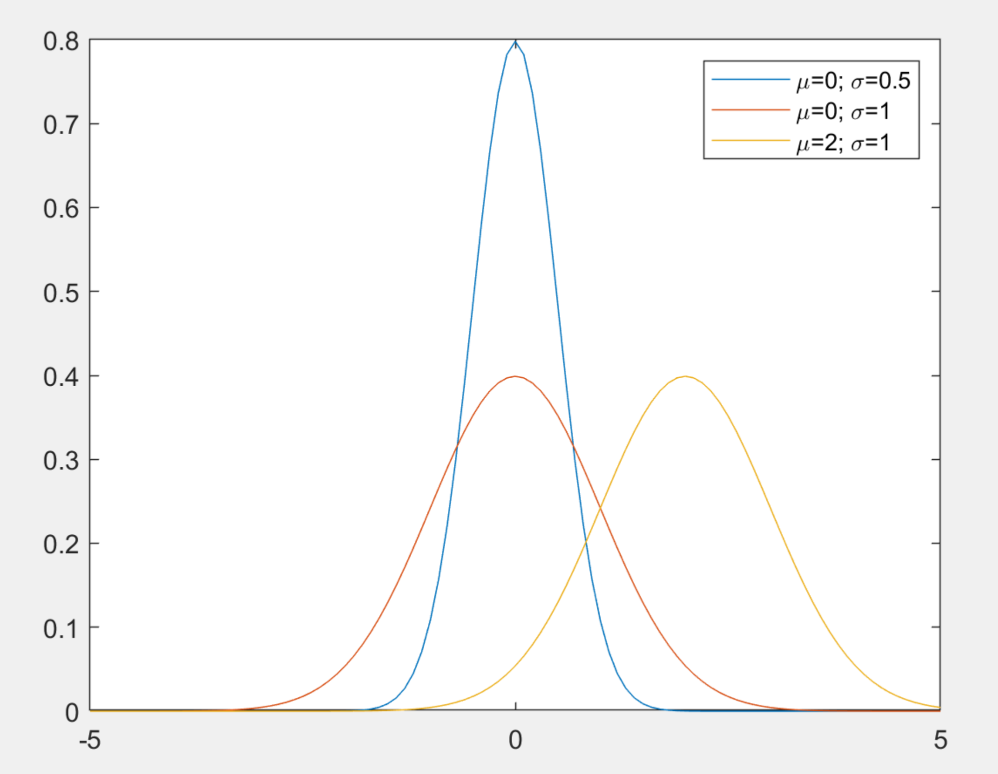
From samples to continuous distribution

- So far: samples with a finite number of observations
- A large number of observations:
  - Following statistical laws
  - From discrete observations to continuous distributions

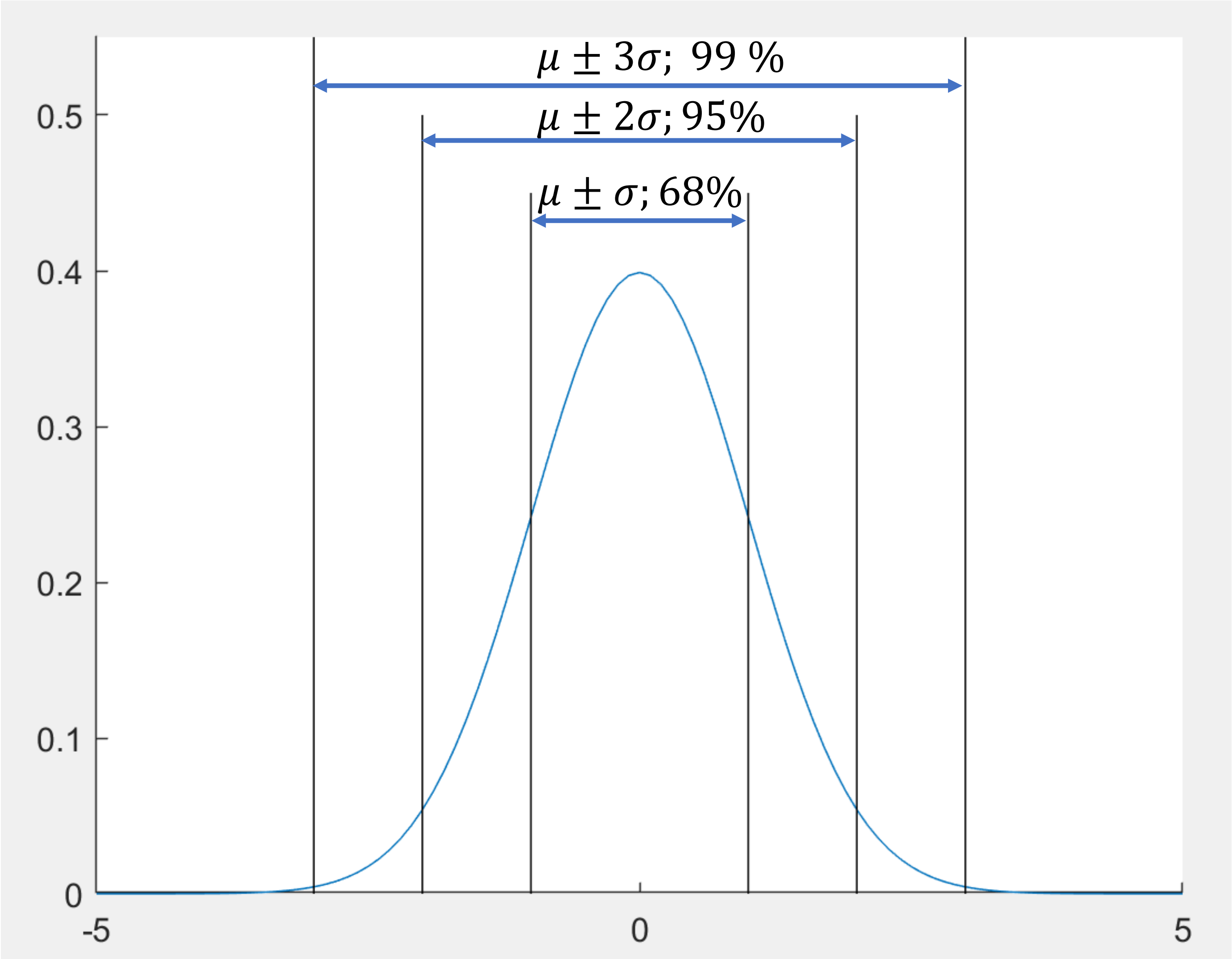


The normal distribution

- The most versatile of all continuous models is the normal distribution, also known as the Gaussian distribution.
- Its parameters are μ and σ, which coincide with mean and standard deviation.
- The probability for a value is never zero but can be infinitely small.

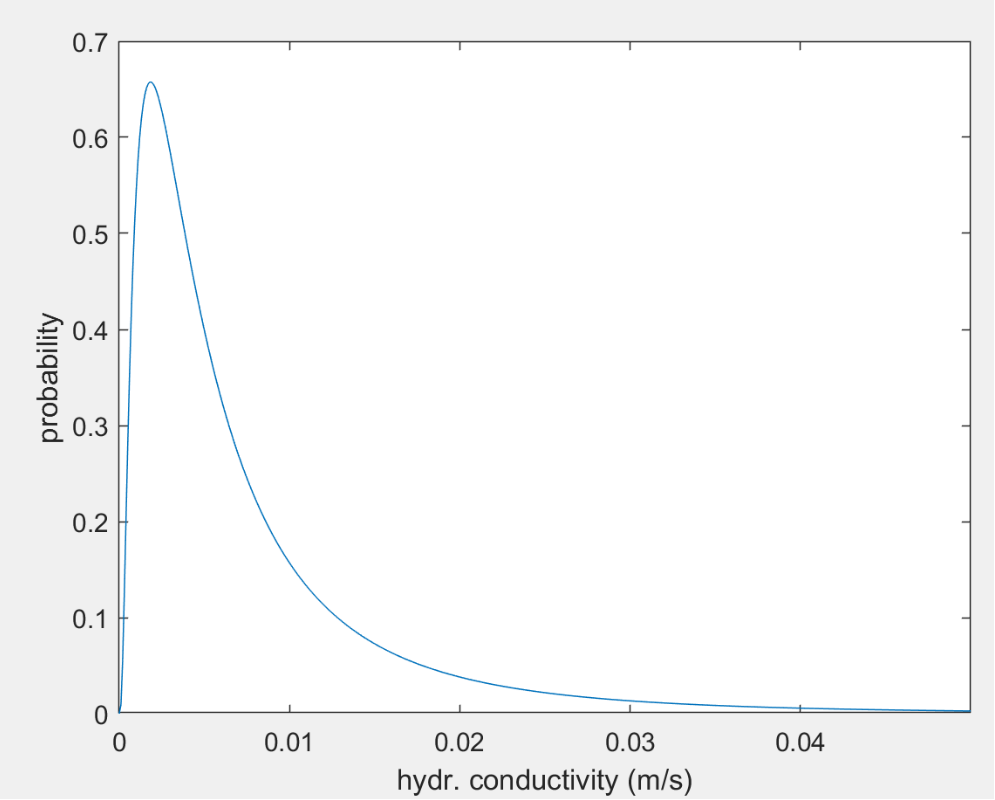


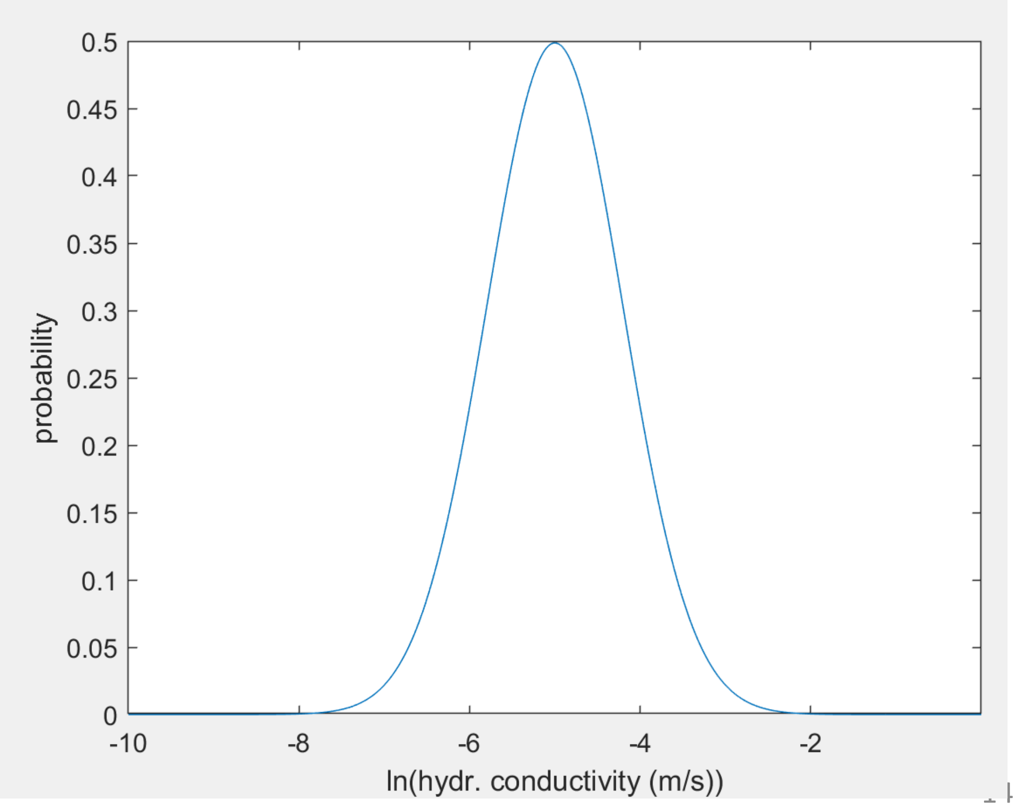
Probability of the normal distribution



Log-normal distribution

- If the logarithm of a quantity can be described by a normal distribution, the quantity is said to follow a log-normal distribution.

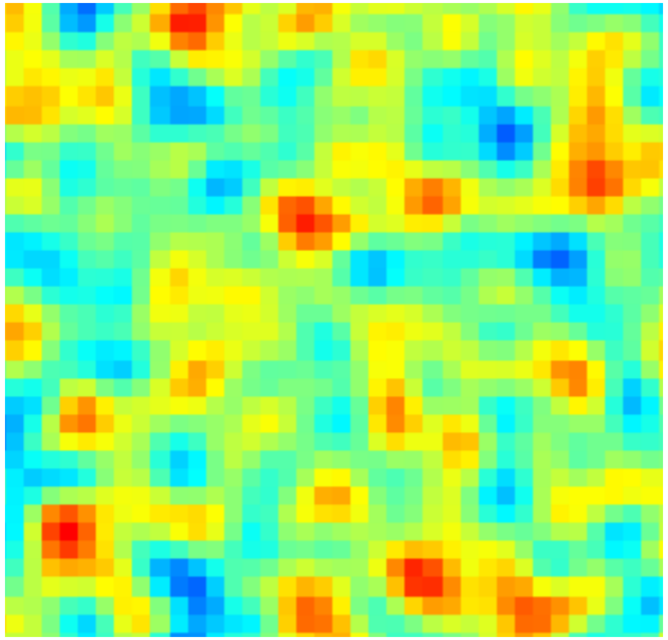




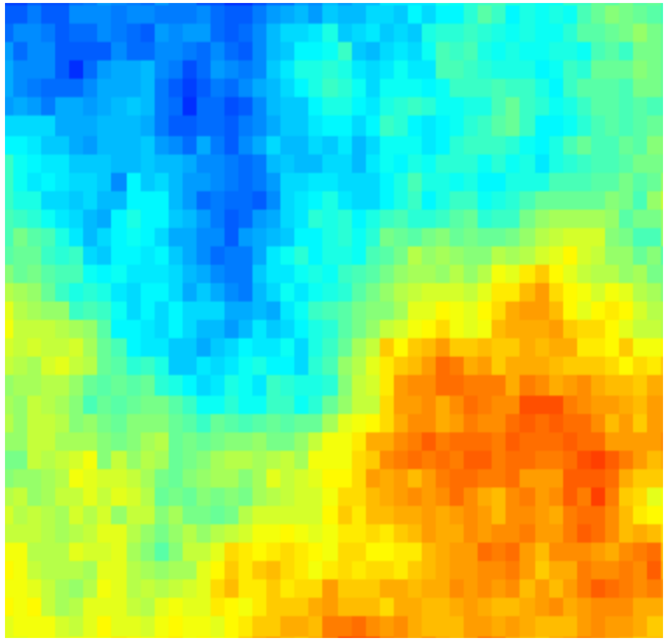
Variogram

- Visualizing spatial variability of a random variable by plotting the semi-varianz vs. distance of both points

Low spatial relation



Strong spatial relation



Lessons learned

- Geostatistics can help incorporating field heterogeneity into a groundwater flow model
- Characterizing a collection of measurements correctly depends on type of data
- Hydraulic conductivity is described usually best by log-normal distribution
- Spatial correlation is important and can be analyzed with a variogram