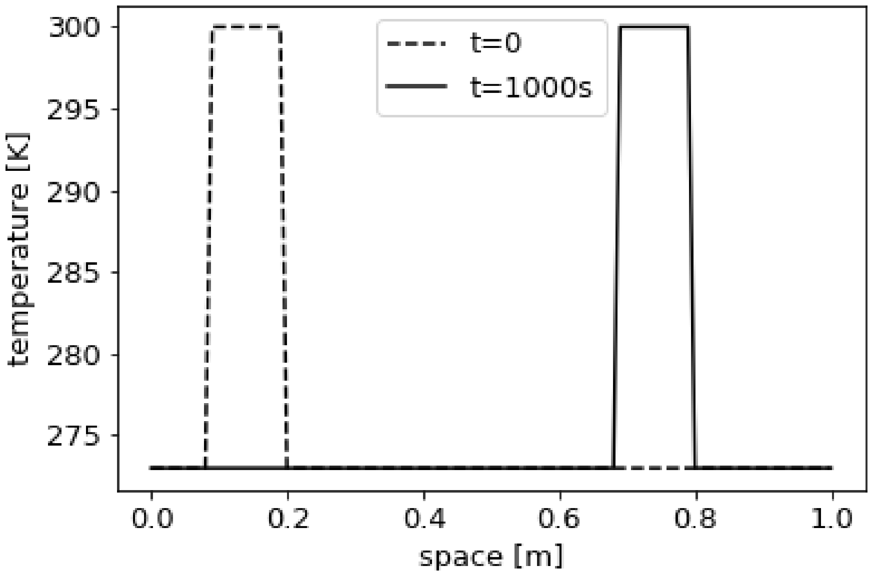
Hydraulic groundwater modeling

- Week 6
- Features of realistic groundwater models

The troublemaker: Advection

- Advection is challenging to simulate accurately in numerical schemes.
  - Some schemes are unconditionally unstable!
  - Numerical diffusion might take place.

- Although CFL is fulfilled:
  - Solutions might violate boundedness
  - Artificial oscillations might occur



The advection equation

- Recap: Advective flux

$$\frac{\partial T}{\partial t} = -\nabla(vT)$$

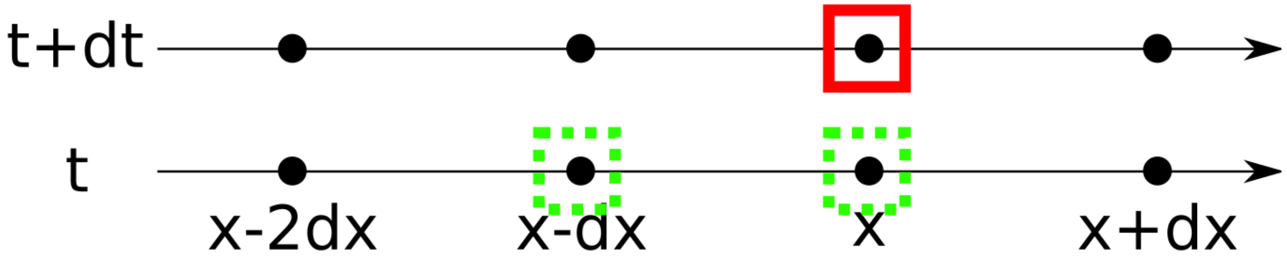
- In 1d and with constant flow velocity v

$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial x}$$

Upwind scheme – 1st order

- The upwind schemes discretizes hyperbolic PDE biased in the direction determined by the sign of the characteristic speed v.

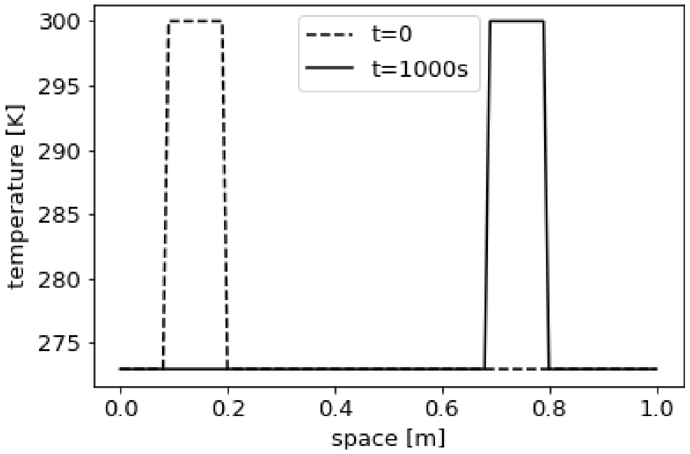
$$\frac{\partial T}{\partial t} = -v\frac{\partial T}{\partial x}$$

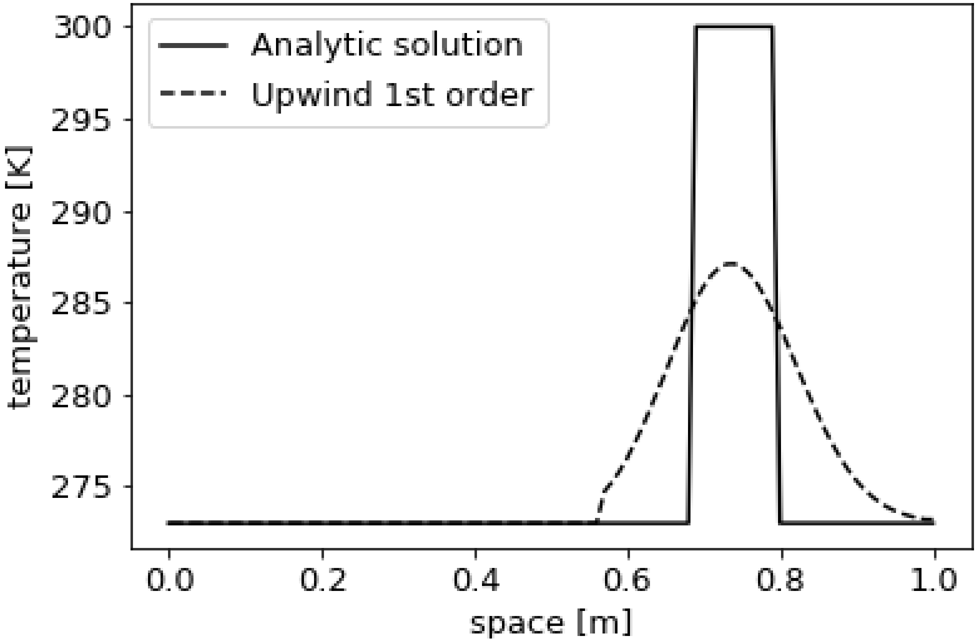


$$\frac{T_x^{t+dt} - T_x^t}{dt} = -v\frac{T_x^t - T_{x-dx}^t}{dx}$$

Upwind scheme – 1st order

- Numerical diffusion = The simulated fluid exhibits a higher diffusivity than the true medium

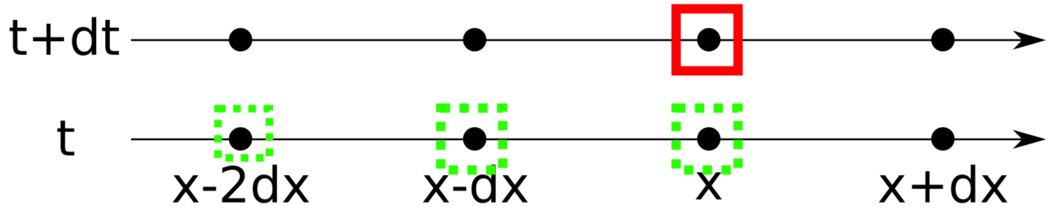


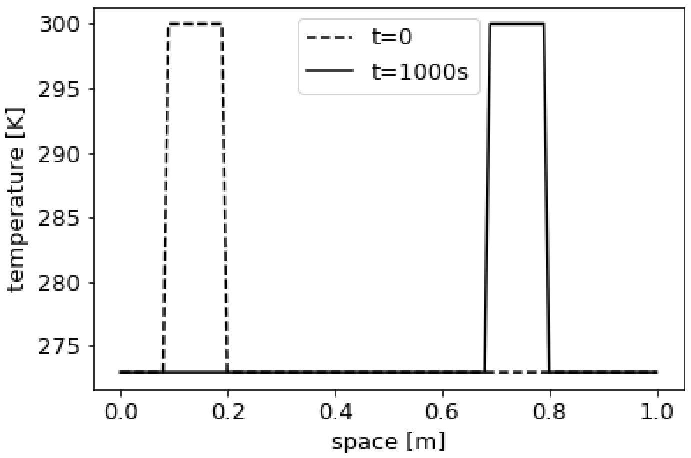


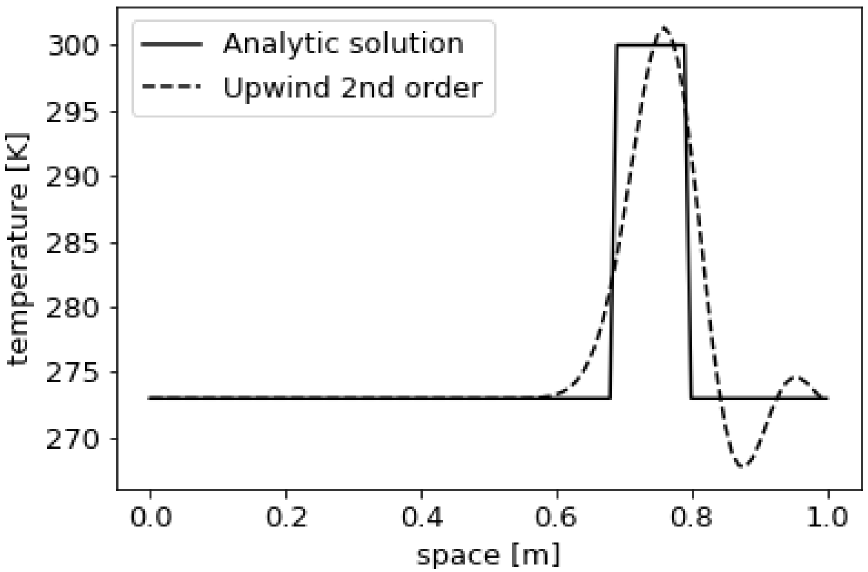
Upwind scheme – 2nd order

- Applying same scheme with an higher order FD approximation

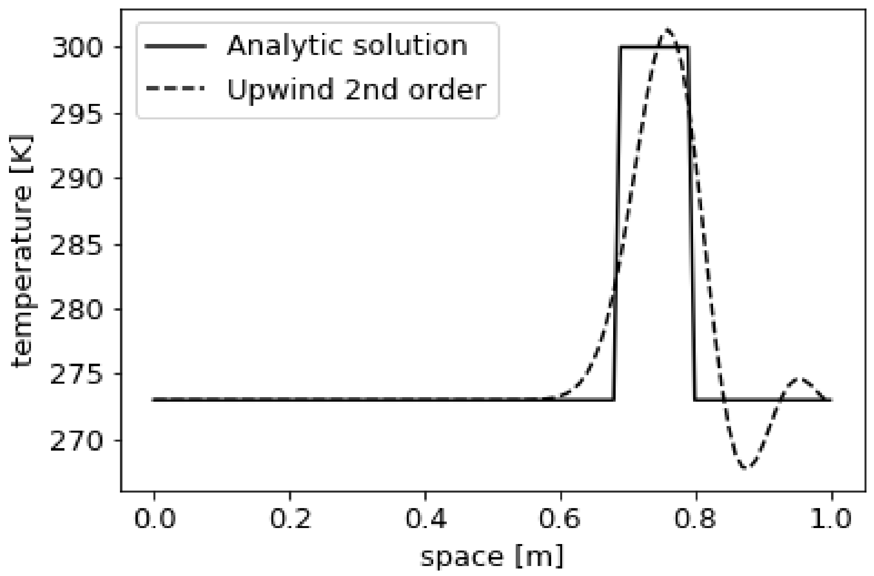
$$\frac{T_x^{t+dt} - T_x^t}{dt} = -v\frac{3T_x^t - 4T_{x-dx}^t + T_{x-2dx}^t}{2dx}$$

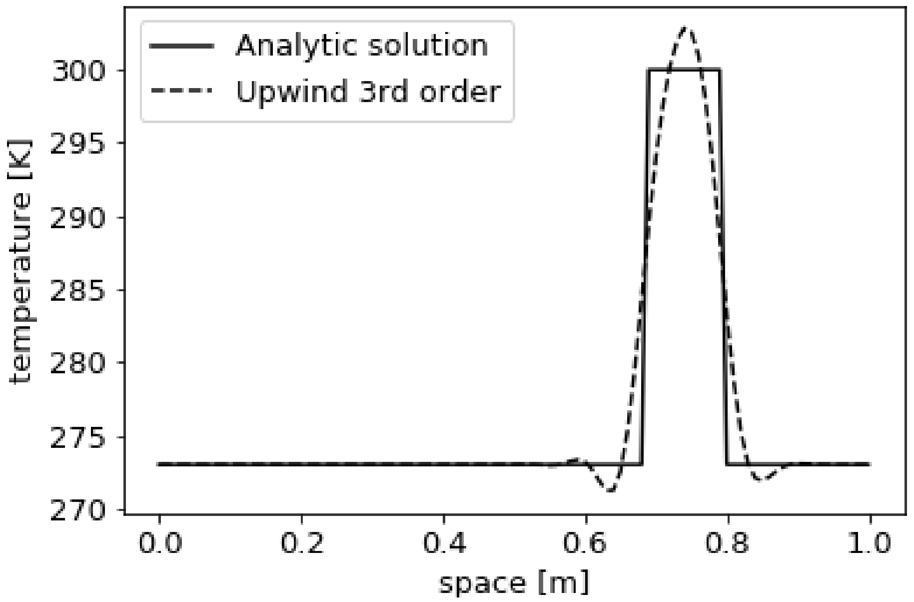






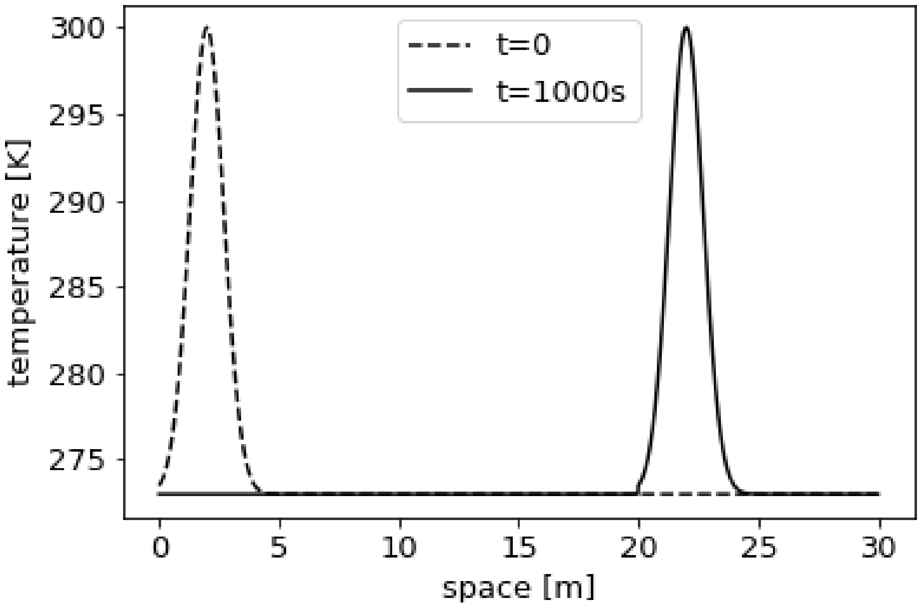
Upwind scheme – 3rd order



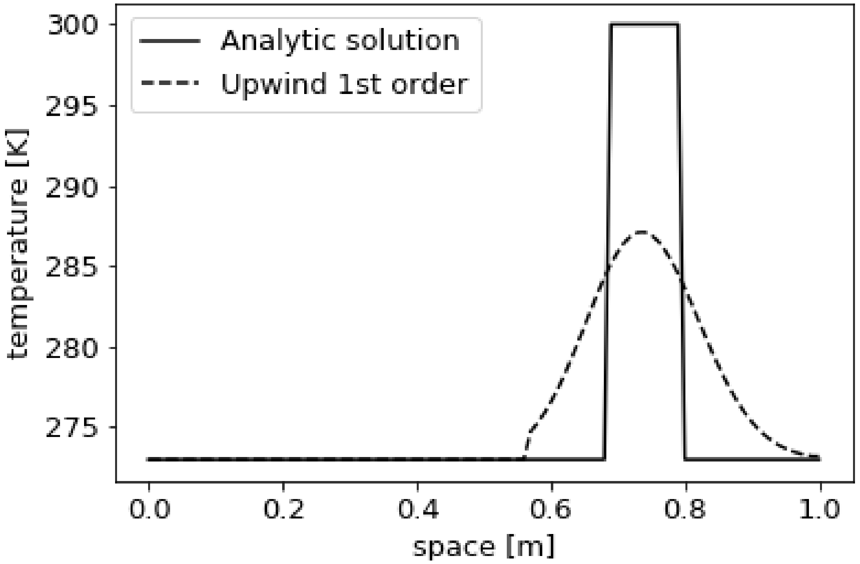


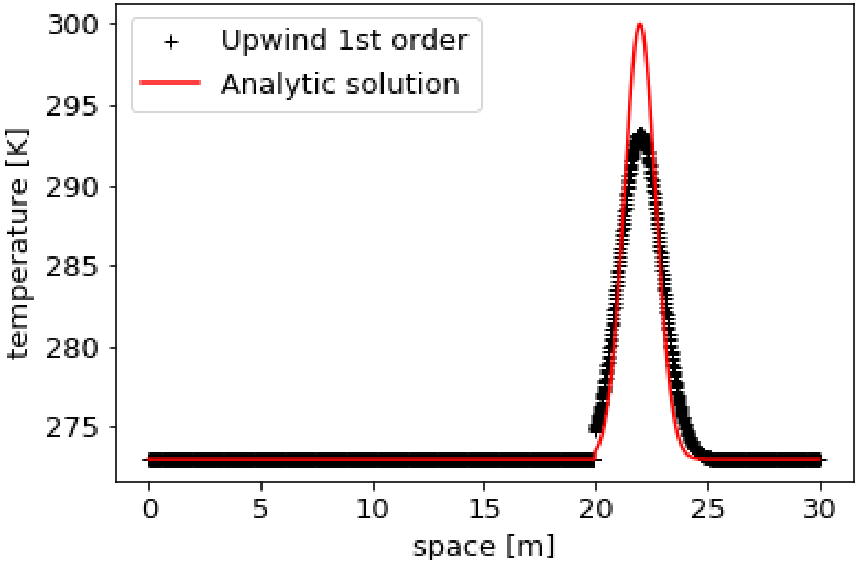
Advection – influence of initial conditions

- Keep everything as before, but vary
  - Shape of Input signal
  - Spatial dimensions

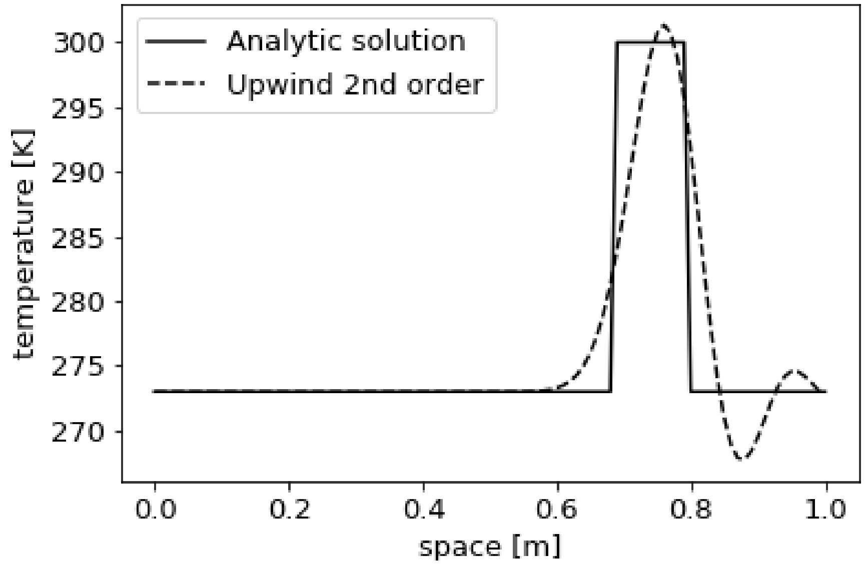


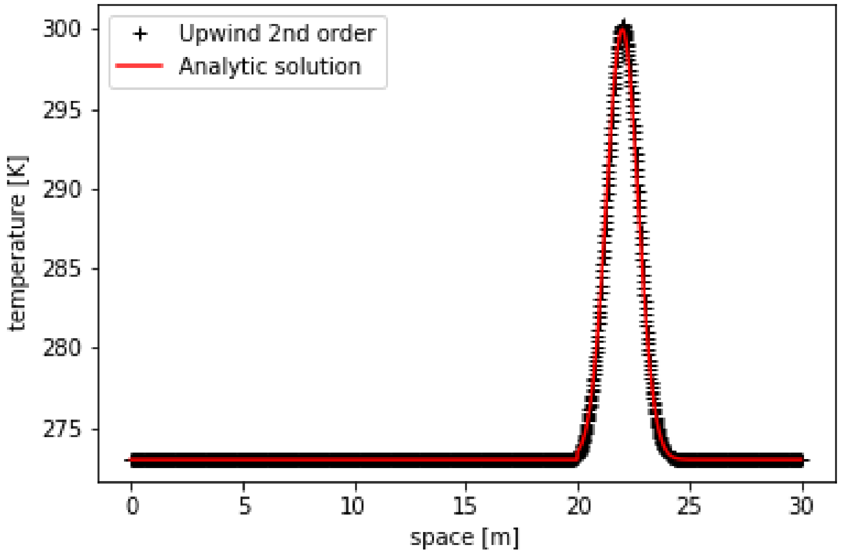
Upwind 1st order – comparison input shape



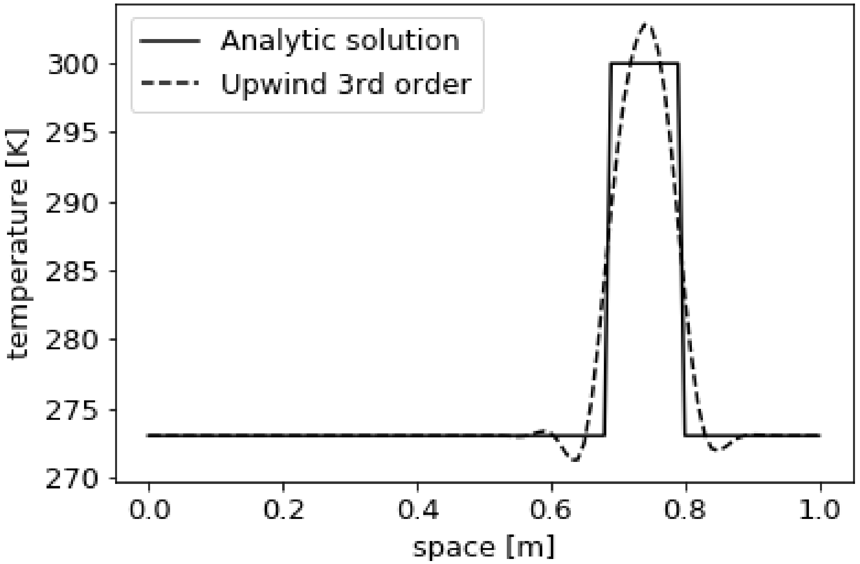


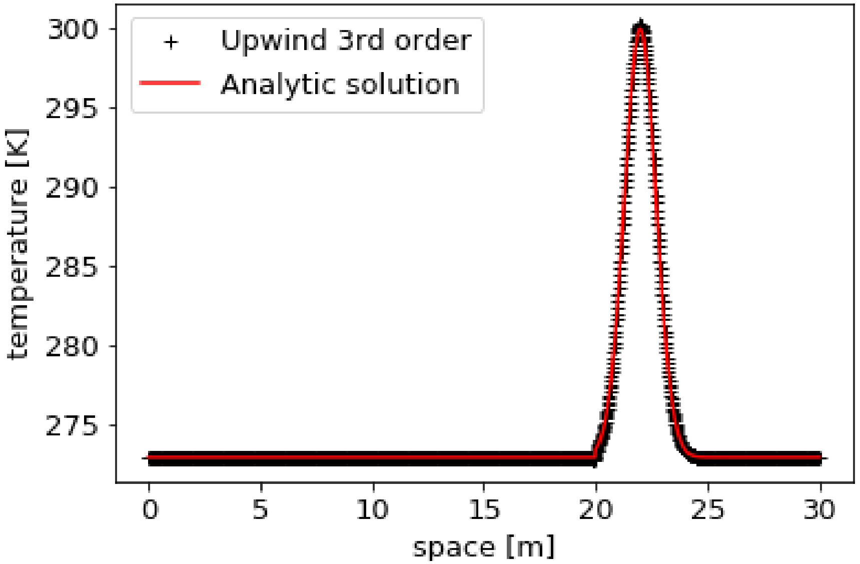
Upwind 2nd order – comparison input shape





Upwind 3rd order – comparison input shape





(Exercise: Discuss accuracy & boundedness for all shown examples)

Eulerian and Lagrangian perspective

- Langragian perspective:
  - Follow an individual fluid parcel as it moves through space and time as sitting in a boat and drifting down a river.
- Eulerian perspective:
  - Focus on specific locations in space through which the fluid flows sitting on the bank at a river side and watching the water pass the fixed location.
- Idea of markers: Combine the use of Lagrangian advecting points (markers, tracers or particles) with an immobile, Eulerian grid.

Markers / Particles / Points

- So far:
  - "Eulerian" simulations employing a fixed mesh

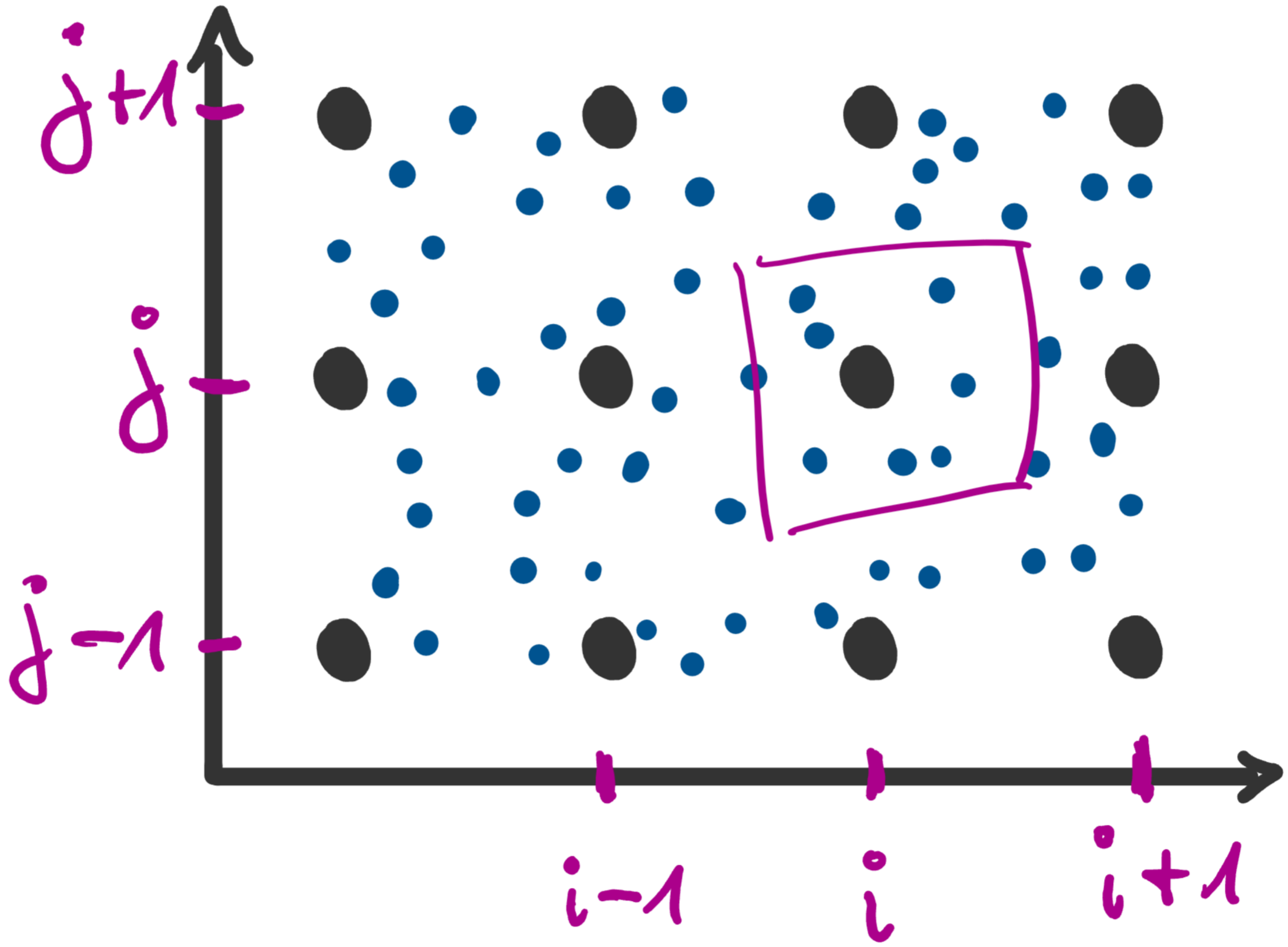
- Other methods:
  - "Lagrangian" simulations feature nodes that may move following the velocity field. (example: mesh free methods).

- Here:
  - Lagrangian markers within a fixed eulerian grid

Particles / Markers

- Place Lagrangian markers within the grid
- The markers are advected with the flow field
- The markers hold physical properties or variables, which are interpolated to the grid nodes and vic versa (if required)
- There is a large number of markers compared to the number of nodes

Interpolation between markers & grid



Summary Markers/Particles/Points

- Updating markers position is a non-trivial task in complex flow fields
  - It requires interpolation of velocity values (and direction) at marker position
- Results of pure advection with markers are not subjected to numerical diffusion
  - There are non-accumulating interpolation errors between markers and nodes.
- BUT: back and forth interpolation between markers and nodes might cause numerical diffusion
- The use of markers can be computationally expansive

Conclusions advection

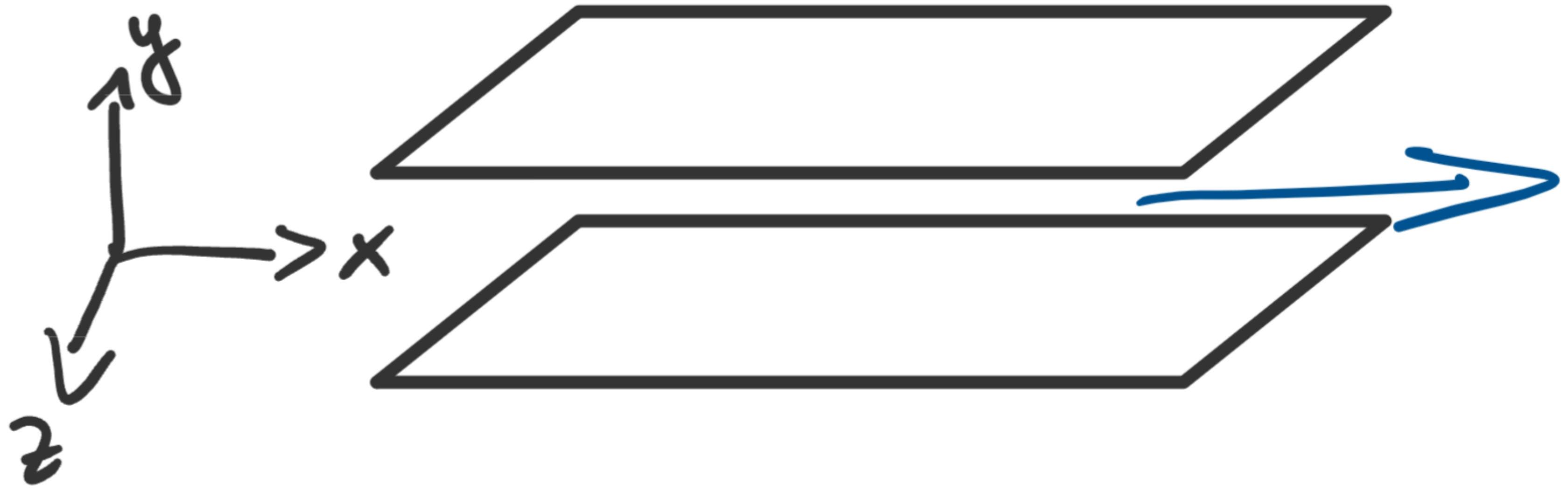
- Advection is challenging to simulate accurately in numerical schemes
- There are various ways to model advection
- The accuracy of the schemes depends on the specific problems
- Large gradients are challenging to preserve
- Smaller time steps might increase diffusion as more iterations are needed

Fractures as discrete features

- Rule of thumb: Fractures increase permeability, reduce porosity
- Fractures as part of the porous REV if their spatial scales overlap
  - fracture length is smaller than the porous subdomain.
- Specific fracture modeling required if there is a fundamentally different flow behavior in the fractures
- Similar effects also by engineering constructions (tunnels, pipes, etc.)

Geometrical description of fractures

- Plane plate model
  - Fractures are wide and thin (aperture much smaller than length)
  - Pressure gradient vertical to fracture surface is negligible
- Represent fractures by a reduced geometrical dimension (1d/2d)

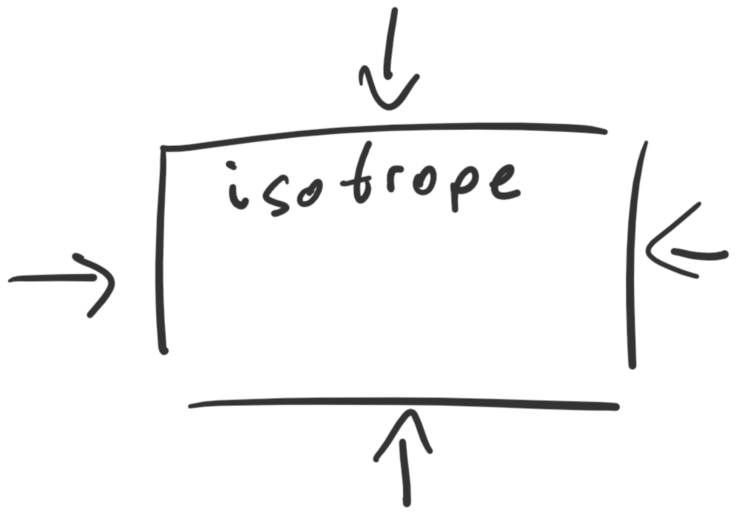


Fracture modeling

- Effective porous media
  - Fractures as part of porous media
  - Modified permeability, porosity, introduced anisotropy, and many more
- Dual Domain
  - Two domains with different parameterization and mass exchange
  - Still based on Darcy‘s law
- Discrete fracture models
  - Fractures are represented along grid lines
  - Computationally expensive pre-processing in the numerical grid generation
- Embedded discrete fracture models
  - Fractures on-top of continuous porous media

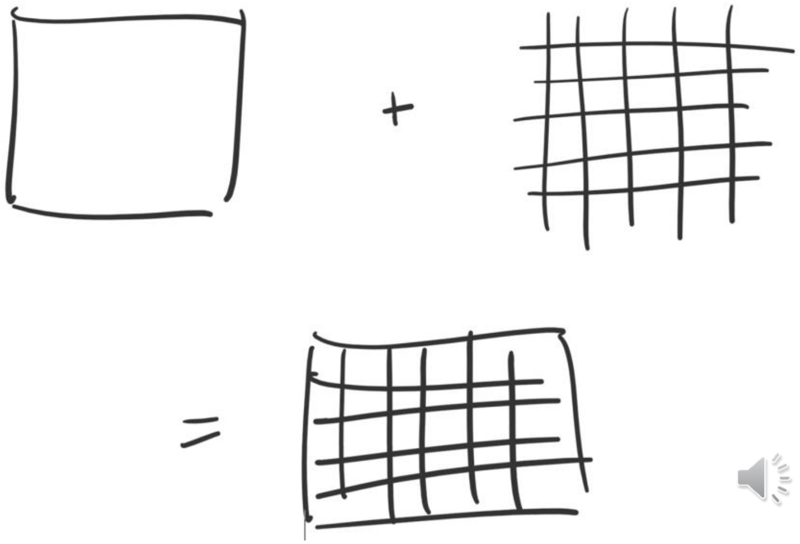
Effective porous media

- Include fractures through parameters of porous media
  - Does assume similar flow behavior as in porous matrix
  - Usually: Increased hydraulic conductivity due to fractures
- Anisotropy in hydraulic conductivity



Dual Domain models

- Since the 1960s various forms and methods exist
- Separate two distinct pore systems with different hydraulic parameters
- The medium is a superposition of those two systems
- Both pore systems interact through an exchange of mass, temperature, solute
- Transferred mass depends on assumptions
  - Such as geometry for fractures, fracture density, …



Discrete Fracture models

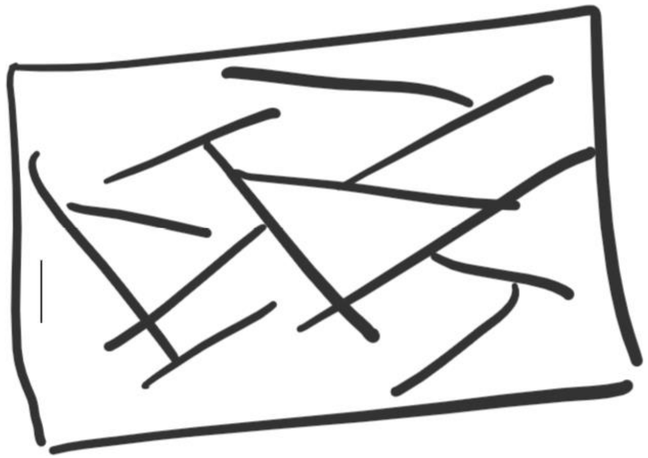
- Account explicitly for the effects of individual fractures
- Some believe:
  - the most accurate method for studying flow through fractured formations
- Drawbacks:
  - computationally cumbersome and excessive data requirement for generating network.
- Parameterization:
  - fracture geometrical parameters are represented with statistical distribution functions or with geostatistical parameters.
- Allows for different flow equations in fractures and matrix!

Fracture network models

- Discrete-fracture-network (DFN) models:
  - fractures do not exchange fluid with the impermeable matrix.
- Discrete-fracture-matrix (DFM) models:
  - Flow occurs in fracture network and matrix
- Equi-dimensional approach:
  - Fractures and matrix are discretized with elements of same dimension
  - High demands on mesh generation
- Lower-dimensional approach:
  - Fractures are represented by elements of lower dimension than the matrix
  - Simple mesh generation, reduced computational costs

Embedded discrete fracture models

- Separation of a reservoir into a fracture and a porous matrix domain
  - A transfer function accounts for coupling effects between the two domains
  - Both domains are computationally independent except for the transfer function
- Fractures:
  - Thin and highly permeable
  - Pressure gradient normal to fracture is negligible
  - A lower dimensional representation of fractures



Lessons learned

- Modeling advection requires special care
  - Avoid steep gradients
  - Numerical diffusion might occur
  - Particle/Marker models might be helfpul but computationally expansive
- Fractures might require special care
  - Modeling decission: Are fractures important?
  - Different ways to include fractures depending on the situation and the available information