Real options analysis and its application in energy sector – exercise

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Presentation outline

1. Introduction to real options analysis
2. Trees versus continuous diffusions
3. Black-Scholes-Merton Formulas
4. Binomial tree
5. Binomial tree – exercise
6. Binomial tree – convergence property
7. Convergence property – exercise

1. Introduction to real options analysis

- **Real options analysis** (ROA) is a sophisticated approach used to evaluate investment opportunities under **uncertainty** by accounting for the value of **managerial flexibility**
- ROA assesses the **potential future opportunities** and **strategic decisions** involved in an investment project
- Unlike traditional discounted cash flow (DCF) analysis, which assumes a fixed set of future cash flows, Real Options Analysis (ROA) **recognizes the value of flexibility** in decision-making
- ROA accounts for the ability to adapt strategies in response to **changing circumstances and new information**
- The approach, known as **real options**, attempts to deal with this problem using **option pricing theory** normally applied for financial assets

1. Introduction to real options analysis

Selected types of real options

- ✓ to **defer**
  - – i.e. to wait before taking an action until more is known or timing is expected to be more favorable (e.g., when to introduce a new product)
- ✓ to **abandon**
  - – i.e. to sell or close down a project if (market) conditions are not favorable (the question is: how much the abandonment option is worth?)
- ✓ to **switch**
  - – i.e. to alter the mix of inputs or outputs of a production process in response to market prices (e.g., the output mix of telephony/internet/cellular services)
- ✓ to **grow**
  - – i.e. to expand the scope of activities to capitalize on new perceived opportunities
- ✓ to **contract**
  - – i.e. to reduce the scale of a project's operation in response to the demand to mitigate losses (e.g. subtracting from a service offered)
- ✓ to **expand**
  - – i.e. to increase the scale of an operation in response to demand; to make further investments and increase the output if (market) conditions are favorable
- ✓ to **choose**
  - – i.e. the managers can choose among two, three or more strategies (e.g., expanding firm's current operations, contacting its operations, or completely abandoning operations)

1. Introduction to real options analysis

Evaluation methods

- Binomial or multinomial lattice models
- Closed-form models with partial-differential equations (Black-Scholes approach)
- Path-dependent stochastic simulations (simulation-based methods)
- Fuzzy pay-off method for real options valuation

2. Trees versus continuous diffusions

Price (1996) in his paper: Optional Mathematics Is Not Optional (in: *Notices of the American Mathematical Society,* 43, 964-971) writes:

"The paper that showed that European option pricing could be put on a rational mathematical basis was Black and Scholes published in 1973. It was so revolutionary that the authors had to submit it to a number of journals before it was accepted. Although there are now numerous approaches to the result, they mostly require specialized methods, including Ito calculus and partial differential equations, and perhaps Girsanov theory and Feynman-Kac methods. ***But it is the binomial method due initially to Sharpe and substantially extended by Cox, Ross, and Rubinstein that made the theory of option pricing accessible to everyone with limited mathematical background."***

- The (binomial) tree has **interesting pedagogical merits**

2. Trees versus continuous diffusions

"… it requires only routine algebraic manipulations, the method is still able to elucidate many of the ideas behind the full theory. Furthermore, all the surprising results mentioned in the opening can be located in this approach. For these reasons it is usually the first method presented in textbooks and finance courses; (…).
The binomial method is, however, much more than a pedagogical breakthrough, since it allows for the development of numerical approximation methods for a wide range of options for which there are no known analytic solutions."

- (Binomial) tree can be used to price complex products such as American options, multiple assets, even path dependent options

2. Trees versus continuous diffusions

Benchmark model in mathematical finance

Consider the standard diffusion (GBM* – Geometric Brownian Motion process) for the price of a stock (S_t)

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t \quad \text{or} \quad S_1 = \begin{cases} S_u = S_0 \cdot u, & with\ probability\ q \\ S_d = S_0 \cdot d, & with\ probability\ 1-q \end{cases}$$

with a no risky bond

$$\frac{dB_t}{B_t} = rdt \quad \text{or} \quad B_1 = B_0 \cdot e^r$$

where:

μ – drift coefficient (expected rate of return)

σ – stock price volatility (standard deviation)

dW_t – Wiener process (Brownian motion), which introduces the random component to the price movement

u, d – "*up*" and "*down*" movement, respectively

S_u, S_d – price of the stock by *u* ("*up*") and *d* ("*down*") movement, respectively

$q, 1-q$ – probability for "*up*" and "*down*" movement, respectively

r – continuously compounded risk-free rate

B_t – bond price

GBM – is a **stochastic process** used to model the random behaviors of asset prices over time. It is widely applied in finance for modeling stock prices and other financial instruments.
Properties:
- The logarithm of the asset price follows a **normal distribution**
- The **process is continuous** (no sudden jumps in the price)
- The future price movement depends only on the current price and not on the past prices (**Markov Property**)

Benchmark model in mathematical finance

Using **replication techniques and no-arbitrage assumption** (law of one price), then the one period model is:

$$C_0 = e^{-r}(\underbrace{\frac{e^r - d}{u - d}}_{q} \cdot C_u + \underbrace{\frac{u - e^r}{u - d}}_{1-q} \cdot C_d) = \frac{1}{1+r} E(C_1)$$

with a standard European call option:

$$C_0 = e^{-r}\left(\underbrace{\frac{e^r - d}{u - d}}_{q} \cdot \max(0, S_0 \cdot u - K) + \underbrace{\frac{u - e^r}{u - d}}_{1-q} \cdot \max(0, S_0 \cdot d - K)\right)$$

$$= \frac{1}{1+r} E(C_1)$$

where:

C_0	– price of the European call option
C_u, C_d	– price of the European call option by *u* ("*up*") and *d* ("*down*") movement, respectively
S_0	– price of stock at time zero
K	– strike price of the stock

Black-Scholes-Merton Formulas

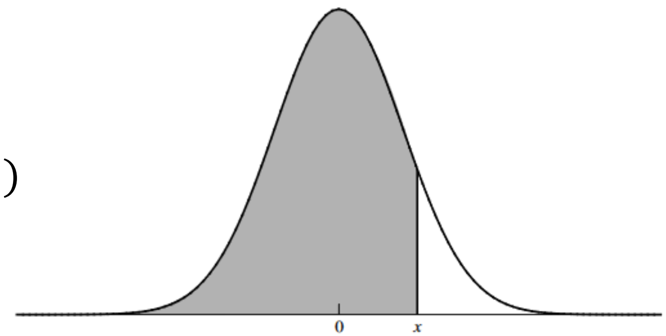
The most famous solutions to the **differential equation** are the **Black-Scholes-Merton formulas** for the prices of **European call and put options**

The formulas are:

$$c = S_0 N(d_1) - Ke^{-rT} N(d_2)$$

and

$$p = Ke^{-rT} N(-d_2) - S_0 N(-d_1)$$



Shaded area represents $N(x)$

where:

$$d_1 = ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)T)/\sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

and

c, p – European call and put price, respectively

$N(x)$ – cumulative probability distribution function for a variable with a standard normal probability distribution

T – time to maturity of the option

All variables are independent of the risk preferences!

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3. Black-Scholes-Merton Formulas

The idea underlying the Black-Scholes-Merton Equation

- The Black-Scholes-Merton equation must be satisfied by the price of any derivative dependent on a non-dividend paying stock
- The idea involves setting up **a riskless self–financing portfolio** consisting of a position in derivative and a position in the stock
- In the absence of arbitrage opportunities, the return from the portfolio must be the **risk-free interest rate**

3. Black-Scholes-Merton Formulas

The assumptions used to derive the BS formulas are as follows:

- The stock price follows the **Geometric Brownian Motion** (GBM) with constant μ and σ
- The short selling of securities with full use of proceeds is permitted
- There are no transaction costs or taxes
- All securities are perfectly divisible
- There are no dividends during the life of the derivative
- There are no riskless arbitrage opportunities
- Security trading is continuous
- The risk-free rate of interest, r, is constant and the same for all maturities

The setting happens to drop out the expected return, μ, in the derivation of the differential equation

➡ If risk-preference do not enter the solution, they cannot affect the solution

Understanding $N(d_1)$ and $N(d_2)$

Interpretation of the term $N(d_2)$

- It is the **probability** that a call option will be exercised in a risk-neutral world

The term $\boldsymbol{N(d_1)}$ term is **not quite so easy to interpret**

- The expression $S_0N(d_1)\ e^{rT}$ is the **expected stock price** at time T in a risk-neutral world when the stock price is greater than K; as just mentioned this has probability of $N(d_2)$

The **expected payoff in a risk-neutral world** is therefore

$$S_0N(d_1)e^{rT} - KN(d_2)$$

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3. Black-Scholes-Merton Formulas

Understanding $N(d_1)$ and $N(d_2)$

Present-valuing this from time T to time zero gives the Black-Scholes-Merton equation for a **European call option**

$$c = S_0 N(d_1) - K e^{-rT} N(d_2)$$

For another way of looking at the Black-Scholes-Merton equation for a value of a **European call option**, note that it can be written as:

$$c = e^{-rT} N(d_2) \, [S_0 e^{rT} N(d_1)/N(d_2) - K]$$

where:

e^{-rT} – is the present value factor

$N(d_2)$ – probability of exercise

$S_0 e^{rT} N(d_1)/N(d_2)$ – expected stock price in a risk-neutral world if option is exercise

4. Binomial tree

Binomial tree method – main idea

- popular numerical technique **for valuating options** and other financial derivatives
- particularly useful because it can **handle a variety of option types** and can accommodate complex features such as **American-style options**, which can be exercised at any time before expiration
- models the **possible price paths of the underlying asset** over discrete time intervals until the option's expiration
- at each step, the asset price can either **move up or down** under certain probabilities, leading to a recombining tree of possible asset prices

Binomial trees from 1 to *T* periods

Sharpe initiated the concept of pricing a call option written on a stock with simple **up** and **down two-state price changes**. Cox, Ross, and Rubinstein (CRR) further developed this concept to obtain a **binomial option pricing model**.

Recombining tree, under S_1 takes value $S_0 \cdot u^t \cdot (1-d)^{T-t}$, where $t = 0, \dots, T$

$$C_0 = e^{-r} \sum_{t=0}^{T} q^t (1-q)^{T-t} \cdot \max(0, S_0 \cdot u^t \cdot (1-d)^{T-t} - K) = e^{-r} E(C_1)$$

where $q = \frac{e^{r\Delta t} - d}{u - d}$

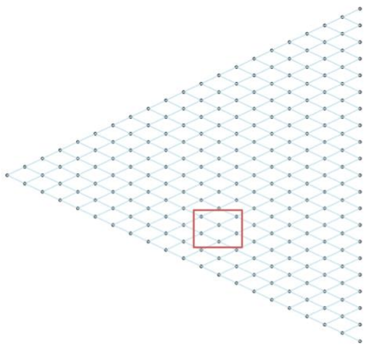
$u = exp(\sigma\sqrt{\Delta t})$

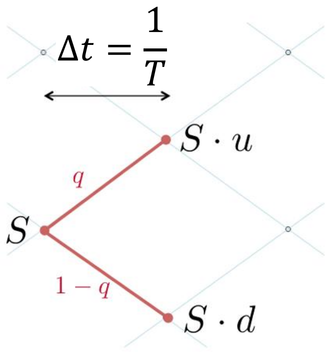
$d = exp(-\sigma\sqrt{\Delta t})$

as $ud = 1$

$\Delta t = \frac{T}{N}$ – length of each time step

N – number of discrete time intervals into which the time to expiration is divided





5. Binomial tree – exercise

Question 1:
Suppose that the price of electricity follows the GBM process with α =0.04 and $\sigma =$ 0.15, and the market price of risk λ =0.3. The initial electricity price $P_{el,0} = 30$ €/MWh, construct a two-timestep tree for the price of electricity in a risk-neutral world.

The Geometric Brownian Motion (GBM) is often used to model the dynamics of stock prices and, in your case, electricity prices. The GBM model for a stock (or electricity price, in this case) is:

$$\frac{dP_{el}}{P_{el}} = \mu dt + \sigma dW_t$$

where:

P_{el} – electricity price

r – risk-free rate, which, in the risk-neutral world, becomes $r = \alpha - \lambda\sigma$

Calculating the risk-free rate:

$$r = \alpha - \lambda\sigma$$

5. Binomial tree – exercise

Without loss of generality assuming each time step is of length 1 year (you can change the length as you wish, but the calculation will still follow the same logic), we have:

$$\Delta t = 1$$

Now, calculate the *up* and *down* factors:

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = e^{-\sigma\sqrt{\Delta t}}$$
$$q = \frac{e^{r\Delta t} - d}{u - d}$$

Then, the two-time step tree for P_{el} becomes:

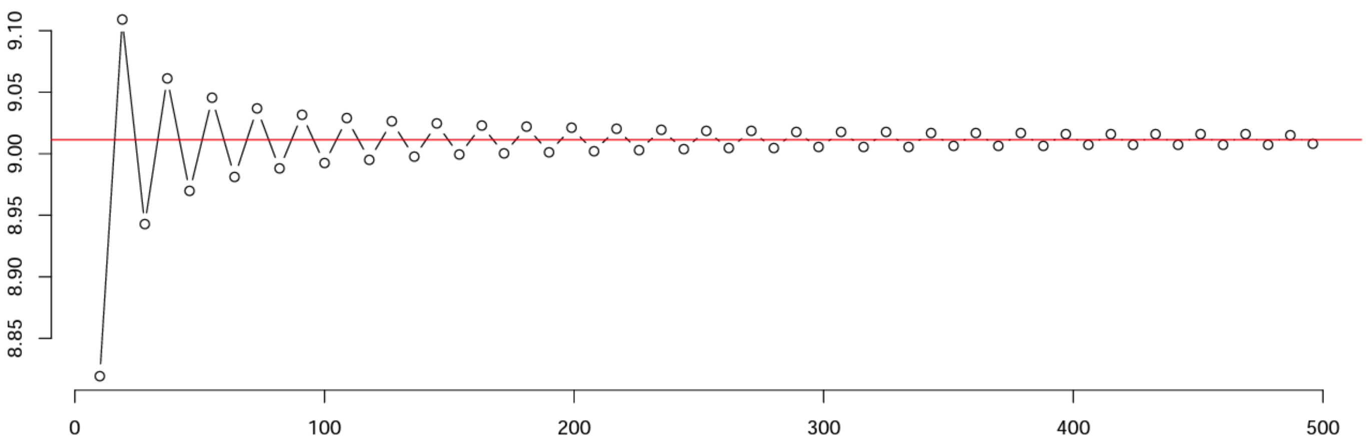
$t = 0$	$t = 1$	$t = 2$
$P_{el} = 30$	$P_{el,u} = 30 \cdot u$	$P_{el,u,u} = 30 \cdot u^2$
	$P_{el,d} = 30 \cdot d$	$P_{el,d,d} = 30 \cdot u \cdot d$
		$P_{el,d,d} = 30 \cdot d^2$

Please try to code it in Python yourself!
Do not forget to include the probabilities of up and down moves

6. Binomial tree – Convergence property

Convergence property

- CRR sacrificed the generality and simplicity to prove that the **binominal tree model converges** to the option pricing model developed by **Black and Scholes** under certain parameters (by matching the first two moments in standard trees as $T \rightarrow \infty$)
- The principle behind is basically that a binomial distribution converges to a normal distribution if $Tq \rightarrow \infty$ as $T \rightarrow \infty$



Source: Cox J.C., Ross S.A., Rubinstein M. (1979). Option Pricing: A Simplified Approach. Journal of Financial Economics 7: 229-263

7. Convergence property – exercise

Question 2:

A consumer is considering purchasing an option for the delivery of EEX baseload electricity. The option provides for a 1-year delivery at a rate of 25 €/MWh. The 1-year futures price for the standard EEX baseload electricity contract, which corresponds to 1 MWh for the defined period, is 24 €/MWh. The risk-free interest rate stands at 5% p.a. with continuous compounding, and the volatility of the futures price is 20% p.a.

- How much is the option worth?
- Provide the value of this option using both the Black-Scholes formula and the binomial tree approach. Are the answers derived from the BS formula and the binomial tree identical?
- Can you explain such differences?

Hint: increase the timesteps of the binominal tree. Draw a graph with the option value as the y-axis and the timesteps of the binominal tree as the x-axis.

The BS formula for a European call option is:

$$c = S_0 N(d_1) - K e^{-rT} N(d_2),$$

where

$$d_1 = ln\left(\frac{S_0}{K}\right) + \left(r + \frac{\sigma^2}{2}\right) \times T)/\sigma\sqrt{T} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

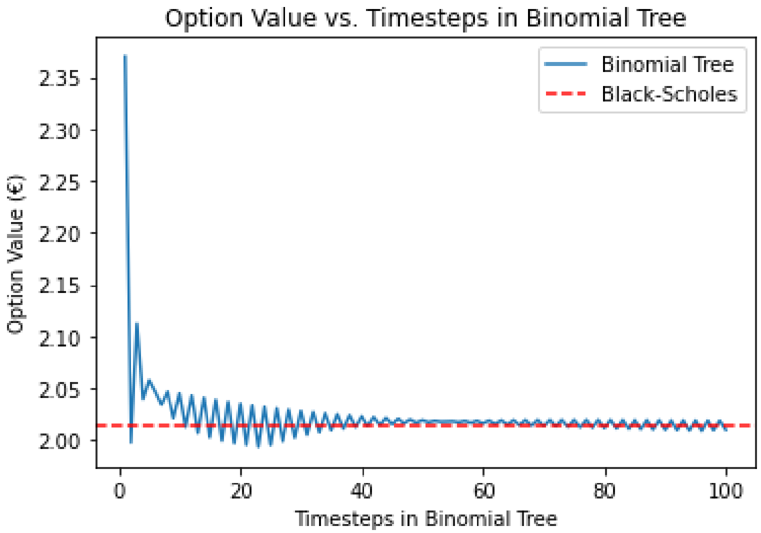
7. Convergence property – exercise

S_0	current futures price, 24 €/MWh
K	strike price, 25 €/MWh
T	time to expiration, 1 year
σ	volatility of the project‘s value, 0.02
r	risk-free interest rate, 0.05
$N(.)$	cumulative distribution function for the standard normal distribution

```python
import numpy as np
import scipy.stats as stats
import matplotlib.pyplot as plt

# Black-Scholes formula
def black_scholes(S, K, r, sigma, T):
    d1 = (np.log(S / K) + (r + 0.5 * sigma ** 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    call_option = (S * stats.norm.cdf(d1) - K * np.exp(-r * T) * stats.norm.cdf(d2))
    return call_option
# Parameters
S = 24
K = 25
r = 0.05
sigma = 0.20
T = 1

# Calculate BS value
bs_value = black_scholes(S, K, r, sigma, T)
print(f"Black-Scholes option value: {bs_value:.2f} €")
```



Please solve the rest of the question yourself!
Please try to code it in Python yourself!

Licensing

Case study "Application of real options analysis for repowering of wind power plants"

Chair of Energy Economics and Management
Institute for Future Energy Consumer Needs and Behavior
Prof. Dr. Reinhard Madlener, Dr. Barbara Glensk, Qinghan Yu M.Sc.
RWTH Aachen University October 2023

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Selected references

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Some important instructions related to the case study

Instruction for NPV calculation

The formula for the NPV is given by:

$$NPV = \sum_{t=1}^{T} \frac{R_t - C_t}{(1+\rho)^t} - Inv_{total}$$

where

R_t – yearly revenue at time t
C_t – yearly cost at time t
ρ – discount rate
Inv_{total} – total investment costs
T – lifetime of the project

Revenue form power generation:

$$R_t = P_{elec} \cdot capacity\ factor \cdot capacity \cdot 8760$$
$$C_t = capacity \cdot O\&M$$

where

P_{elec} – electricity price at time t
$O\&M$ – yearly operation and maintenance cost

Some important instructions related to the case study

Static estimation of electricity price

The formula:

$$\overline{P_{elec}} = \frac{\int_0^T P_{elec,0} \cdot e^{\mu t} dt}{T}$$

where

$P_{elec,0}$ – initial electricity price at time 0

μ – growth rate of the electricity price

Some important instructions related to the case study

Instruction for coding

For the coding, you could consider the following steps:

(1) Initialization: Import necessary libraries and define given parameters

(2) Function for NPV Calculation: define a function for NPV calculation using the formula provided above

(3) Average Electricity Price Estimation: Create a function/write the formula to compute the average electricity price over the range of years using the GBM model

(4) Yearly Revenue and Costs Calculation: Calculate yearly revenue and costs

(5) Compute and Display the NPV: Call the function in step 1 to compute the project's NPV, and finally print your result

Some important instructions related to the case study

Instruction for coding option to defer

You should first complete the two exercises from the exercise unit to familiarize yourself with the mathematics and coding of binominal trees, and then proceed to solve case study.

Here are some general instructions:

(1) Construct an 11-year price tree for electricity price

(2) Calculate the present value of cash flows at each node from year 1 to 11 (we are waiting before year 0 to 1, so there is no income in between) based on the price of electricity and other input parameters

$$PV = (R - c) \cdot \Delta t \cdot e^{t}$$

where:

PV – present value

R – yearly revenue at each note

C – yearly cost at each note

Δt – length of time step

t – time index

Some important instructions related to the case study

Instruction for coding option to defer

(3) Compute the option value at each node using backward induction. Sum the present values from t= 11 until the decision time point t=1, and compare at each nodes of year 1:

$$OV = max(summed_{PV} - Inv_{initial} - Inv_{deferred}, 0)$$

where:

OV	– option value
$Inv_{initial}$	– initial investments
$Inv_{deferred}$	– deferred investments

(4) Create a range of volatilities and calculate the option value using the steps above. Observe how changes in volatility affect the value of the option to defer. You may want to consider writing a function of the option price to simplify the process.